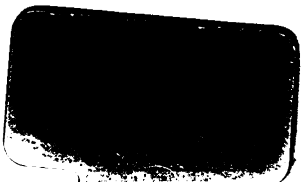

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TEXT-BOOKS OF SCIENCE

ADAPTED FOR THE USE OF

ARTISANS AND STUDENTS IN PUBLIC AND SCIENCE SCHOOLS

MACHINE DESIGN

BY THE SAME AUTHOR.

WROUGHT IRON BRIDGES AND ROOFS. Lectures delivered at the Royal Engineer Establishment, Chatham. With Examples of the Calculation of Stress in Girders and Roof Trusses by Graphic and Algebraic Methods.

EXAMPLES OF MACHINE DETAILS. A Series of Working Drawings of Machine Details, printed in colours. Prepared for the Science and Art Department, South Kensington.

SHORT LOGARITHMIC AND OTHER TABLES. Comprising Logarithms of Numbers; Antilogarithms; Natural and Logarithmic Trigonometrical Functions; Functions of Numbers; Conversion Tables for English and Metric Measures; and Tables of the Weights of Materials. Intended to facilitate Practical Calculations and for solving Arithmetical Problems in class.

O

THE ELEMENTS

OF

MACHINE DESIGN

AN INTRODUCTION TO THE
PRINCIPLES WHICH DETERMINE THE
ARRANGEMENT AND PROPORTIONS OF THE PARTS
OF MACHINES, AND A COLLECTION OF
RULES FOR MACHINE DESIGN

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THE THIRD SCIENCE is that of MACHINE DESIGN. This has been removed by Redtenbacher from its incorrect position as a part of Applied Mechanics, and established on a footing of its own. Its province is to show how the parts of the machine are to be proportioned so as to resist deformation. In order to accomplish this fully, they must be considered both with reference to the external forces acting on the machine, and the corresponding molecular forces within its substance.

The former are assumed as determined by theoretical mechanics (for example, the steam pressure upon a piston, or the water pressure on the vanes of a turbine); these define the requirements of the parts as to strength. The latter, the molecular forces, transmit the force action from part to part (for example, from the piston rod to the connecting rod, or from toothed wheel to toothed wheel), and cause also friction and wear. The science of Machine Design applies the results of research in these two directions to the special problems with which it deals. When it solves these problems in accordance with technological requirements, it forms a really technical science.

REULEAUX, *Theoretische Kinematik.*

PREFACE.

THIS little book was begun some four or five years since, when the want of any English text-book of machine design had become very apparent to the author. Its progress has been delayed by the pressure of other work, and it has been completed in rather restricted intervals of leisure. Nevertheless, no labour has been spared in condensing into the smallest compass the information at the author's disposal, and in endeavouring to render the treatment of the subject simple and clear. If the student's path has in any degree been rendered easy, it is because a good deal of labour has been expended on the roadway.

Long experience has enabled engineers to proportion special machines in a very perfect way, and no great improvement can be expected from a theoretical study of the strength of their different parts. The empirical rules current in the drawing office, are sufficient for the construction of ordinary machines. The present treatise is not merely a collection of such rules. Its primary object is to explain the principles which are available as guides in machine construction. So far as it succeeds in this, it will place the draughtsman in the best position to make use of the facts which come under his notice in the workshops and the drawing office, and will enable him to apply that experience in dealing with new materials, with new forms of construction, and with novel

conditions of force and speed. In addition, this text-book contains a selection of practical rules and empirical proportions, for various parts of machines. These are not intended to override the draughtsman's own judgment and experience. All such rules have a more or less limited application, and the most that can be done is, to indicate how such matters can best be dealt with. The author has endeavoured to avoid excessive minuteness in giving the empirical proportions of machine parts, and he has usually left a certain range of choice open. That which appears least desirable, in a text-book of this kind, is to reduce designing to mere rule of thumb. In good designing, judgment, foresight, knowledge and science must be constantly brought to bear.

Many rules for machine design are rational in form, but are affected by arbitrary coefficients, intended to allow for contingencies which are neglected. In such cases, the arbitrary part of the rule has, in this treatise, generally been distinguished from the rational part. Thus, in dealing with shafting, the diameter is determined by the law of torsional resistance, but the coefficients, in the rules ordinarily given, are made up of two parts, one belonging to the rational formula for torsional resistance, the other, an arbitrary factor of safety, which is intended to make allowance for the undetermined bending action. In this treatise, the two parts of the coefficient are kept separate, so that it may be seen what amount of undetermined straining action is actually allowed for.

To some students this treatise may appear to contain an excessive amount of mathematical work. This is partly due to the fact that much has been condensed in a small space, and that the symbolical expression of the reasoning is the simplest and briefest. But the mathematics employed are, with few exceptions, of a very simple kind, and ought to present no difficulty to anyone fairly acquainted with ordi-

nary algebra and trigonometry. Real difficulty is most likely to occur where the student is imperfectly acquainted with applied mechanics, and this difficulty can only be removed by studying some of the numerous excellent treatises we possess on that subject. Of these none is more accurate or complete than Rankine's 'Millwork.' The use of the rules given has been made easy by copious numerical tables.

To some it may appear that the book is too theoretical. But every engineer acts upon a theory of some kind, in proportioning machines. If a piston rod for a 48-inch cylinder is made of twice the diameter of that for a 24-inch cylinder, a theory of the piston rod's resistance is acted upon, and in fact a theory which is only approximate and safe within certain limits, for the resistance of the rod is not independent of its length, and the forces due to the inertia of the piston which act on the rod are not proportional to the piston's diameter. Hence, theory is essential to any systematic treatment of the subject, and all that ought to be required is that the theory should be accurate and free from useless refinements. It is not legitimate, or safe, to render the solutions of practical problems easier, by ignoring some essential conditions. Even when all the elements of the problem cannot in practice be taken into the reckoning, it is still important that the designer should bear them in mind.

In order to avoid constant repetition, a uniform plan is adopted, as to the units employed, which is only departed from in a few cases for special reasons. Wherever there is no express statement to the contrary, the units adopted are as follows:—

Dimensions are in inches.

Loads or forces are in lbs.

Stresses are in lbs. per sq. in.

Fluid pressure is in lbs. per sq. in.

Velocities and accelerations are in feet per second.

Work is in foot lbs. per second.

Speeds of rotation are in revolutions per minute, or in angular velocity per second.

Statical moments (as bending and twisting moments), are in inch lbs.

A more consistent and scientific system of units could easily be adopted, but it would involve a departure from the modes of reckoning current in the workshop.

It is perhaps too much to expect that all errors have been eliminated, and the author will be obliged to any reader who will communicate to him mistakes that are discovered, or cases in which the rules appear to fail.

Thanks are due to Mr. Heys of Manchester, who has carefully read through the chapter on toothed gearing, and made some suggestions noted in the text; to Messrs. Pearce of Dundee, who supplied data of their rope gearing; and to Messrs. Tullis of Glasgow, who afforded information about leather belting. Some years since the author made himself acquainted with the works of Armengaud, Redtenbacher and Reuleaux, and although he has referred to them but little while actually writing this book, he no doubt owes much to them. Redtenbacher and Reuleaux must be considered to have done more than any other writers, to reduce machine design to a science.

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Errata.

P. 52, eq. 21. It should have been mentioned that this equation is only applicable to bars not fixed in direction at the ends.

P. 121, eq. 13. For l^2 read n^2.

Pp. 153–154, figs. 123, 124. The sectional shading of the steps is inaccurate. The upper step is of gun-metal, the lower shell step of cast iron.

P. 203, line 11, for C read c.

ELEMENTS OF MACHINE DESIGN.

CHAPTER I.

MATERIALS USED IN MACHINE CONSTRUCTION.

CAST IRON.

1. UNDER the term cast iron is included a series of materials varying greatly in appearance, in strength, and in the purposes to which they are applicable. At one end of the series is white cast iron, a silvery and very hard material; at the other end is found grey cast iron, which is much softer, and which owes its dark appearance to particles of graphite. Cast iron contains 3 to 5 per cent. of carbon. In white iron this is entirely combined with the iron; in grey iron 0.6 to 1·5 per cent. is combined, and the remainder, 2·9 to 3·7 per cent., crystallises separately as graphite. The harder white varieties are used only for conversion into wrought iron. The greyer and softer qualities are used also for the production of castings. The greyest iron is said to be less fusible than the whiter varieties, but it is more fluid when molten and it expands at the moment of solidification, so as to form accurate and smooth castings. On the other hand it is deficient in strength. Hence most castings are composed of a mixture of the greyest iron with other and less grey varieties. The larger the casting and the stronger it requires to be, the less is the proportion of the greyest or No. 1 iron which is used.

Cast-iron machine parts are formed by pouring the melted cast iron into moulds. A *pattern* is first made of the exact shape of the casting required. A *mould* is then formed from this in foundry sand or loam. Then the molten iron is poured into the mould. After solidification the sand is cleared away.

The patterns are commonly made of yellow pine, or when small of mahogany. Metal patterns are used when a great number of similar castings are required. As the cast iron contracts about $\frac{1}{8}$th of an inch per foot in each direction, the pattern is made larger than the required casting in that proportion. The amount of contraction varies with the quality of the iron and the size of the casting, and this sometimes gives rise to much difficulty and trouble. Passages and apertures in castings which are so small that the sand would not resist the scouring action of the flowing metal, are formed of loam, in wooden moulds termed core-boxes, and are baked before being used. Simple cylindrical parts can be moulded in loam, without the use of a core-box. Thus, the core of a pipe is formed of loam, plastered on to a hollow metal core-bar. By rotating the core-bar and strickling off the superfluous loam with a sharp-edged board, the exact cylindrical form is obtained. The moulds for large cylinders are formed of loam, plastered over roughly built brick cylinders, and strickled to the required form.

Although simple forms are more easily moulded and cast than more complex forms, the skill of the moulder enables him, when necessary, to mould castings of very complicated and difficult shapes. Hence, the cast parts of machines are more complicated in form than those which are forged. Castings, however, do not retain an altogether sharp and accurate impression of the mould. The corners of castings are usually somewhat blunt and ragged, deep hollows partially filled up, and straight lines slightly twisted. Hence, for appearance sake, castings should have broad

and rounded surfaces with well-rounded edges and filleted hollows. Architectural mouldings are not suitable for castings.

2. Cast iron is stronger than wrought iron in compression, and much weaker in tension. Hence, it is more suitable for compressed than for stretched machine parts. Within a limited range of stress, it is tougher than wrought iron, or undergoes a greater deformation. But its range of deformation is not great. Hence, it is not so safe as wrought iron when subjected to suddenly applied or impulsive forces.

When castings are contracted for, it is sometimes stipulated that test bars shall be cast, at the same time and of the same metal as the castings. These bars are often $3\frac{1}{2}$ ft. × 2 ins. × 1 inch. When laid on supports 3ft. apart, they should carry from 24 to 32 cwt. at the centre before breaking.

3. The special difficulty and danger in the use of cast iron is its liability to be put into a state of internal strain, in consequence of its contraction when cooling. That contraction varies with the size and thickness of the casting, and with the quality of the iron. Thus it has been found that thin locomotive cylinders contract only $\frac{1}{16}$th of an inch per foot. Heavy pipe castings and girders contract $\frac{1}{8}$th inch in 12 inches, or $\frac{1}{8}$th inch in 15 inches. Small narrow wheels contract as little as $\frac{1}{25}$th inch per foot, while large and heavy wheels contract $\frac{1}{10}$th inch per foot or more. If some parts of a casting contract more than others, the thick parts, for instance, more than the thin parts, the casting is twisted and strained. If some parts of a casting solidify while others are still fluid, the former attain nearly their final dimensions, while the contraction of the latter has still to be effected. That contraction therefore strains the parts already set, and their resistance to deformation gives rise to strains in the parts which are contracting. Thus a condition of initial strain is induced, sometimes great enough to fracture the casting

without the application of any external force, and in all cases reducing the effective strength of the casting. The danger of initial strain is less, when the form of the casting is simple and the thickness uniform and not excessive. It appears that the initial strain is to some extent gradually removed by molecular yielding, the alteration going on for months after the casting is made.

Suppose a casting of the form shown at *A*, Fig. 1. The thin side would solidify, while the greater body of heat in the thick part still retained it in a fluid condition. When the thick part contracted, it would necessarily curve the bar and induce compression in the thin part, and a corresponding tension in the thick part. In a panel of the form shown at *B*, with a thin but rigid flange, the contraction of the

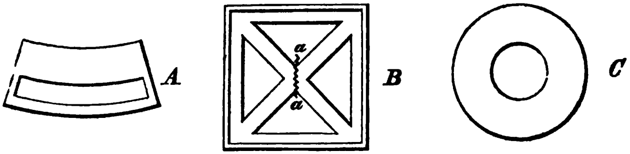


Fig. 1.

diagonals takes place more slowly than that of the rim surrounding them, and is very liable to cause fracture at *a a*. In a thick cylinder such as a press cylinder, Fig. 1, *C*, the outer layers solidify and begin contracting first. The contraction of the inner layers, after that of the outer layers is completed, induces compression in the outer layers, and the rigidity of the outer layers, causing a resistance to the contraction of the inner layers, puts them into tension. Such a cylinder will not bear so great a bursting pressure as if there were no initial strain. In fact, to obtain the greatest resistance to an internal bursting force, the reverse distribution of initial stress is necessary. This has sometimes been obtained by a water core, or hollow core having a water circulation through it. The interior is then cooled most rapidly. Compression of the

inner layers and tension of the outer layers is the result of this mode of cooling. Castings in the form of wheels and pullies often give much trouble. In pullies, which have a thin but rigid rim, the rim contracts first, and the subsequent contraction of the arm breaks it by tension along the line *a a a*, Fig. 2. In some cases, however, the rim breaks across near the arm, at *a b*. This appears to be due to the arms setting first. They then form a rigid abutment resisting the contraction of the rim, and bending stress is produced in the rim, causing fracture to begin outside and extend inwards.

It is because of these incalculable initial strains, that cast iron is an unreliable material, where great strength is

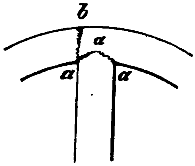


Fig. 2.

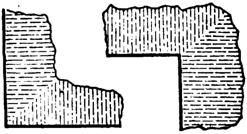


Fig. 3.

required, in structures of irregular form. The danger may be partially removed by the skill of the founder, who, by various devices, ensures as far as he can an uniform rate of cooling. But generally cast-iron structures must have excessive dimensions in order to ensure safety.

At sharp corners, a plane of weakness is formed, in consequence of the way in which the crystals arrange themselves, normally to the surfaces through which heat is transmitted. This is one reason why all corners should be well rounded. Fig. 3 shows roughly the crystalline structure.

4. *Chilling.*—When castings are rapidly cooled during solidification, the separation of the graphite from the iron is prevented. The casting has then a silvery fracture and is extremely hard. To effect this chilling, as it is termed, the mould is made of a thick block of cast iron, the surface in contact with the molten iron being protected by a wash

of loam. The iron mould abstracts the heat much more rapidly than a sand mould.

5. *Malleable Cast Iron.*—This is made by surrounding a casting with oxide of iron or powdered red hæmatite, and keeping it at a high temperature, for a time varying with the size of the casting, from two or three, to thirty or forty hours, or even longer; part of the carbon is eliminated, and the casting is converted, partially or wholly, into a tough material resembling wrought iron. Malleable castings stand blows much better than ordinary castings, but they should only be hammered when cold. The decorative parts of iron-work and pinions of wheels are often thus treated.

Wrought Iron.

6. Wrought iron is a silvery metal, fusing with difficulty, moderately hard, very strong, and, what is of great importance, very tough. At a temperature of 1,500° or 1,600° Fahr. it softens, and can then be welded. Wrought iron is used for parts of machines requiring strength and toughness, and of simple form. Such parts are first shaped roughly by hammering or rolling, and if accuracy of form is required, they are then reduced to exact dimensions by cutting tools. Large wrought-iron structures are built up by riveting. Wrought iron easily oxidises, and must be polished bright and oiled, or painted.

The different qualities of wrought iron are commercially distinguished as merchant bar, best iron, double best, and treble best. These terms refer to the amount of working the iron has received in manufacture, and are only rough indications of quality. To ensure a given quality the iron used should be tested. Its strength is usually determined by subjecting it to tensile stress. Its ductility and toughness may be deduced from its elongation and contraction of area before rupture. Workmen test its toughness by bending it over a sharp corner with the hammer.

7. The forms in which wrought iron is most easily procured are the following :—

Bar iron. Round bars $\frac{1}{8}$ in. to 7 ins. diameter. Square iron up to 5 ins. or 6 ins. each side. Flat iron from $\frac{1}{4}$ in. thick and $\frac{1}{2}$ in. to 6 ins. wide, to $1\frac{1}{2}$ in. thick and 3 to 10 ins. wide. Lengths usually 20 to 30 ft.

Plates $\frac{3}{16}$ to $1''$ thick, and usually not exceeding 24 sq. ft. area. Angle iron, Tee iron, and double Tee iron, in bars, usually not exceeding 8 ins. in the sum of the widths.

Various other forms are made, as half round iron, channel iron, grate bar iron.

The quality of wrought iron varies greatly, and for some purposes strength is most important, while for others capability of being worked under the hammer without cracking or losing strength is more important. The following is a rough classification of the qualities usually met with :—

(*a*) Iron easily worked hot, and hard and strong when cold ; used for rails.

(*b*) Common iron, used for ships, bridges, and sometimes for shafting.

(*c*) Single, double and treble best iron, from Staffordshire and other parts, where similar qualities are made. The single or double best is used for boilers. Double and treble best are used for forging.

(*d*) Yorkshire iron, from Lowmoor, Bowling, or other forges where only fine qualities are made. The best Yorkshire iron is very reliable, and uniform in quality. It is used for tyres, for difficult forgings, for furnace plates exposed to great heat, for boiler plates which require flanging, &c.

(*e*) Charcoal iron. Very ductile, and of the best quality.

The inferior qualities of bar iron will often bear a stress of 26 tons per sq. in. without breaking, but they elongate only about 8 per cent. in a length of 8 ins. The best qualities often bear only 24 or 25 tons per sq. in. before breaking, but they elongate 20 to 25 per cent. in an 8-inch

length. Plates are usually weaker, and elongate less than bars. The presence of slag in the inferior kinds of iron causes them to crack in forging.

8. *Case Hardening.*—The surface of wrought iron may be hardened by partially converting it into steel. This can be effected to a slight extent by making the surface bright, heating it to a red heat, then rubbing it with prussiate of potash, and quenching in water. It is far more completely effected by heating the iron in a close box, filled with bone dust and cuttings of horn and leather.

9. *Cold Rolled Iron.*—Wrought iron, rolled cold under great pressure, gets a smooth polished surface, and is found to have a greatly increased tenacity. Its ductility and toughness are, however, much diminished. Hammering iron when cold produces a similar effect. Annealing, or heating the iron to red heat and allowing it to cool slowly, restores it to its original condition.

Steel.

10. Under the term steel are included materials differing greatly in quality. The softer kinds, which contain least carbon, approach wrought iron in character, having equal or greater toughness, greater strength, and the same capacity of welding. The harder qualities have less toughness, but much greater strength, and are less easily welded. All materials to which the term steel is strictly applicable may be defined as capable of being cast into a malleable ingot, and they are rendered harder and less tough, and have a higher limit of elasticity, when heated to red heat and suddenly cooled. When hardened, they can again be softened by heating to a temperature determined by the colour of the oxide which forms on the surface, the process being termed tempering or 'letting down.' Steel of soft quality is used for the same purposes as wrought iron; but it is more liable to injury than the latter, in the various processes of forging,

punching, &c., to which it is subjected in the workshop. It is also more variable in quality, and in large masses is liable to concealed defects. In small masses, steel is generally preferable to wrought iron, and the difficulties attending its use will probably be overcome.

Steel is more fusible than wrought iron, and can be cast roughly to the form required, but not so easily and accurately as cast iron. When cast it is porous, and contains cavities, and it is only after being forged that its soundness can be relied on. Sir J. Whitworth has introduced a method of casting under pressure, which may probably remove the difficulties of producing sound steel castings. In welding steel, it is important that the two pieces to be united should contain the same amount of carbon. If they do not, their welding temperatures are different. The smithing of steel is more difficult than that of wrought iron, and it is more liable to injury from over-heating, and for this reason iron rivets are preferable to steel rivets. If pure iron were combined with carbon, the physical properties of the steel produced would depend on the proportion of carbon combined with the iron, the mildest steels containing about 0·15 to 0·4 per cent. of carbon, and the hardest 1·4 to 1·6. Actual steel, however, contains other ingredients, such as silicon and phosphorus, which influence its physical properties, having a similar effect, in hardening the steel, to a certain amount of carbon.

COPPER.

11. Copper is a red, soft, malleable material, which can be forged cold, but cannot be welded. When cast it takes the mould badly. It is not very strong, and is very expensive. Hence it is chiefly used for pipes and other parts, which require to be bent cold, and for the fireboxes of locomotives.

BRONZE OR GUN-METAL.

12. Bronze or gun-metal is harder and less malleable

than copper. It is fusible, and makes excellent castings. It varies in quality according to the proportion of tin. Thus:—

Soft gun-metal contains .	8 tin	to 92	copper.
Hard gun-metal . . .	18	„ 82	„
Bell metal	24	„ 76	„

Some zinc is often added to facilitate casting. Ordinary bronze is not uniform in texture. Whitish spots of alloy, rich in tin, are distributed through the mass. It has been found that when it is rapidly cooled after casting, the composition is more uniform, the density greater, and the strength and toughness are increased. This rapid cooling is best effected by using thick cast-iron moulds or chills, the process being analogous to the chilling of cast iron. The best alloy for guns contains 8 to 10 parts of tin and 100 of copper. Such an alloy, when cast in sand moulds, breaks with about 11 tons per sq. in. of tension, and its limit of elasticity is reached at 5·6 tons. Cast in chills, its strength is 17·6 tons, and its limit of elasticity is raised to 6·7 tons per sq. in.

The friction between bronze and wrought iron is moderate and regular, and the bronze being softer wears most rapidly. Hence it is very suitable for the steps upon which rotating pieces are supported. Gun-metal for bearings often contains 82 per cent. of copper and 18 per cent. of tin. The softest bronze is used for cocks and small fittings. Bronze is tougher than cast iron, and is sometimes used for gearing subjected to shocks.

Brass.

13. Brass contains about 85 parts of copper to 35 of zinc. A little lead is often added. Common brass for cheap brass-work contains equal parts of copper and zinc. Muntz metal, which can be rolled hot, contains 66 parts of copper, 33 of zinc, and 1 of lead. It is used for sheathing plates and locomotive tubes. Brass is cheaper, and less strong and tough than gun-metal. It is used for bearings and for cocks and fittings.

White Brass.

14. Various alloys have been used for bearings containing large proportions of tin or lead. The alloys in which tin is the chief ingredient contain 40 to 90 parts of tin, 5 to 17 parts of antimony, and 1·5 to 17 parts of copper. Alloys in which lead is the chief ingredient contain 66 to 88 parts of lead with 4 to 20 parts of antimony and 12 to 20 parts of tin. The object of trying these very various alloys is to obtain a metal for bearings which is cheaper and softer than ordinary gun-metal, and which works with less friction. The friction depends on the way in which the step wears. If it is soft and of uniform texture, and wears with a smooth and polished surface, the friction may be expected to be small.

The following table gives particulars of some alloys which have been used for railway and other bearings.

Parts by Weight.

Lead	70	—	42·5	37·5	—	—	—	84
Zinc	—	82	42·5	—	—	—	—	—
Tin	—	—	—	37·5	66·7	90	85	—
Antimony . . .	20	11	15	25·0	11·1	7	10	16
Copper	10	7	—	—	22·2	3	5	—

Some of these alloys are fusible at a low temperature, and are cast in position round a smooth mandril. Then they do not require turning. Most of them are too soft to be used alone for large bearings; in such cases, a thin sheet of the alloy is cast in recesses in an ordinary gun-metal step. One objection to very soft alloys is that they crush, and clog the oil channels.

Phosphor Bronze.

15. Under this name an alloy has been introduced composed of copper and tin with a small proportion of phos-

phorus, which is likely to be of great service in machine construction. It appears to owe its properties in part to the great care exercised in its manufacture, and the accurate proportions of the constituents. Its qualities can be varied at will, so that it may be either very strong and hard, or with less strength, soft and very tough. Unlike ordinary bronze, it can be remelted without deterioration of quality.

As to its strength and ductility, various tests show a tenacity of from 50,000 lbs. per square inch in the softer qualities, to 75,000 in the hardest. The elastic limit of the former is about 11,000 lbs., and of the latter 56,000. The former elongate 30 per cent. or more before fracture, and the latter 3 to 4 per cent. The contraction of area at fracture ranges from 4 to 30 per cent. Unannealed wire (16 B. W. G.) broke with from 102 tons per square inch to 151 tons per square inch, and the same wire after annealing carried from 48 to 74 tons per square inch.

In some Belgian experiments, railway axle bearings of phosphor bronze were found to wear much longer than gun-metal bearings, and this bronze has also been used for the large crank bearings of marine engines, where ordinary gun-metal has failed. Its great strength and toughness render it especially suitable for gearing subjected to shocks. It has been used in place of steel for tools, in gunpowder factories, and it can be drawn into wire and used for rigging and perhaps for wire-rope belting.

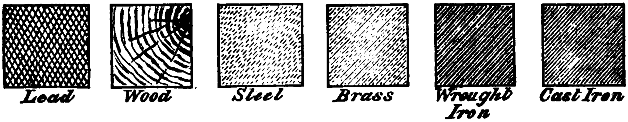


Fig. 4.

Fig. 4 shows the sectional shading adopted in this Treatise to indicate the materials most commonly used.

CHAPTER II.

ON THE STRAINING ACTIONS TO WHICH MACHINES ARE SUBJECTED.

16. *Load.*—By the load on any member of a machine, is meant the aggregate of all the external forces acting on it; including the weight of the member itself or of other parts supported by it, the forces of friction or inertia called into play by its motion, and the pressures transmitted through it.

We may distinguish (1) the *useful load*, or the forces arising out of the useful work transmitted; (2) the *prejudicial resistances* due to friction of parts of the machine, or to work expended otherwise than at the working point. (3) Forces due to the weight of parts of the machine. (4) Forces of inertia due to changes in velocity of motion. (5) Centrifugal forces due to changes of direction of motion. (6) Forces due to alterations of temperature.

In each part of the machine the straining action varies with the fluctuations of the useful load, and with the variations of velocity and position of the different parts of the machine. Each member of the machine must be capable of sustaining the maximum straining action for that part of the machine. If, in consequence of variation of position or velocity, the kind of straining action is different at different times, it must be capable of sustaining the maximum straining action of each kind. Lastly, as will be more fully explained presently, the amount of variation of the straining

action affects the endurance of the material, and therefore requires also to be considered.

17. *Factor of Safety.*—In most cases, some of the kinds of straining action above enumerated produce a comparatively small effect, and are for simplicity omitted, in estimating the forces to which a part of a machine is subjected. In many cases, some of the forces producing straining action cannot be determined. The aggregate amount of straining action due to the forces which are taken into consideration, is therefore multiplied by a *factor of safety*, before estimating the strength of the piece; and in this way a rough allowance is made for the straining action due to those forces which are neglected, and a margin of resistance is provided for contingencies, not foreseen in designing the structure. The amount of that factor of safety is fixed by practical experience in similar cases.

18. *Steady or Dead Load, and variable or Live Load.*—A steady load is one which is invariable during the life of the structure, and which produces a permanent and unvarying amount of straining action. The weight of a fixed part of a machine is such a dead load. A variable or live load is a load which is alternately imposed and removed, and which produces a constantly varying amount of straining action. A steady load can generally be very exactly estimated, and when the load is entirely of this kind, a comparatively low factor of safety affords a sufficient guarantee of security. A live load is often less easily estimated, and a load of this kind produces much more injurious effects on a structure than a dead load of the same amount. Hence, for a double reason, a higher factor of safety must be used for a live than for a dead load.

A suddenly applied load is a load imposed on an unstrained structure, without velocity, but at one instant. Practical cases only approximate to these conditions. Such a load accumulates, in deflecting or elongating the structure, a certain amount of energy of motion which is ultimately

expended in increasing the deformation beyond the amount due to a steady load. If the stress does not exceed the limit of elasticity of the material, a suddenly applied load produces nearly twice the stress, which would be produced by the same load resting on the structure.

If the load impinges on the structure with an amount of energy of motion previously accumulated, the stress produced will exceed that due to the same load applied steadily, to an extent which depends on the original energy of motion. Such a load may be termed an impulsive load.

19. *Strain.*—Every load which acts on a structure produces a change of form which is termed the strain due to the load. The strain may be either a vanishing or elastic deformation, that is, one which disappears when the load is removed; or a permanent deformation or set, which remains after the load is removed. In general, machine parts must be so designed that, under the maximum straining action, there is no sensible permanent deformation.

Stress or Strength.—The molecular forces, or forces acting within the material of a structure, which are called into play by external forces, and which resist its deformation, are termed stresses. In most materials the stresses are sensibly proportional to the strains, so long as no considerable permanent set is produced. If, however, the straining action is so great as to produce a permanent change of form, then the stresses increase less rapidly than the strains. For any given material subjected to gradually increasing straining action, there is found to be a certain limit, more or less clearly marked, within which the stresses are sensibly proportional to the strains, and beyond which that proportionality sensibly ceases. The intensity of stress corresponding to that limit, or stress per unit of area when that limit is reached, is called the elastic strength of the material. A bar subjected to straining action producing that intensity of stress, is said to be strained to the elastic limit. The condition that a machine or structure should

suffer no permanent deformation, enables us to fix on the intensity of stress corresponding to the elastic limit, as the maximum stress to which any machine part should in any case be subjected.

If a machine part is subjected to a steady load only, whose amount can be completely and exactly determined, it will be sufficiently strong, if the intensity of stress due to that load is not greater than the elastic strength of the material. If the straining action can only be partially determined, or if the straining action is variable, then the stress corresponding to that straining action must be less than the elastic strength of the material.

Safe Working Intensity of Stress.—If the stress corresponding to the elastic limit is divided by the factor of safety, we get the permissible working intensity of stress, due to those straining actions, which are taken into account in estimating the strength of the structure. Although this is usually termed the greatest safe intensity of stress (or for brevity greatest safe stress), it is, in most cases, less than the real intensity of the stresses induced by the actual straining actions. The resistance corresponding to the greatest safe intensity of stress may be termed the working strength of the piece.

20. *Ultimate Strength.*—If the straining action on a bar is gradually increased till the bar breaks, the load which produces fracture is called the ultimate or breaking strength of the bar. That ultimate strength is for different materials more or less roughly proportional to the elastic strength. Now experiments on ultimate strength are much more easily made than experiments on the elastic strength, and the results are more definite. For most materials there are more numerous experiments on ultimate than on elastic strength; and for certain forms of machine parts only experiments on ultimate strength are available. Thus we know nothing, either theoretically or experimentally, of the elastic strength of cylindrical boiler flues, but only that at a certain limit of

straining action they collapse altogether. In such a case, we may ensure the safety of a structure, by taking care to multiply the actual straining action by a factor sufficiently large to allow, not only for unforeseen contingencies and the neglected causes of straining action, but also for the difference between the elastic and ultimate strength. The actual straining action multiplied by this factor, which is still termed a factor of safety, is then equated to the ultimate strength of the structure. The value of the factor of safety must, as in other cases, be determined by practical experience. There is, as yet, no theory of the resistance of materials in circumstances in which the stresses are not proportional to the strains; but, in all cases of ultimate strength, the limit has been passed within which that proportionality continues. Hence, in determining the ultimate strength of structures, we are dependent on empirical formulæ, derived from experiments necessarily extending over a limited range of cases. In applying such formulæ to cases beyond the range of the experiments, there is always a doubt whether or not they are strictly true. For this reason, it is less satisfactory to determine the strength of a structure in this way, than to examine the relation of the load to the elastic strength.

21. *On the Peculiar Action of Live Loads.*—The researches of Wöhler, since repeated by Spangenberg, show that the safety of a structure, subjected to a varying amount of straining action, depends on the *range of variation* of stress to which the structure is subjected, and on the number of repetitions of the change of load. It has been, hitherto, assumed that it depends only on the maximum intensity of the stress; but this must now be considered to be erroneous. Every machine, subjected to a constant variation of load, must be designed to resist a practically infinite number of changes of load. In order that it may do so, the greatest intensity of stress must be less than for a steady load, and less in some

proportion which depends on the amount of variation the stress undergoes in its successive changes.

A steady load has already been defined as one which remains invariable during the life of the structure. Let the intensity of stress required to fracture a given material, under a steady load, be denoted by K, so that K is what is commonly termed the breaking strength of the material. In designing a machine part to sustain a steady load, the greatest safe stress is commonly taken at about $\frac{1}{2}$ to $\frac{1}{3}$ K. With a live or variable load, it has been usual to take a higher factor of safety, and to restrict the working stress to $\frac{1}{4}$ or $\frac{1}{6}$ K, or to some other limit, ascertained by practical experience in special cases. Wöhler's researches show that this is not a scientific way of dealing with the question. Suppose that under the action of the live load the stress varies from $\sigma_{\text{max.}}$ to $\sigma_{\text{min.}}$, and that the range of variation $= \Delta = \sigma_{\text{max.}} - \sigma_{\text{min.}}$ In using this expression, if tensile stresses are reckoned positive, compressive stresses must be reckoned negative, so that, if the two stresses are of different sign, the range of stress is equal to their sum $[\sigma_{\text{max.}} - (-\sigma_{\text{min.}}) = \sigma_{\text{max.}} + \sigma_{\text{min.}}]$. Let the number of changes of load be indefinitely great. Then Wöhler's researches show that fracture will occur, for some value of $\sigma_{\text{max.}}$ less than K, and so much smaller, the greater the range of stress Δ. Hence, in designing a structure for such a varying load, the ultimate strength is to be taken at some value $k < \text{K}$, which is to be determined with reference to Δ.

For example, Wöhler found that a bar was equally safe to resist varying bending, and tensile straining actions, repeated for an indefinite time, when the maximum and minimum stresses had the following values :—

For Wrought Iron.

	Pounds per sq. in.
In tension only	+ 18713 to + 31
In tension and compression alternately . . .	+ 8317 to − 8317

For Cast Steel.

	Pounds per sq. in.
In tension only	+ 34307 to + 11436
In tension and compression alternately . .	+ 12475 to − 12475

These results are sufficient to show that, as the range of stress increases, the maximum stress should be reduced. Unfortunately, Wöhler's experiments, although extensive, do not furnish decisive rules for practical guidance. They afford an explanation of the apparently high factors of safety, which, in certain cases, experience has shown to be necessary, but they are not complete enough to indicate precisely the factor of safety to be chosen in different cases. Nor indeed could rules be obtained, without the most careful comparison of the results of researches, of the kind begun by Wöhler, with the actual stresses found to be safe in practice, in a great variety of cases.[1]

Let, as before, K be the breaking strength per square inch, for a gradually applied load, for any given material; k, the breaking strength, for a variable load, repeated an indefinitely great number of times, and producing alternately the stresses $\sigma_{\text{max.}}$ and $\sigma_{\text{min.}}$ Let $\Delta = \sigma_{\text{max.}} - \sigma_{\text{min.}}$

Then Wöhler's experiments appear to suggest a rule of the following kind, as giving the relation between k and K

$$k = \frac{\Delta}{2} \pm \sqrt{(\mathrm{K}^2 - n\,\Delta\,\mathrm{K})},$$

where the + sign is to be taken if Δ is +, and the − sign if Δ is −. This, however, must be regarded at present as an empirical rule only, based on experimental results. The value of n appears to be about 1·5 for iron, and not very different for steel.

The cases most useful to consider are:—(1) When the load changes from a maximum intensity to zero, the stress

[1] Wöhler's experiments agree with and confirm the earlier experiment of Sir W. Fairbairn, communicated to the Royal Society, on the effect of continuous changes of load on a riveted girder.

remaining of the same sign; (2) When the load changes from one direction to the opposite direction, so as to produce equal stresses of opposite sign. In the former case $\Delta = \sigma_{\text{max.}} = k$; in the latter case $\Delta = 2\,\sigma_{\text{max.}} = 2\,k$. By solving the equation above we get:—

For Case I.; $k = \text{K}\,(\sqrt{13} - 3) = \cdot 6056\,\text{K}$.

For Case II.; $k = \frac{1}{3}\,\text{K}$.

Thus, for instance, wrought iron, with an ultimate strength of 54,000 lbs. per sq. in., would safely bear, under a steady load, from 27,000 to 18,000 lbs. per sq. in. With a load, such as that in Case I., its ultimate strength would be k=32,700 lbs. per sq. in., and the greatest safe load, with the same factors of safety, would be 16,350 to 10,900 lbs., which agrees fairly well with experience of structures subjected to tension only. For such a load as that in Case II. the ultimate strength would be k=18,000 lbs., and the greatest safe stress 9,000 to 6,000 lbs. per sq. in., which is not very different from the stress allowed in axles and similar parts subjected to constant alteration of the direction of the straining action.

22. *Straining Action due to Power transmitted.*—When HP horses' power are transmitted through a link or connecting rod moving with velocity v, in ft. per second, the straining force, parallel to the axis of the rod, due to the work transmitted, is

$$\text{P} = \frac{550\ \text{HP}}{v}\ \text{lbs.}$$

There will be in this case other straining actions, due to the reactions of the supports of the link, if the link is not moving parallel to its axis.

When HP horses' power are transmitted through a rotating piece, making N revolutions per second, the moment of the straining force, about the centre of the piece, is given by the equation

$$\text{M} = \frac{550\ \text{HP}}{2\,\pi\,\text{N}} = 1050{\cdot}4\ \frac{\text{HP}}{\text{N}}\ \text{inch lbs.}$$

Straining Effects due to Variations of Velocity.—When a heavy body is accelerated or retarded, straining actions are produced, due to its inertia. If a mass, of weight w, suffers an acceleration dv, in the time dt, the reaction or force opposed to acceleration is

$$-\frac{w}{g} \cdot \frac{dv}{dt}.$$

Straining Effects due to Change of Direction of Motion.—When a mass is forced to move in a curved path, it exerts, in consequence of its inertia, a force equal and opposite to the constraint which deflects it. If a mass of weight w moves in a circular path of radius r, with the angular velocity α, its centrifugal force, which is equal and opposite to the force deviating it, is

$$\frac{w}{g} a^2 r.$$

23. *Resilience. Resistance to Impulsive Loads.*—The quantity of work expended in deforming a bar (provided the stress does not exceed the elastic limit) is equal to the product of the deformation, and the mean load producing it. Thus, if a bar is elongated or deflected a feet, by a force gradually increased from nothing to P, the work done in deformation is $a \times \frac{P}{2}$ in ft. lbs. A heavy body of weight W moving with velocity v has $\frac{W}{g} \cdot \frac{v^2}{2}$ ft. lbs. of work stored in it. Hence the relation between the impulsive load and the resistance of the bar, when the direction of the impulse coincides with the direction of the deformation, is

$$\frac{W}{2g} v^2 = \tfrac{1}{2} P\, a.$$

If a bar is twisted, the work done is equal to half the twisting moment, multiplied by the angle of torsion.

The work done in deforming a bar up to the elastic limit is termed the resilience of the bar.

CHAPTER III.

RESISTANCE OF STRUCTURES TO DIFFERENT KINDS OF STRAINING ACTION.

PHYSICAL CONSTANTS FOR ORDINARY MATERIALS.

24. THE Table given on p. 24 shows the elastic strength and ultimate strength of different materials, as determined by experiment, when the stress is simple tension, compression, or shearing stress. The values given are average values selected from the most trustworthy experiments.[1] In different specimens of the same material, and even in different pieces of the same bar or plate, there are often considerable differences of elasticity and strength, and the judgment of the engineer must be relied on, in deciding how far average values of this kind are applicable in any given case. The first two columns of the Table relate to direct stresses produced by straining actions normal to the sections considered. The third relates to tangential stress produced by forces parallel to the section. The elastic strength is that stress per unit of area at which the strains cease to be sensibly proportional to the stresses, and cannot in practice be determined with great exactness. The ultimate strength is the intensity of stress at the moment preceding rupture, and this also depends in some degree on the manner of carrying out the experiment. The more rapid the loading of the bar, and the less vibration induced,

[1] The Tables of Grashof, Rankine, Weisbach, Kirkaldy, and Reuleaux have been consulted, in selecting the values in this Table.

the greater is the load carried before rupture ensues. Nevertheless, if the experiment is carried out with proper care, the ultimate strength is a definite measure of the properties of the material. The elastic and ultimate strength are expressed in lbs. per sq. in.

A modulus of elasticity is the ratio of the intensities of stress and strain of some given kind, when the elastic limit is not passed. Thus, the modulus of direct elasticity of a material is the ratio of the stress p, per unit of section of a bar, to the elongation or compression l, per unit of length, produced by the stress. That is the modulus of direct elasticity $= \mathrm{E} = \frac{p}{l}$ where p is expressed in lbs. per sq. in., and l in inches per inch of length. The bar is supposed to be free laterally. The modulus of transverse elasticity is the ratio of the shearing stress q per unit of area to the distortion n; the distortion being measured by the tangent of the difference of the angles of an originally square particle before and after the stress is applied. Hence the modulus of transverse elasticity $= \mathrm{G} = \frac{q}{n}$. The ratio $\frac{\mathrm{G}}{\mathrm{E}}$ for ordinary materials of construction is about $\frac{1}{3}$ or $\frac{2}{5}$.

25. *Working Stress.*—It has been pointed out that the working stress of a material must be less than the elastic strength, to allow for straining actions which cannot be taken into account, for imperfections of workmanship and for other sources of danger. The Table on p. 25 gives values of the ordinary working stress allowed in designing machinery, in which the load is of the nature of a live or varying load. Parallel with these have been placed theoretical values of the working stress in the following cases:—(1) Structures subjected to tension alone; (2) Structures subjected to compression alone; (3) Structures subjected to both tension and compression of equal intensity. (See Art. 21.)

TABLE I.—*Ultimate and Elastic Strength of different Materials. Moduli of Elasticity in lbs. per sq. in.*

Material.	Breaking Strength.			Elastic Strength.			E.	G.
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.		
	30,500	130,000	...	...	...	...	23,000,000	...
Cast iron	17,500	95,000	28,500	10,500	21,000	7,900	17,000,000	6,300,000
	10,800	50,000	...	...	...	...	14,000,000	...
	67,000	...	...	...	...	...	...	...
Wrought iron bars	57,600	50,000	50,000	24,000	24,000	20,000	29,000,000	10,500,000
	33,500	...	...	...	...	...	...	...
Wrought iron plates, with fibre	50,700	...	...	...	...	...	25,000,000	...
,, ,, across fibre	46,100	...	...	...	...	...	27,000,000	...
,, ,, mean	48,400	...	...	20,000	20,000	15,000	26,000,000	9,500,000
	100,000	...	...	...	...	...	...	...
Soft steel, unhardened	80,000	...	...	35,000	...	26,500	30,000,000	11,000,000
	60,000	...	...	...	...	...	...	...
Soft steel, hardened	120,000	...	...	70,500	...	53,000	30,000,000	11,000,000
	150,000	...	...	...	...	...	...	...
Cast steel, untempered	120,000	...	...	80,000	...	64,000	30,000,000	11,000,000
	84,000	...	...	...	...	...	...	...
Cast steel, tempered	...	...	...	190,000	...	145,000	36,000,000	13,000,000
Copper	33,000	58,000	...	4,300	3,900	2,900	15,000,000	5,600,000
Brass, yellow	17,500	10,500	...	6,950	...	5,200	9,170,000	3,440,000
	52,000	...	...	...	...	...	...	...
Gun-metal	36,000	...	...	6,200	...	4,150	9,900,000	3,700,000
	23,000	...	...	...	...	...	...	...
Muntz metal	49,000	...	...	...	...	...	...	...
Phosphor bronze	58,000	...	...	19,700	...	14,500	14,000,000	5,250,000
Cast zinc	7,500	...	...	3,200	...	...	...	...
Lead	1,900	7,300	...	1,500	...	...	720,000	270,000
Tin	4,700	...	...	...	...	...	...	...
Wood, pine	12,000	6,000	650	...	...	...	1,400,000	90,000
,, oak	15,000	10,000	2,300	...	...	...	1,500,000	82,000
Leather	4,200	...	...	...	...	...	25,000	...

TABLE II.—*Ordinary Working Stress.*

Material.	Safe limit of stress in lbs. per sq. in.			Theoretical limit of stress.			Weight of a cub. ft. in lbs.
	Tension f_t	Compression f_c	Shearing f_s	Tension.	Compression.	Tension and Compression.	
Cast iron	3,600	10,400	2,700	5,250	28,500	3,000 ?	450
Wrought-iron bars	10,400	10,400	7,800	17,280	15,000	9,000	480
,, ,, plates	10,000	10,000	7,800	14,520	...	8,000	487
Soft steel, untempered	17,700	17,700	13,000	24,000	...	13,000	480
Cast steel, untempered	52,000	52,000	38,500	36,000	...	20,000	496
Copper	3,600	3,120	2,300	9,900	17,400	5,500 ?	550
Brass	3,600	...	2,700	5,250	3,150	3,000	518
Gun-metal	3,120	...	2,400	10,800	...	6,000	546
Phosphor bronze	9,870	...	7,380	17,400	...	9,700	...

When three values are given in Table I. for the same material, they are maximum, mean, and minimum values.

RESISTANCE TO SIMPLE TENSION AND COMPRESSION.

26. A bar is in tension or compression when the load acts parallel to its axis, and the stress on any section of the bar is uniformly distributed or not, according as the line of action of the load does or does not pass through the centre of figure of that section. Cases in which the stress is a varying stress will be treated as cases of compound stress. At present only cases of uniformly distributed stress are considered.

Let A B, fig. 5, be a section, of area a (in sq. ins.), on which a load P (in lbs.) acts, normally to the section. Then the intensity of normal or direct stress is $f=\frac{P}{a}$ (in lbs. per sq. in.). If the section C D is not perpendicular to the direction of the load, let θ be the angle between the normal to the section and the direction of P. Then the stress on C D consists of a normal or direct stress $f_n=f\cos^2\theta$, and a tangential or shearing stress $f_t=f\sin\theta\cos\theta$.

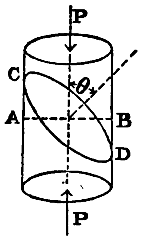


Fig. 5.

Very long bars bend under the action of a longitudinal compressive force, and must be treated by special rules.

The extension or compression l of a bar L inches in length is given by the equation

$$l=\frac{f}{E}L \qquad \text{(1)}$$

where f is the intensity of stress on sections perpendicular to the axis of the bar, and E is the modulus of direct elasticity. This equation is not applicable if the stress f exceeds the limit of elasticity.

Resistance of thin Cylinders to an internal bursting Pressure.—Consider a thin cylindrical shell of diameter d, length l, and thickness t, in inches, subjected to a uniform internal pressure of p lbs. per sq. in. Let the cylinder be cut by a

diametral plane $a\ b\ c\ d$, fig. 6. The resultant force P acting on either side of that plane $= p \times$ area $a\,b\,c\,d$. Hence, $P = p\ d\ l$. The molecular tensions which resist the bursting force act at $a\ b$ and $c\ d$, and are equal to the intensity of stress induced $\times$ area of $a\,b$ and $c\,d$. Putting f for the intensity of tensile stress, the total force resisting the bursting pressure is $2\ f\,t\,l$. Equating the load and resistance

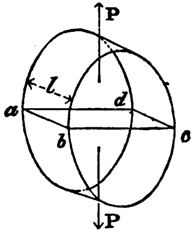


Fig. 6.

$$2\,f\,t\,l = p\,d\,l$$

$$f = \frac{p\,d}{2\,t} \qquad . \qquad . \qquad . \qquad (2)$$

If the cylinder consists of riveted plates, the section $a\,b\,c\,d$ should be taken, so as to pass through the rivet holes. Then the area of the rivet holes must be deducted from $2\ t\,l$, before equating the internal and external forces. If the cylinder is thick relatively to its diameter, the mean stress is unaltered, but the inner layers are more severely strained than the outer layers. In that case the thickness necessary to resist a bursting pressure p, with a maximum intensity of stress f, is found by Grashof to be

$$t = \frac{d}{2}\left\{-1 + \sqrt{\frac{3f + 2p}{3f - 4p}}\right\} \qquad . \qquad . \qquad . \qquad . \qquad (3)$$

it being assumed that p is less than $\frac{3}{4}f$. If $\frac{p}{f}$ is a small ratio—

$$t = \frac{d\,p}{2\,f}\left(1 + \tfrac{3}{4}\frac{p}{f}\right) \text{ very nearly} \qquad . \qquad . \qquad . \qquad . \qquad (3a)$$

In a thin spherical shell, the tension is half as great as in a thin cylindrical shell of the same diameter and thickness, exposed to the same pressure.

Resistance to Bending.

27. A bar is subjected to simple bending when the following conditions are fulfilled :—(1) The axis of the bar

is straight; the axis of the bar being a line connecting the centres of figure of parallel transverse sections; (2) The bar is symmetrical about a plane passing through the axis; (3) All the external forces act in such a plane of symmetry normally to the axis. If these conditions are not fulfilled, the action of the straining forces is more complex, and some cases in which this happens will be considered under the head of compound stress.

Consider the case represented in fig. 7, where, in the lower figure, the flexure is exaggerated for the sake of clearness. In this case, a bar originally straight, and having transverse sections symmetrical about the plane of the paper, in which the bending forces act, is subjected to flexure, under the action of two equal couples of forces applied to its ends. Then the curvature from *c* to *d* is circular, and the effect of the bending is to lengthen the upper parts of the bar, and to shorten the lower parts. If the flexure is very small, so that the straining forces are sensibly parallel,

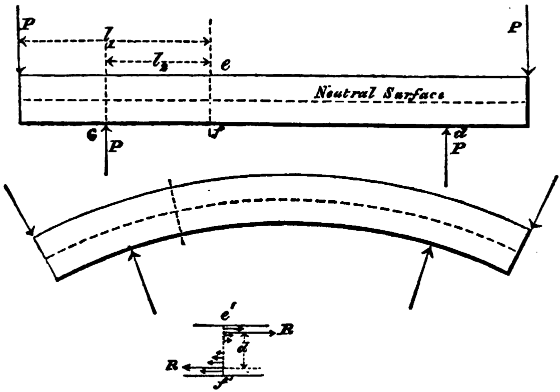


Fig. 7.

a plane normal to the paper, through the axis of the bar, will divide the parts in tension from those in compression. The

length of the bar measured along that surface will be unaltered by the flexure, and hence it is termed the neutral surface.

The amount of the bending action, at any section *ef* of the bar, is measured by the resultant moment of the straining forces on either side of that section, which is termed the bending moment. Taking the forces to the left of *ef*, the bending moment is $P\,l_1 - P\,l_2$. The molecular forces in the bar, developed by the external forces, form at any section a couple, whose moment is equal and opposite to the bending moment, and which is termed the moment of resistance of the section. The action of the molecular forces is represented at *e′f′*. The tensions above and the compressions below the axis have resultants R, R, whose moment is $R\,d$. Equating this to the bending moment

$$P\,(l_1 - l_2) = R\,d \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In other cases the action is a little more complex. Suppose the force P acts at the end of a bar (fig. 8) solidly fixed at the other end, and let it be required to find the straining action at *ef*. Equilibrium is not disturbed, if we introduce two equal and opposite forces P′, P″ in the direction *ef*. Then the action of P on the section *ef* is equivalent to that of a couple, P, P″, and an unbalanced force P′. The couple has a moment $P\,l$, which produces simple bending, and is in equilibrium

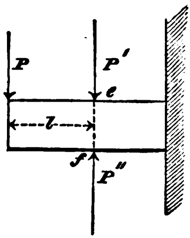


Fig. 8.

with a couple formed by molecular forces at *ef*, parallel to the axis of the bar, precisely similar to those described in the previous case. The remaining force P′ produces a shearing stress on the section *ef*. The two actions are independent, and the bar must be strong enough to resist both the bending moment and the shearing force. In a large number of cases the amount of material necessary to resist the bending moment is much more than sufficient to

resist the shearing force, so that the latter may be left out of consideration.

If several forces act to the left of ef, we may take their resultant, and then proceed as if only a single force required to be dealt with.

28. Let M be the moment of the external forces on one side of any transverse section, or bending moment, estimated relatively to the section.

Z = the modulus of the section, that is a function of the dimensions of the section, which is proportional to the moment of resistance of the section. The value of Z for various sections is given in the following tables.

f_t and f_c = the greatest safe working stress in tension and compression for the material of the bar. Then the bar will be safe, if

$$\left.\begin{array}{r} \text{M is not greater than } f_t Z \\ \text{and also is not greater than } f_c Z \end{array}\right\}$$

If we put fZ for the lesser of the two values of the moment of resistance, the bar will be of adequate strength when

$$M = fZ \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the greatest stress due to bending is

$$f = \frac{M}{Z} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

29. When the section is not symmetrical about the neutral surface, Z has two values, Z' corresponding to the part above, and Z'' to the part below the neutral surface. Then two cases arise:—

(1) The part of the bar above the neutral axis in tension, the part below in compression

$$\text{Moment of resistance} = f_t Z' \text{ or } f_c Z''.$$

(2) The part of the bar above the neutral axis in compression, the part below in tension

$$\text{Moment of resistance} = f_c Z' \text{ or } f_t Z''.$$

The smaller of the two values is to be taken in either case. If the straining forces act successively in opposite directions, the least of the four values is the effective moment of resistance.

In many cases, bars subjected to bending are necessarily uniform in section. Then it is only necessary to consider the greatest bending moment, and to design the section of the bar for that moment. In other cases, the bar varies in section, and the moment of resistance at each section must be, at least, equal to the bending moment at that section. The section at which the bending moment is greatest is sometimes termed the dangerous section. The Table on p. 31 gives, for various loads and modes of support, the greatest bending moment; the position of the dangerous section; the greatest shearing force; and the working load corresponding to a given moment of resistance $f z$, at the dangerous section.

30. *Continuous Beams.*—When a beam rests on more than two supports, the ordinary statical conditions of equilibrium do not suffice to determine the reactions of the supports. Recourse may then be had to a relation due to Clapeyron, which is termed the Theorem of three moments. Let fig. 9 represent two consecutive spans of a beam

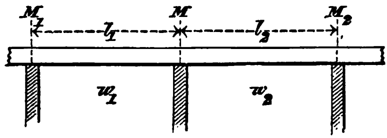
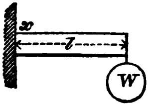
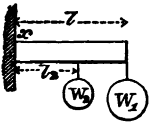
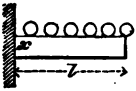
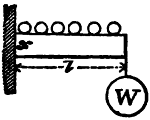
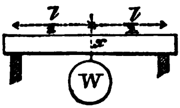
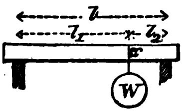


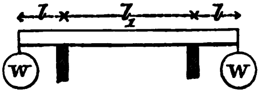
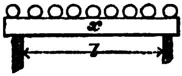
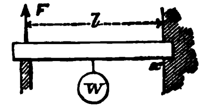
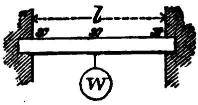
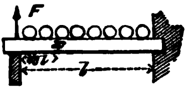
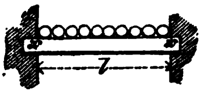
Fig. 9.

resting on several supports, at the same level; let l_1, l_2, be the lengths of the spans; w_1, w_2, the loads per unit of span; M_1, M, M_2, the bending moments over the supports. Then

$$8\,(l_1+l_2)\,M+4\,l_1\,M_1+4\,l_2\,M_2=w_1\,l_1^3+w_2\,l_2^3 \quad . \quad . \quad (7)$$

TABLE III.—*Bending Moment and Shearing Force corresponding to different Loads and for different modes of Support.*

		Greatest bending moment (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
	BEAMS FIXED AT ONE END.				
I.		$W\,l$	$W=\frac{fz}{l}$	W	Loaded at free end.
II.		$W_1 l_1 + W_2 l_2$		$W_1 + W_2$	More than one load.
III.		$w\frac{l^2}{2}$	$w=\frac{2fz}{l^2}$	$w\,l$	Uniform load, *w* lbs. per in. run.
IV.		$\frac{w\,l^2}{2}+W\,l$		$w\,l+W$	Load partly uniform, partly concentrated.
	BEAMS SUPPORTED AT BOTH ENDS.				
V.		$W\frac{l}{4}$	$W=4\frac{fz}{l}$	$\frac{W}{2}$	Loaded at centre.
VI.		$W\frac{l_1 l_2}{l}$	$W=\frac{fz\,l}{l_1 l_2}$	$W\frac{l_1}{l}$ & $W\frac{l_2}{l}$	Load not at centre.

		Greatest bending moment (at x)	Working load for given moment of resistance	Greatest shearing force	Remarks
VII.		W l	$W=\frac{fz}{l}$	W	Two equal couples. Uniform bending moment, and no shearing force between the supports.
VIII.		$\frac{wl^2}{8}$	$w=\frac{8fz}{l^2}$	$\frac{wl}{2}$	Uniformly distributed load. w lbs. per in. run.
	BEAMS FIXED AT ONE END AND SUPPORTED OR FIXED AT THE OTHER.				
IX.		$\frac{3}{16}$ W l	$W=\frac{16}{3}\frac{fz}{l}$		Load at centre. Pressure on support $=F=\frac{5}{16}$ W,
X.		$\frac{Wl}{8}$	$W=\frac{8fz}{l}$		Load at centre. Equal bending moments at ends and centre.
XI.		$\frac{wl^2}{8}$	$w=\frac{8fz}{l^2}$		Greatest bending moment at fixed end.
XII.		$\frac{wl^2}{12}$	$w=\frac{12fz}{l^2}$		Greatest bending moment at ends.

This theorem furnishes, for a beam of n spans, $n-1$ equations. In addition to these, the condition that a beam simply supported at the ends, has no bending moment at the ends, furnishes two additional equations, $M_0=0$, $M_n=0$. There are then $n+1$ equations, to determine the $n+1$

bending moments at the points of support. By then reversing the ordinary process, the reactions can be found, from the bending moments and loads. The following are some of the simplest results of applying this theorem to beams uniformly loaded with w lbs. per inch run.

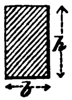
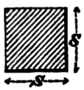
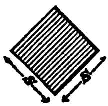
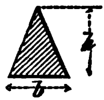
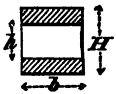
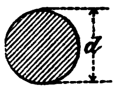
	Reactions at supports
Beam of 2 equal spans	$\frac{3}{8} w l$; $\frac{5}{4} w l$; $\frac{3}{8} w l$.
" 3 " . . .	$\frac{4}{10} w l$; $\frac{11}{10} w l$; $\frac{11}{10} w l$; $\frac{4}{10} w l$.

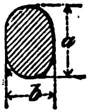
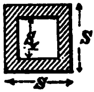
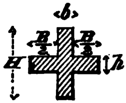
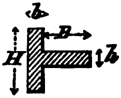
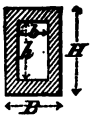
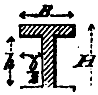
31. *Relative Economy of different forms of Section.*—The weight of a bar is proportional to its sectional area, its resistance to bending to its section modulus. Of two bars of different forms, subjected to the same loading, that will be the more economical of material which, with a given value of the modulus of resistance z, has the lesser sectional area A. Hence, the more economical the form of the bar, the greater will be the ratio $\frac{Z}{A}$.

In a prismatic bar, of circular or rectangular section, only the material at the extreme top and bottom of the section is fully strained. Nearer the neutral surface the material is less strained, and at the neutral surface it is not strained at all by the direct stresses due to bending. Such a bar would be made stronger, by removing some of the material from the neighbourhood of the neutral surface towards the top and bottom of the section. We thus arrive at the excellent form of section known as the I or double T section. The material is chiefly collected in the top and bottom flanges, which bear nearly the whole of the direct stresses due to bending; the remainder forms a vertical web, whose chief function is to resist the shearing force.

TABLE IV.—*Area and Modulus of different forms of Section.*

The plane of bending is supposed parallel to the side of the page. Where two values are given for the modulus, z' is applicable to the upper part, z'' to the lower part, of the section.

	Form of section		Area of section A	Modulus of section Z
I.	Rectangle		$b\,h$	$\frac{1}{6}\,b\,h^2$
II.	Square		s^2	$\frac{1}{6}\,s^3$
III.	Square		s^2	$0{\cdot}118\,s^3$
IV.	Triangle		$\frac{1}{2}\,b\,h$	$z' = \frac{1}{24}\,b\,h^2$ $z'' = \frac{1}{12}\,b\,h^2$
V.	Pierced rectangle		$b\,(H - h)$	$\frac{1}{6}\,\frac{b}{H}\,(H^3 - h^3)$
VI.	Circle		$\frac{\pi}{4}\,d^2 = {\cdot}785\,d^2$	$\frac{\pi}{32}\,d^3 = {\cdot}0982\,d^3$

	Form of section		Area of section A	Modulus of section Z
VII.	Hollow circle		$\frac{\pi}{4}(d_2^2-d_1^2)$	$\frac{\pi}{32}\frac{d_2^4-d_1^4}{d_2}$
VIII.	Ellipse		$\frac{\pi}{4}ba$	$\frac{\pi}{32}ba^2 = \cdot0982\,ba^2$
IX.	Hollow ellipse		$\frac{\pi}{4}(ba-b_1a_1)$	$\frac{\pi}{32}\frac{ba^3-b_1a_1^3}{a}$
X.	Hollow square		$s^2-s_1^2$	$\frac{1}{6}\frac{s^4-s_1^4}{s}$
XI.	Cross or Tee	 	$b\mathrm{H}+\mathrm{B}h$	$\frac{b\mathrm{H}^3+\mathrm{B}h^3}{6\mathrm{H}}$
XII.	Hollow rectangle or I with equal flanges	 	$\mathrm{BH}-bh$	$\frac{\mathrm{BH}^3-bh^3}{6\mathrm{H}}$

32. *Flanged Sections when both Flanges are strained to the Working Limit.*—In order that the stress at the stretched edge of the bar may be at the working limit of tensile resistance, and the stress at the compressed edge may be at the working limit of compressive resistance, we must have

$$f_t z' = f_c z'' \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where z' is the modulus, corresponding to the part in tension, and z'', that corresponding to the part in compression, and f_t, f_c, are the working intensities of tensile and compressive resistance. If $f_t = f_c$, then $z' = z''$, or the modulus must be the same, both for the upper and lower parts of the section, and this will be the case when the section is symmetrical about the neutral surface.

If f_t and f_c are not equal, both edges cannot be fully strained when $z' = z''$, and the material is not used in the most economical way. In that case, it is better to adopt a section unsymmetrical with respect to the neutral surface.

Let a_t be the area of the tension flange, a_c the area of the compression flange, and a the area of the web of a beam of I-shaped section. Let h be the depth, from centre to centre, of the flanges. Then the required condition is nearly fulfilled, when

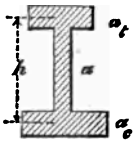


Fig. 10.

$$f_t a_t = f_c a_c$$

$$a_t = \frac{M}{f_t h} \text{ and } a_c = \frac{M}{f_c h} \quad . \quad . \quad . \quad . \quad (9)$$

Further, if F is the total shearing force, and f_s the safe shearing resistance, the strength of the web is sufficient when,

$$a = \frac{F}{f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

For practical reasons, especially in cast beams, a has to be made of larger area than is given by this equation.

In the foregoing equations the resistance of the web to bending is neglected. If this is taken into account we get the following equations, which are taken from Rankine :—

CASE I.—f_c greater than f_t, as in cast iron.

$$a_t = \frac{f_c}{f_t} a_c + \frac{f_c - f_t}{2 f_t} a \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and the modulus of the section is

$$z_c = h \left\{ a_c + \left(2 - \frac{f_t}{f_c}\right) \frac{a}{6} \right\} \text{ for compression ;}$$

$$z_t = h \left\{ a_t + \left(2 - \frac{f_c}{f_t}\right) \frac{a}{6} \right\} \text{ for tension.}$$

CASE II.—f_t greater than f_c, as in wrought iron and steel.

$$a_c = \frac{f_t}{f_c} a_t + \frac{f_t - f_c}{2 f_c} a \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and the modulus of the section is

$$z_t = h \left\{ a^t + \left(2 - \frac{f_c}{f_t}\right) \frac{a}{6} \right\} \text{ for tension ;}$$

$$z_c = h \left\{ a_c + \left(2 - \frac{f_t}{f_c}\right) \frac{a}{6} \right\} \text{ for compression.}$$

In consequence of the tendency in wrought iron beams to a vertical or lateral buckling of the compressed flange, the working stress, in compression, is sometimes taken $\frac{2}{5}$th less than the working stress in tension. In cast iron, the working resistance, in compression, may be taken three to four times as great as the resistance in tension.

33. *Distribution of Bending Moment and Shearing Force.* —When the magnitude and position of the greatest bending moment and shearing force are known, the bending moment and shearing force, at any other point, can often be found

very easily, by a simple graphic construction. Let O A be equal, on any scale, to the span of a beam. If, at any point, a, a perpendicular is erected, and $a\,b$ is made equal, on any scale, to the bending moment at the section of the beam which corresponds to a, b is a point in a curve, termed the curve of bending moments. Similarly, if $a\,b$ were made equal to the shearing force at a, then b would be a point in a curve of shearing force. The curves might be constructed, by finding the moments and the shearing forces at a sufficient number of points, setting them off on a diagram in the way just described, and then connecting the points of the curve, so found, by a line. In the simpler cases of loading, however, these curves can be more simply constructed. When the curve (which in some cases becomes a straight line) is drawn, the moment, or shearing, force at any point is obtained by scaling off the ordinate corresponding to that point. The following Table (p. 40), gives the form of the curves of bending moment and shearing force, in the simpler cases. The cases given in this Table should be compared with the corresponding cases in Table III.

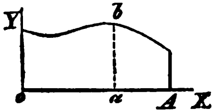
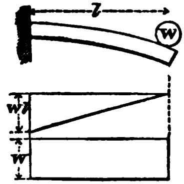
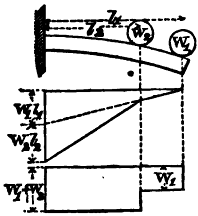
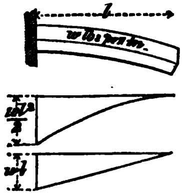
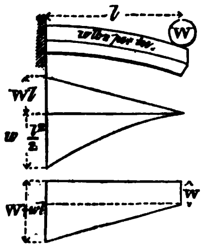


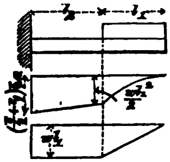
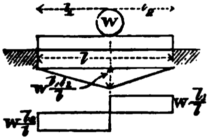
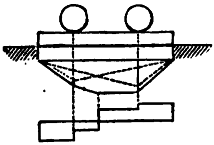
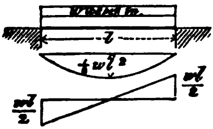
Fig. 11.

34. *Beams of Uniform Resistance to the direct Stresses due to Bending.*—Except in one special case, the bending moment varies at different points in the length of the beam. At the point where the bending moment is greatest, the section must be designed for that maximum moment. For practical reasons, it is frequently necessary to make the beam, or bar, uniform, and then the section where the bending moment is greatest determines the section of the rest of the bar. In other cases, the section of the bar may be diminished in parts where the bending moment is less, and material is then economised. The best distribution of material, so far as the direct stresses are concerned, is that which fulfils the condition

$$M = f\,Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

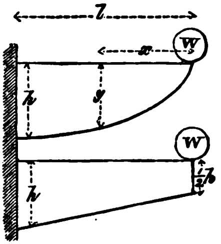
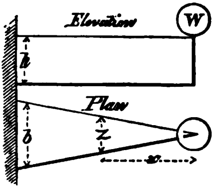
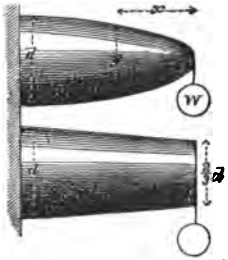
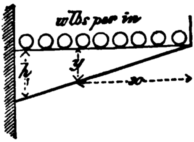
TABLE V.—*Distribution of Bending Moment and Shearing Force.*

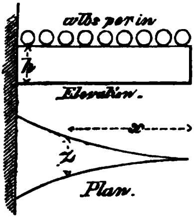
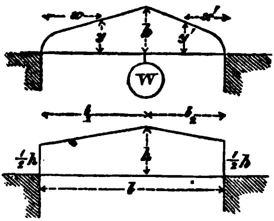
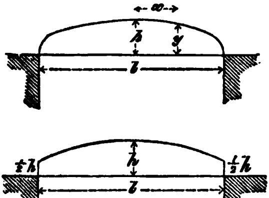
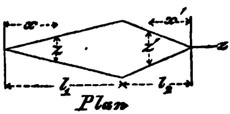
	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
		BEAMS ENCASTRE AT ONE END.		
I.	Load at free end		Straight line	Straight line
II.	Two loads		Broken line	Broken line
III.	Uniform load		Parabola, vertex at free end, axis vertical	Straight line
IV.	Uniform load and load at free end		Obtained by combining the curves in I. and III.	

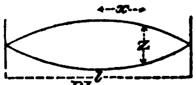
	Loading	Diagram of load, and of bending moment and shearing force curves	Bending moment curve	Shearing force curve
V.	Partial uniform load		Parabola, with vertex at free end and straight line	Broken line
		BEAMS SUPPORTED AT BOTH ENDS.		
VI.	Single load		Broken line	Broken line
VII.	Two loads		Obtained by adding ordinates due to each separate load	
VIII.	Uniform load		Parabola, vertex at centre, axis vertical	Straight line

for every transverse section, M being the bending moment at any section, and Z the modulus of that section. Beams so designed are often termed beams of uniform strength. The theoretical form thus obtained requires, in some cases, to be modified, for practical reasons. Approximate forms fulfilling the necessary conditions are given with the theoretical forms in the following Table. Table VI. gives some examples, partly selected from Reuleaux.

TABLE VI.—*Forms of Beams of Uniform Strength.*

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
BEAMS FIXED AT ONE END.					
I.	Load at free end		Rectangle of uniform breadth, b, and variable depth, y	Straight line and parabola. Approximate form, a truncated pyramid	$y^2 = \frac{6\,W}{bf}x$
II.			Rectangle of uniform depth, h, and variable breadth, z	Straight lines forming a wedge	$z = \frac{6\,W}{h^2 f}x$
III.			Circle of variable diameter, y	Cubic parabola. Approximate form, a truncated cone	$y^3 = \frac{32\,W}{\pi f}x$
IV.	Uniform load		Rectangle of uniform breadth, b, and variable depth, y	Straight lines forming a wedge	$y^2 = \frac{3\,w}{fb}x^2$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
V.	Uniform load		Rectangle of uniform depth and variable breadth, z	Parabolas with vertex at free end	$z=\frac{3\,w}{f\,h^2}x^2$
		BEAMS SUPPORTED AT EACH END.			
VI.	Single load		Rectangle of uniform breadth, b, and variable depth, y	Two parabolas and straight line. Approximate form, truncated pyramids	$y^2=\frac{6\,W\,l_2}{b\,f\,l}x$ $y'^2=\frac{6\,W\,l_1}{b\,f\,l}x'$
VII.	Uniform load		Rectangle as above (w lbs. per inch uniformly distributed)	Ellipse and straight line. Approximate form, circular arc and straight line	$y^2=\frac{3\,w\,b}{16\,f}$ $(l^2-4\,x^2)$
VIII.	Single load		Rectangle of uniform depth, h, and variable breadth, z	Straight lines	$z=\frac{6\,W\,l_2}{h^2 f\,l}x$ $z'=\frac{6\,W\,l_1}{h^2 f\,l}x'$

		Longitudinal elevation of beam	Form of transverse section	Bounding lines of elevation or plan	Equation for determining the dimensions
IX.	Uniform load	 Plan	Rectangle as above. (Load, *w* lbs. per inch, uniformly distributed)	Two parabolas	$z = \frac{3(l^2 - 4x^2)w}{4fh^2}$

A beam, supported at each end, is equivalent to two beams encastre at the point where the bending moment is greatest. The forms given for beams encastre at one end, may be used for each segment of a beam supported at both ends.

Resistance to Shearing.

35. A shearing force is one acting in the plane of the section of a bar which is being considered. Thus, the pressure of the cutting edges of an ordinary shearing machine, fig. 12, induces a shearing stress in the plane *a b*. The mean intensity of the shearing stress is the shearing force P, divided by the

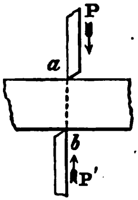


Fig. 12.

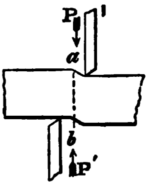


Fig. 13.

area *a* of the section *a b* of the bar. In the case shown, the forces P P act exactly in the plane of the section, and the shearing stress is uniformly distributed, and at all parts of the section $= P \div a$. But if the forces P P do not act exactly in the plane of the section, the bar tends to bend as well as to shear (fig. 13). The effect of this is to alter the distri-

bution of the shearing force at all sections between *a* and *b*. Near the middle of the section the shearing stress is greater than the mean shearing stress, and at the upper and lower boundary of the section it becomes zero. A rivet connecting two plates (fig. 14) is almost always in shear, and a bolt is very often so. A cotter or key is similarly intended to resist shear. In these cases, the shearing forces do not act in the plane of the section *b c*, but along the centres of the plates connected. In consequence, however, of the rigidity and friction of the edges *a b* and *c d* of the plates, the points of application of the forces P P′ on the surface of the rivet

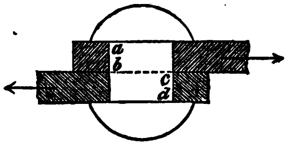


FIG. 14.

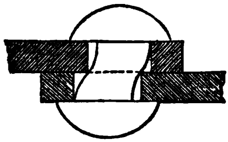


FIG. 15.

may very nearly approach the plane *b c*, and then the shearing stress is uniformly distributed on *b c*. If, however, the rivet fits very loosely in the rivet holes, fig. 15, the rivet bends and the distribution of stress becomes more or less unequal. For rivets, it is usual to assume that they fit their holes tightly, and that the shearing stress is simply $f_s = \text{P} \div a$. But for bolts and cotters, it is safer to assume that the stress is unequally distributed, and the maximum stress may then reach the values

$f_s = \frac{3}{2}\,\frac{\text{P}}{a}$ if the section is rectangular and P perpendicular to one side.

$= \frac{4}{3}\,\frac{\text{P}}{a}$ if the section is circular or elliptical.

$= \frac{9}{8}\,\frac{\text{P}}{a}$ if the section is square and P acts parallel to a diagonal.

RESISTANCE TO TORSION.

36. A bar is subjected to simple torsion when two equal and opposite couples act upon it in two planes perpendicular to its axis, instead of being, as in the case of bending, in the plane of the axis. When the bar is subjected to straining action of this kind, any two transverse sections rotate slightly relatively to each other, and on any one transverse section, the stress is a simple tangential or shearing stress, varying in intensity from the centre of the bar, where it is

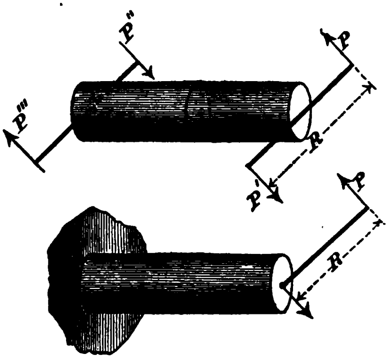


FIG. 16.

zero, to the circumference, where it is greatest. Of the two couples one, P P′, is usually due to motive forces applied to the bar. The other, P″ P‴, is due to the passive reaction of the parts to which the bar is attached, or to the resistances which are being overcome. Further, of the two forces P, P′, constituting the former couple, one of the two forces, for instance, P′, may be due to the reaction of a support or bearing of the shaft, and it then acts at the centre of the shaft, as shown in the lower figure.

The amount of straining action at any section *a b*, is measured by the moment of the couple on either side of the section. In this case that moment, termed the twisting

moment, is $T = PR$. If several couples act on one side of the section, the algebraic sum of the moments of all those couples is to be taken, right-handed couples being considered positive, and left-handed couples negative.

When the bar is kept in rotation overcoming a resistance, and the amount of work transmitted is known, the twisting moment is easily found. Let HP be the number of horses' power transmitted, N the number of revolutions of the bar per minute. Then the work expended in inch lbs. per minute is $12 \times 33000 \times HP$, and this is equal to the twisting moment in statical inch lbs., multiplied by the angular motion of the bar in the same time, or to $T \times 2\pi N$. Hence

$$T = \frac{12 \times 33000 \times HP}{2\pi N} = 63024 \frac{HP}{N} \text{ inch lbs.} \quad . \ (14)$$

The moment of resistance of any section to torsion, is proportional to the greatest stress at any part of the section, and to a function of the dimensions, which is termed the modulus of the section with respect to torsion. Let f be the greatest shearing stress, and z_t the modulus :—

$$T = f z_t . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

For cylindrical bars of diameter d,

$$z_t = \frac{\pi}{16} d^3 = \frac{d^3}{5 \cdot 1} = 0 \cdot 196\, d^3.$$

For hollow cylindrical bars having d_1, d_2, for outside and inside diameters,

$$z_t = \frac{\pi}{16} \cdot \frac{d_1^4 - d_2^4}{d_1} = 0 \cdot 196 \frac{d_1^4 - d_2^4}{d_1}.$$

For bars of square section, s being the side of the square,

$$z_t = 0 \cdot 281\, s^3.$$

It is sometimes necessary to know the angle through which one end of a bar rotates, relatively to the other end, when subjected to torsion. For a cylindrical bar, let d be

the diameter, l the length and G the modulus of transverse elasticity; then the angle of torsion is, in circular measure,

$$\theta=\frac{2\,T\,l}{G Z_t d}=\frac{2\,f\,l}{G\,d} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where f is the greatest actual stress due to the twisting moment.

Resistance to Combined Compression and Bending.

37. When a bar, of moderate length, is subjected to a thrust, acting in the direction of its axis, the stress on each cross section is a simple compression. The working strength of the bar is given by the equation

$$P_1=f\,A \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where P_1 is the greatest safe load, A the sectional area of the bar, and f the safe working compressive resistance of the material.

38. If, however, the bar is of great length, it gives way, ultimately, under the action of a thrust, by lateral bending; the stress, at the section where fracture occurs, being a compound stress, due both to the longitudinal compression, and the curvature of the bar. Rules for the ultimate resistance of long bars to compression were first obtained, experimentally, by Professor Hodgkinson, and these rules have been very generally used in this country, in designing compression bars. These rules are inconvenient in form, and they can only be extended to many cases of common occurrence, by theoretical assumptions, which are probably only approximately true. A rule more convenient in form than Hodgkinson's was proposed by Professor Gordon, and was afterwards modified by Professor Rankine, so as to be applicable to bars of all forms of section. The reasoning on which this rule is based is, however, not satisfactory. All these rules are intended to give the ultimate strength of the bars, and in applying them, it is necessary to divide the resistance thus calculated by an arbitrary factor of safety. Actual compression bars are not intended to be loaded

beyond their elastic limit; and hence it may be urged, with some reason, that the theoretical rules of Euler, which Hodgkinson discarded, as not agreeing with his experiments on ultimate strength, are more strictly applicable to the circumstances in which compression bars are used, than Hodgkinson's rules. They are also simpler, and include all cases. Euler's rules assume the elasticity of the bar to be unimpaired. In that case, no increase of the load would *directly* cause bending, but a point is reached at which the equilibrium of the bar becomes unstable. With less loads, the bar, if bent, will restore itself to straightness by its elastic resistance to bending; with greater loads, it is unable to do so, and if any flexure is produced, however slight, that flexure will be increased by the action of the load, until the bar breaks. The greatest load for which the bar remains stable, is the measure of the strength of the bar. The working load may be $\frac{1}{n}$th of the load thus calculated, where n is a factor of safety.

Let E be the modulus of direct elasticity of the material of the bar; I, the moment of inertia of the section of the bar, estimated with respect to an axis passing through the centre of gravity of the section, and at right angles to the plane in which the bar most easily bends; λ, the length of an arc of the curved bar, measured between two points of contrary flexure. Then

$$P=\pi^2 \frac{I E}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where P is the greatest load, consistent with the stability of the bar. The greatest safe load is

$$P_2=n P \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where $n=5$ for wrought iron, 6 for cast iron and 10 for wood.

Let Z be the smallest modulus of the section of the bar,

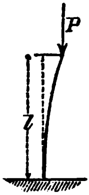
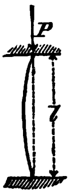
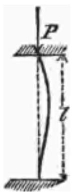
values of which are given in Table IV.; r the distance from the centre of gravity of the section to the edge of the bar, measured parallel to the plane of bending

$$I = z\,r \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Moment of Inertia = I =

For a circular section (diam. $= d$)	$0{\cdot}0491\,d^4$
For an annular section (diams. $= d_1, d_2$) .	$0{\cdot}0491\,(d_1^4 - d_2^4)$
For a square section (length of side $= s$)	$\frac{1}{12}\,s^4$
For a rectangular section (longer side b, shorter h) .	$\frac{1}{12}\,b\,h^3$

TABLE VII.—*Strength of Long Compression Bars. Greatest Load consistent with Stability.*

		Mode of fixing.	Length of arc λ in terms of total length	Greatest load P consistent with stability
I.		One end free, the other fixed.	$\lambda = 2l$	$\frac{\pi^2}{4} \frac{EI}{l^2}$
II.		Both ends free, but guided in the direction of the load.	$\lambda = l$	$\pi^2 \frac{EI}{l^2}$
III.		One end fixed, the other free, and guided in direction of the load.	$\lambda = \frac{l}{\sqrt{2}}$	$2\pi^2 \frac{EI}{l^2}$
IV.		Both ends fixed in direction.	$\lambda = \frac{l}{2}$	$4\pi^2 \frac{EI}{l^2}$

39. In applying the above rules, it is assumed that the length is so great that $P_2 < P_1$. For bars of moderate length, when—

λ is less than $24\,d$ or $28\,h$ for wrought iron;
„ „ $10\,d$ or $11\frac{1}{2}\,h$ for cast iron;
„ „ $11\frac{1}{2}\,d$ or $13\,h$ for wood;

the rules in Table VII. are not applicable. The following empirical rules, suggested by Grashof, may then be used:

$$P_3 = \frac{k_1\,A\,I}{C\,A\,l^2 - I} \text{ or } = \frac{k_2\,A\,I}{C\,A\,l^2 + I} \quad . \quad . \quad (21)$$

Where A=area of section; I=moment of inertia of section as before; l=length; C a constant; k_1 and k_2, the safe working stress in tension and compression; P_3 is the greatest safe load, the lesser of the two values being always taken.

	$C =$	$k_1 =$	$k_2 =$
Steel . . .	·00009	14220	14220
Wrought iron .	·00009	8500	8500
Cast iron . .	·00027	2840	11360
Wood . . .	·00022	850	700

40. *Resistance of thin cylinders to an external collapsing pressure.*—When a thin cylinder, rigidly supported at the ends, is subjected to a uniform external pressure, it gives way by buckling, or collapse. There is, at present, no theory of this mode of yielding, but experiments were made by Sir W. Fairbairn, from which the following rules were deduced.

Let t be the thickness, d the diameter, and l the length between the rigidly-supported ends of a cylinder, subjected to a uniform external pressure of p lbs. per sq. in. Then collapse takes place when

$$p = 9672000\frac{t^{2.19}}{l\,d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

This can only be solved by logarithms; and in a logarithmic form the equation becomes

$$\log. p = 6{\cdot}9855 + 2{\cdot}19 \log. t - \log. (l\,d) \quad . \quad . \quad . \quad (23)$$

All the dimensions are in inches. A formula, which is probably still more reliable, was deduced from the same experiments by M. Love. His formula is

$$p = 5358150\frac{t^2}{l\,d} + 41906\frac{t^2}{d} + 1323\frac{t}{d}. \qquad (24)$$

The limits of length for which these formulæ are applicable are not known. The factor of safety may be 6 or 8.

Compound Stress.

41. I. *Tension or compression, combined with bending.*—When a force P, fig. 17, acts in a plane passing through the axis of the bar, and parallel to that axis, the stress on transverse sections of the bar is equivalent to that due to a direct tension or compression P, and a bending moment P r. The greatest stress is then

$$f = \mathrm{P}\left(\frac{1}{\mathrm{A}} + \frac{r}{\mathrm{Z}}\right). \qquad (25)$$

where A is the transverse sectional area, and Z the modulus of the section. The greatest stress will be tensile or compressive, according as P tends to extend or to compress the bar.

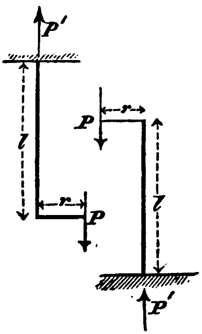


Fig. 17.

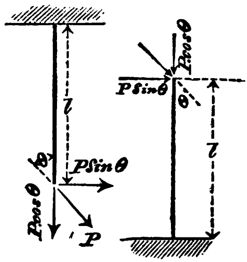


Fig. 18.

II. When the force P acts as above, but not parallel to the axis of the bar, fig. 18, its direction will intersect that axis, at some distance l from the point of support. At that point, resolve the force P into its components. The com-

ponent P Cos θ produces a simple tension, or compression; the other component P Sin θ produces bending; the greatest bending moment being P l Sin θ. Then, the greatest stress at the section nearest the point of support is

$$f = \text{P}\left(\frac{\text{Cos}\,\theta}{\text{A}} + \frac{l\,\text{Sin}\,\theta}{\text{Z}}\right) \quad . \quad . \quad . \quad . \quad (26)$$

42. III. *Combined twisting and bending.*—Let the force P act in a plane perpendicular to the axis of the bar, at a distance r from the axis, and at a distance l from the point of support. The force P will give rise to a parallel reaction P_1 at the point of support, and the bar will be subjected to a wrenching moment $\text{P}\sqrt{(r^2+l^2)}$. It will not affect the conditions of equilibrium, if we introduce two opposite forces P', P'_1, each equal to P or P_1. Then the wrenching moment will be seen to be equivalent to a simple twisting moment, due to P and P', and a bending action, due to P_1 and P'_1. The twisting moment is $\text{T}=\text{P}\,r$, and the greatest bending moment is $\text{M}=\text{P}\,l$.

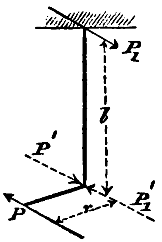


Fig. 19.

Let M_e be a simple bending moment, which would produce an effect on the bar, equivalent to that due to the combined bending and twisting action. Then the theory of elasticity furnishes the two following values of M_e, according as we have regard to the greatest stress or the greatest strain induced in the bar

$$\text{M}_e = \tfrac{1}{2}\text{M} + \tfrac{1}{2}\sqrt{(\text{M}^2+\text{T}^2)} \quad . \quad . \quad . \quad . \quad (27)$$
$$= \tfrac{3}{8}\text{M} + \tfrac{5}{8}\sqrt{(\text{M}^2+\text{T}^2)} \quad . \quad . \quad . \quad . \quad (27\,a)$$

The former value will be used in this Treatise. It can be put in the simpler approximate forms—

$$\left.\begin{aligned} \text{M}_e &= 0\cdot98\,\text{M} + 0\cdot2\,\text{T} \quad \text{if } l > r \\ &= 0\cdot7\,\text{M} + 0\cdot48\,\text{T} \quad \text{if } r > l \\ &= 0\cdot914\,\text{M} + 0\cdot414\,\text{T} \text{ if } r \text{ and } l \text{ are unknown} \end{aligned}\right\} (28)$$

The greatest safe load in the above case is

$$\text{P}=\frac{2fz}{l+\sqrt{(l^2+r^2)}}=\frac{fz}{al+br}\text{ nearly} \quad . \quad . \quad . \ (29)$$

where a and b are the numerical values of the constants in the approximate formulæ given above.

STRENGTH OF FLAT PLATES.

43. I. A flat plate, of thickness t, is supported, but not fixed, on a circular support of radius r, and is uniformly loaded with p lbs. per sq. in. (fig. 20). Then the greatest stress is

$$f=\frac{5}{6}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \ (30)$$

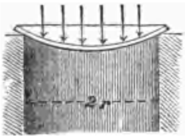


Fig. 20.

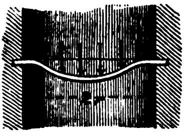


Fig. 21.

II. A circular flat plate, of radius r and thickness t, is encastré at the edge, and is uniformly loaded with p lbs. per sq. in. (fig. 21). Then the greatest intensity of stress is

$$f=\frac{2}{3}\frac{r^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \ (31)$$

III. A circular plate, of radius r and thickness t, is supported at the edge, and loaded with a concentrated load P, applied at a circumference, the radius of which is r_0 (fig. 22). The greatest stress is

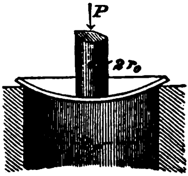


Fig. 22.

$$f=\left(\frac{4}{3}\log.\frac{r}{r_0}+1\right)\frac{\text{P}}{\pi t^2} \quad . \quad . \quad . \quad . \ (32)$$

$\dfrac{r}{r_0}=$	10	20	30	40	50
$\frac{4}{3}\log.\dfrac{r}{r_0}+1=$	4·07	5·00	5·53	5·92	6·22

The above rules are due to Grashof.

IV. *Strength of stayed surfaces.*—A flat plate, of thickness t, is supported uniformly by stays arranged in lines (fig. 23). Distance of stays from centre to centre $=a$, uniform load $=p$ lbs. per sq. in. The greatest stress in the plate is

$$f=\frac{2}{9}\frac{a^2}{t^2}p \quad . \quad . \quad . \quad (33)$$

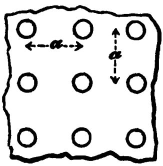


Fig. 23.

Each stay supports $p\,a^2$ lbs.

V. A rectangular plate, of thickness t, length l, and breadth b, is encastré at the edge, and loaded uniformly with p lbs. per sq. in. The greatest stress is

$$f=\frac{1}{2}\frac{l^4}{l^4+b^4}\frac{b^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad (34)$$

VI. A square plate, s inches in length of side, is similarly supported and loaded. The greatest stress is

$$f=\frac{1}{4}\frac{s^2}{t^2}p \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

Note to § 40.—The Author has recently re-examined Fairbairn's experiments, and has obtained the following formula for the ultimate resistance to collapse of flues having longitudinal and cross joints

$$p=15{,}547{,}000\frac{t^{2.35}}{l^{0.9}\,d^{1.16}} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

CHAPTER IV.

ON FASTENINGS.

RIVETED JOINTS.

44. THE simplest fastening is the rivet, employed to unite wrought iron, soft steel or copper plates. A rivet is virtually a bolt, with the head, body and nut in one piece. It is a permanent fastening, only removable by chipping off the head. Bolts are most often used with the straining force parallel to the axis, so that the bolt is in tension; but rivets are almost always placed at right angles to the straining force, so as to be in shear. They are not reliable in tension.

A rivet is formed of round bar, and, when ready for use, has the form shown in fig. 24. It is parallel for about half its length, and very slightly tapers for the remainder. The head is cup-shaped, or more often, pan-shaped, as shown. For iron or steel plates, the rivets are of very soft uniform iron, and are made in rivet-making machines of various kinds, being pressed, while red hot, in suitable dies. When used, the rivets are again heated to red heat, placed in the rivet hole in the plates to be connected, and then the second head is formed by hand, or by machine. In hand riveting, the tail of the rivet is held

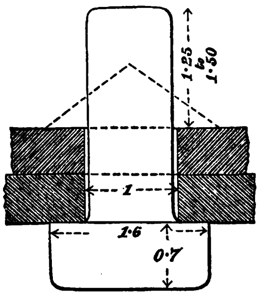


Fig. 24.

up, while the head is formed by two riveters working with hammers, and the head is either made conical by the hammers alone, or finished by the aid of a cup-shaped die, called a snap. In machine riveting, the rivet is pressed between two dies, actuated by a lever, or by steam or hydraulic pressure. Machine riveting causes the rivet to fill up the holes more perfectly than hand riveting, but is more liable to form the head eccentrically to the rivet. Steel rivets have been used, but the steel is injured by the heating to which the rivet is subjected, and the best practice is to use iron rivets for steel plates, although this sacrifices part of the advantage of using steel plates. Very good soft iron may be riveted cold.

Rivet holes are most commonly made by punching. This somewhat rough process is objectionable, on two grounds. The spacing of the rivet holes is not perfectly accurate, and the metal round the hole is more or less injured by the compression at the moment of punching. With very soft, ductile iron plates, it is believed that the injury done in punching is comparatively small, if the punch is sharp, and the hole in the die block not too large. But, with steely iron or steel plates, the injury is serious, the plates being weakened 15 to 30 per cent. The injury is due, not to cracks formed in the plate, but to the pressure straining it beyond its limit of elasticity, and thus altering its homogeneous character and power of equal elongation under strain. The metal near the hole becomes more rigid than that further off, and hence, when the joint is subjected to strain, the stress is not uniformly distributed on the metal between the rivet holes. Hence, steel plates should be annealed after punching, or the holes should be punched smaller than the size of the rivet, and then enlarged by a cutting tool (such as a rymer) to the full size.

To obviate the objections to punching, the holes are sometimes drilled. The process of drilling is, in most cases, more expensive than punching, but the holes are

more accurate in size and spacing. On the other hand, the sharp, square edge of a drilled hole appears to be slightly unfavourable to the resistance of the rivet. Sir W. Fairbairn showed that the strength of the joint was increased by slightly rounding the edges of the hole.

When the riveting is done at red heat, the contraction of the rivet, in cooling, nips the plates powerfully, and causes considerable tension on the rivet. In very long rivets, this may cause fracture of the rivet, and to prevent this the tail end is cooled before placing it in the rivet hole. In ordinary riveting, the contraction is advantageous in securing staunchness of the joint. Further, the contraction creates a frictional resistance to slipping between the plates, which enables the joint to sustain a considerable force, even when the rivets do not fit the holes. The tension in the rivet may be estimated at 21,000 lbs. per square inch of its section, and the friction due to this would be about 7,000 lbs. per inch of rivet section. Experiments show a still greater friction, but if the tension in the rivet exceeds the elastic limit, its permanence cannot be relied on. English engineers entirely neglect the friction, in estimating the strength of the joint, the reasons assigned being that the amount of tension in the rivet is not ascertainable, and that vibrations and other causes, tending to slightly elongate the rivet, may, in course of time, destroy it altogether. If, however, a moderate estimate is made of the friction, there is no sufficient reason why it should not be taken into account, in cases where it may seem desirable.

The staunchness of the joint, or its power of resisting the tendency to leak, when subjected to steam or water pressure, depends on the nearness of the rivets to the edge of the plate, and their nearness together. The metal between two rivets is in the position of a beam subjected to uniform pressure, and tending to deflect. If the joint is not naturally staunch, it may be rendered so by caulking, that is, burring down a narrow strip at the edge of the plate by a

chisel (fig. 25). In the best boiler work, the plates have their edges planed before riveting, and this renders caulking much more easy. It is a point of cardinal importance, in boilerwork, to arrange the joints, so that all can be properly caulked.

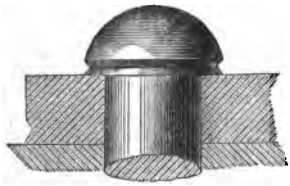


Fig. 25.

The experiments of the Manchester Steam Users' Association, have shown that machine-riveted work is somewhat stronger than hand-riveted work, and this would appear to be due to the greater friction of the machine-riveted joint.

45. *Forms of rivets.*—Fig. 26 shows an ordinary hand-formed rivet, with conical head. Fig. 27, a machine-formed rivet, with cup or snap head. Fig. 28, a countersunk rivet. This last is only used when the surface of the plate must be fair, and without projections ; it weakens the plate more

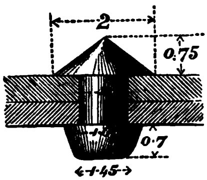


Fig. 26.

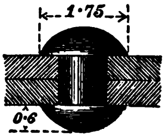


Fig. 27.

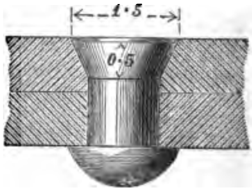


Fig. 28.

than an ordinary rivet, and it is less reliable. The conical head is formed entirely by hand hammers, and is used where there is a restricted space for hammering ; but it is thinner at the shoulder, and less reliable, than the cup form. The cup-shaped head is finished by a die or snap, which requires the use of a sledge hammer.

The numbers on the figures are proportional to the diameter of the rivet, and give good ordinary proportions, although it must be remembered that the sizes used by

different engineers vary more or less. To form the head, a length equal to about the diameter is required in countersunk riveting, and $1\frac{1}{4}$ to $1\frac{1}{2}$ times the diameter in ordinary riveting.

Lap and butt riveting.—When one plate is made to overlap the other, and one or more lines of rivets are put through the two, the riveting is lap riveting. When the plates are kept in the same plane, and a cover plate is put over the joint and riveted to each, the riveting is butt riveting.

Single and double riveting.—If there is one line of rivets in lap riveting, or one line on each side of the joint in butt riveting, the joint is single riveted. If there are two lines in lap, or two lines on each side of the joint in butt riveting, the joint is double riveted.

46. *Size of rivets for plates of different thickness.*—Let t=thickness of plate, d=diameter of rivet, f_s=resistance of plate to shearing, f_c=resistance of punch to crushing. The area sheared by the punch is $\pi\, d\, t$, and the resistance to shearing is $\pi\, d\, t\, f_s$. The strength of the punch is $\frac{\pi}{4} d^2 f_c$. Hence, if

$$\pi\, d\, t\, f_s \text{ is greater than } \frac{\pi}{4} d^2 f_c,$$

$$\text{or if } d \text{ is less than } 4\, t \frac{f_s}{f_c},$$

the punch will crush before the plate shears. If $f_c = 4 f_s$, d must not be less than t, or the plate cannot be punched. To allow a margin of safety for the punch, the rivet diameter is rarely less than one and a-half times the thickness of the plate. The diameter of rivets in practice ranges from

$$d = \tfrac{3}{4}\, t + \tfrac{3}{8} \text{ to } \tfrac{7}{8}\, t + \tfrac{3}{8}$$

and a very simple and convenient rule is

$$d = 1{\cdot}2\sqrt{t} \qquad \ldots \ldots \quad (1)$$

$t=$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
$d=$	0·60	0·67	·73	·79	·85	·90	·95	1·04	1·12	1·20
$=$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{15}{16}$	$1\frac{1}{16}$	$1\frac{1}{8}$	$1\frac{3}{16}$

For rivets connecting a number of plates, we may take

$$d = \frac{T}{8} + \frac{5}{8} \qquad \ldots \ldots \quad (2)$$

where T is the total thickness of the plates. Rivets of more than $1\frac{1}{8}$ inch diameter are rarely used.

47. *Overlap of plates and pitch of rivets.*—If the rivet hole is too near the edge of the plate, the latter is liable to be burst through in punching; and if the pitch of the rivets, or distance from centre to centre, is too small, a crack extends between them. A practical limit is thus fixed for the minimum overlap and pitch. A good rule is, that the distance from the edge of the rivet hole to the edge of the plate, or to the edge of the next rivet hole, should not be less than the diameter of the rivet. We thus get the proportions shown in fig. 29, as the minimum proportions of overlap and pitch. The unit is the diameter of the rivet.

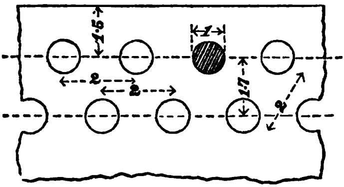


Fig. 29.

Resistance of Riveted Joints.

48. Let d=diameter of rivets.
p=pitch of rivets.
t=thickness of plates.
l=semi-overlap.
f=working tension of material of plates.

Let f_c=working resistance to crushing of plates or rivets.

f_s=shearing resistance of rivets.

T=resistance of a strip of the joint of width p.

Modes of fracture of riveted joints.—Consider, for simplicity, a simple, single-riveted lap joint, subjected to tension. Since each rivet supports a strip of plate, whose width is p, we may consider such a strip, independently of the rest. Such a strip, subjected to tension, might fracture in four ways.

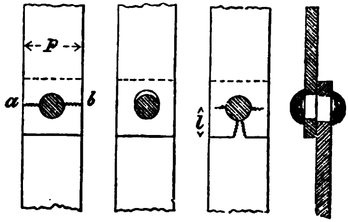


Fig. 30. Fig. 31. Fig. 32. Fig. 33.

(1.) The plate may tear across, along the line of minimum section $a\,b$ (fig. 30). The area of the plate at $a\,b$ is $(p-d)\,t$, and the resistance to tension is $f\,(p-d)\,t$.

(2.) The plate and rivet may be crushed, as shown in fig. 31, and this will render the joint loose and insecure. The area of the plate or rivet supporting the pressure, estimated normally to the pressure, is $d\,t$, and this is called the bearing area. The resistance to crushing is $f_c\,d\,t$.

(3.) The plate may break across in front of the rivet (fig. 32), the action being similar to the transverse fracture of a bar, fixed at the ends, and loaded at the centre. The bending moment is about $\frac{1}{8}\,\mathrm{T}\,d$. Equating this to the moment of resistance of the section of the plate

$$\mathrm{T}=\frac{1}{3}\,\frac{(2\,l-d)^2\,t\,f}{d}.$$

(4.) The rivet may shear across (fig. 33). The area resisting shear is $\frac{\pi}{4}\,d^2$, and the resistance to shearing is $\frac{\pi}{4}\,d^2 f_s$.

Values of the working stress in riveted joints.—For boilers of wrought iron, the greatest safe tensile stress f is taken at 7,500 to 9,000 lbs. per sq. in., which allows a margin against corrosion. In iron bridge and girder work, $f=11,200$ lbs. For steel plates, $f=\frac{4}{3}$ its value for iron. These numbers allow about 10 per cent. for loss of strength, due to the punching of the plates. If the rivets were of the same material as the plates, and if the latter were uninjured in punching, f_s for iron rivets should be taken at $\frac{4}{5}$ths of the safe tensile stress. Rivet iron is, however, somewhat stronger than plate iron, and hence it is better to take f_s, equal to the value of f, for iron. The resistance to crushing is more difficult to assign, because the compressed metal is partially supported by unstrained metal surrounding it. Probably $f_c=1\frac{1}{2}$ to $2f$, where f is the tensile resistance of the iron. The latter value will be chosen in the following calculations. Hence, for iron rivets in iron plates

$$f=f_s=\tfrac{1}{2}f_c\,;$$

for iron rivets in steel plates

$$f_s=\tfrac{3}{4}f\,;\ f_c=1{\cdot}5f.$$

49. *Proportions of joint.*—Equating each of the resistances previously found (§ 48) to T, and inserting the values just assigned to the limiting stress, we get

Iron Plates, Iron Rivets	Steel Plates, Iron Rivets	
$\mathrm{T}=(p-d)\,tf$	$\mathrm{T}=(p-d)\,tf$	(3)
$=2fdt$	$=1{\cdot}5fdt$	(4)
$=\frac{1}{3}\dfrac{(2l-d)^2\,tf}{d}$	$=\frac{1}{3}\dfrac{(2l-d)^2\,tf}{d}$	(5)
$=\cdot785\,d^2f$	$=\cdot588\,d^2f$	(6)

four equations for determining the values of p, d, t, and l.

Theoretical overlap.—From equations (4), (5), and (1)

$$\left.\begin{aligned} l &= \frac{d}{2} + 0{\cdot}92\sqrt{d} \text{ for iron plates} \quad . \quad . \quad . \quad . \\ &= \frac{d}{2} + 0{\cdot}8\sqrt{d} \text{ for steel plates} \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (7)$$

$d=$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	
$l=$	·90	1·04	1·17	1·30	1·42	iron plates
$=$	·81	·94	1·07	1·18	1·30	steel plates
$1{\cdot}5\,d=$	·75	·94	1·12	1·31	1·50	

The minimum value of l in § 47 is $1{\cdot}5\,d$. The values of l, just found, do not differ much from the minimum value, $1{\cdot}5\,d$, previously assigned, but it will be seen that a slight increase above the minimum value is desirable, in some cases, with rivets of the size given in equation (1). As the amount of overlap does not influence any of the other dimensions, it will not be necessary to consider further equation (5).

Single Riveting.

50. *Joint with equal tearing and shearing resistance.*—From equations (3) and (6), we get, for single riveting,

$$\left.\begin{aligned} \frac{p}{t} &= {\cdot}785\left(\frac{d}{t}\right)^2 + \frac{d}{t} \text{ for iron plates} \quad . \quad . \quad . \quad . \\ &= {\cdot}588\left(\frac{d}{t}\right)^2 + \frac{d}{t} \text{ for steel plates} \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (8)$$

If there are n rows of rivets in lap, or $2\,n$ rows in butt riveting, the shearing resistance is increased n times, and the tearing resistance is unaltered. Then,

$$\left.\begin{aligned} \frac{p}{t} &= {\cdot}785\,n\left(\frac{d}{t}\right)^2 + \frac{d}{t} \text{ iron plates} \quad . \quad . \quad . \quad . \\ &= {\cdot}588\,n\left(\frac{d}{t}\right)^2 + \frac{d}{t} \text{ steel plates} \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (9)$$

These proportions are independent of the rule for the size of the rivets, and are applicable, whatever size is chosen.

Theoretical joint of equal tearing, bearing and shearing resistance.

	Iron Plates	Steel Plates	
From (4) and (6)	$d=2{\cdot}55\,t$	$d=2{\cdot}55\,t$	(10)
From (3) and (6)	$p={\cdot}785\,\frac{d^2}{t}+d$	$p={\cdot}588\,\frac{d^2}{t}+d$	
	$=3\,d$	$=2{\cdot}5\,d$	(11)
Efficiency of joint	$\frac{2}{3}$	$\frac{3}{5}$	(12)

This joint is the strongest single-riveted joint which can be constructed, with the assigned limits of working stress, but the rivets are larger than is usual in practice. With the ordinary size of rivets, the three conditions can no longer be exactly fulfilled, but one of the resistances will be in excess, and the efficiency of the joint will be a little diminished. The efficiency of the joint is the ratio of the strength of the joint to that of the solid plate.

Ordinary single-riveted joint.—Equal tearing and shearing resistance. Introducing in the equations (8) the diameter of rivet given in equation (1), we get

	Iron Plates	Steel Plates	
From (1)	$d=1{\cdot}2\sqrt{t}$	$1{\cdot}2\sqrt{t}$	(13)
From (8) and (1)	$p=d+1{\cdot}13$	$d+0{\cdot}85$	(14)
Efficiency of joint $=\frac{p-d}{p}=$	$\frac{1{\cdot}13}{d+1{\cdot}13}$	$\frac{0{\cdot}85}{d+0{\cdot}85}$	(15)

In both cases, the efficiency diminishes as the plates become thicker. The bearing resistance is in excess, if $d>0{\cdot}56$ inches in either case.

The following Table gives proportions calculated by these rules. They agree well with practice for plates of $\frac{3}{8}''$ to $1''$ thickness.

Iron Plates, Iron Rivets				Steel Plates, Iron Rivets			
Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joint	Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joint
$\frac{5}{16}$	·670 = $\frac{11}{16}$	1·82 = $1\frac{13}{16}$	·621	$\frac{5}{16}$	$\frac{11}{16}$	1·54 = $1\frac{9}{16}$	·552
$\frac{3}{8}$	·735 = $\frac{3}{4}$	1·87 = $1\frac{7}{8}$	·606	$\frac{3}{8}$	$\frac{3}{4}$	1·58 = $1\frac{9}{16}$	·538
$\frac{7}{16}$	·790 = $\frac{13}{16}$	1·94 = $1\frac{15}{16}$	·598	$\frac{7}{16}$	$\frac{13}{16}$	1·66 = $1\frac{11}{16}$	·512
$\frac{1}{2}$	·849 = $\frac{7}{8}$	1·98 = 2	·571	$\frac{1}{2}$	$\frac{7}{8}$	1·70 = $1\frac{11}{16}$	·501
$\frac{5}{8}$	·949 = $\frac{15}{16}$	2·08 = $2\frac{1}{16}$	·543	$\frac{5}{8}$	$\frac{15}{16}$	1·80 = $1\frac{13}{16}$	·472
$\frac{3}{4}$	1·04 = $1\frac{1}{16}$	2·17 = $2\frac{3}{16}$	·521	$\frac{3}{4}$	$1\frac{1}{16}$	1·89 = $1\frac{7}{8}$	·450
$\frac{7}{8}$	1·12 = $1\frac{1}{8}$	2·25 = $2\frac{1}{4}$	·500	$\frac{7}{8}$	$1\frac{1}{8}$	1·97 = 2	·431
1	1·20 = $1\frac{1}{4}$	2·33 = $2\frac{5}{16}$	·485	1	$1\frac{1}{4}$	2·05 = $2\frac{1}{16}$	·415

Fig. 34 shows an ordinary single-riveted lap joint. Fig. 35 a similar butt joint. The pitch is given in the Table

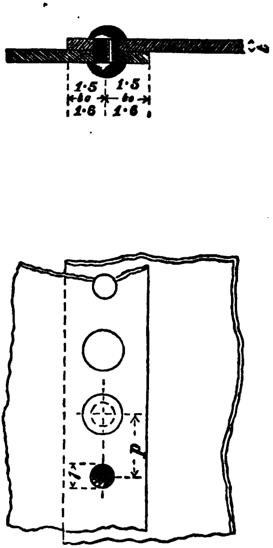


Fig. 34.

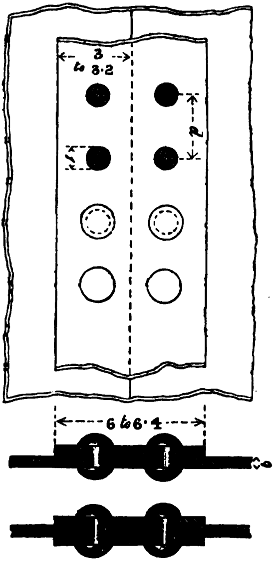


Fig. 35.

above. The other dimensions are given by the proportional

numbers, the unit for which is the diameter of the rivet. The objection to a lap joint is, that the straining force in one plate is not directly opposed to that in the other, but forms with it a couple tending to bend the joint. Grooving of boiler plates is, indirectly, due to this bending action. The objection also applies to the butt joint, with a single cover, when in some positions. The butt joint, with two cover plates, is free from bending action. Butt joints are preferable to lap joints, for the longitudinal joints of boilers, and the cross joints are also sometimes made with a cover strip, welded into a ring and shrunk on.

Double Riveting.

51. For a double-riveted joint, with equal tearing and shearing resistance, equations (3), (4), and (6) become

Iron Plates, Iron Rivets	Steel Plates, Iron Rivets	
$T=(p-d)\,tf$	$T=(p-d)\,tf$	. . . . (16)
$=4fdt$	$=3fdt$	. . . . (17)
$=1{\cdot}57\,d^2f$	$=1{\cdot}176\,d^2f$	. . . (18)

Theoretical joint with equal bearing, tearing and shearing resistance.

	Iron Plates	Steel Plates	
From (17) and (18)	$d=2{\cdot}55\,t$	$d=2{\cdot}55\,t$	. (19)
From (16) and (18)	$p=1{\cdot}57\dfrac{d^2}{t}+d$	$p=1{\cdot}176\dfrac{d^2}{t}+d$	
	$=5\,d$	$=4\,d$	. (20)
Efficiency of joint $=\dfrac{p-d}{p}$	$=\frac{4}{5}$	$\frac{3}{4}$	. . (21)

The objection to this joint is the same as to the corresponding single-riveted joint, that the rivets are larger than

is usual in practice. If the size of the rivets is reduced, the bearing resistance will be in excess.

Ordinary joint equal tearing and shearing resistance.—Using equation (1) for the diameter of the rivets, we get:—

	Iron Plates	Steel Plates	
	$d = 1{\cdot}2\sqrt{t}$	$d = 1{\cdot}2\sqrt{t}$	. (22)
	$p = d + 2{\cdot}26$	$p = d + 1{\cdot}69$	. (23)
Efficiency of joint $= \dfrac{p-d}{p} =$	$\dfrac{2{\cdot}26}{d+2{\cdot}26}$	$\dfrac{1{\cdot}69}{d+1{\cdot}69}$	. (24)

The following Table gives proportions calculated by these rules:—

Iron Plates, Iron Rivets				Steel Plates, Iron Rivets			
Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joint	Thickness of Plates	Diameter of Rivets	Pitch of Rivets	Efficiency of Joint
$\frac{3}{8}$	$\frac{3}{4}$	3	·75	$\frac{3}{8}$	$\frac{3}{4}$	$2\frac{7}{16}$	·69
$\frac{7}{16}$	$\frac{13}{16}$	$3\frac{1}{16}$	·73	$\frac{7}{16}$	$\frac{13}{16}$	$2\frac{1}{2}$	·67
$\frac{1}{2}$	$\frac{7}{8}$	$3\frac{1}{8}$	·72	$\frac{1}{2}$	$\frac{7}{8}$	$2\frac{9}{16}$	·66
$\frac{9}{16}$	$\frac{7}{8}$	$3\frac{1}{8}$	·72	$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{9}{16}$	·66
$\frac{5}{8}$	$\frac{15}{16}$	$3\frac{3}{16}$	·71	$\frac{5}{8}$	$\frac{15}{16}$	$2\frac{5}{8}$	·64
$\frac{3}{4}$	$1\frac{1}{16}$	$3\frac{5}{16}$	·69	$\frac{3}{4}$	$1\frac{1}{16}$	$2\frac{3}{4}$	·61
$\frac{7}{8}$	$1\frac{1}{8}$	$3\frac{3}{8}$	·66	$\frac{7}{8}$	$1\frac{1}{8}$	$2\frac{13}{16}$	·59
1	$1\frac{1}{4}$	$3\frac{1}{2}$	·64	1	$1\frac{1}{4}$	$2\frac{15}{16}$	·57

The efficiency of the iron-plate joint is about 70 per cent., and that of the steel joint about 65 per cent. on the average. The gain of strength in double riveting, as compared with single riveting, is, on the average, 15 per cent. It certainly seems desirable to consider whether somewhat larger rivets could not be used; the efficiency of the joint would then be greater, and its strength would be more nearly equal to that of the plate.

Fig. 36 gives the proportions of a double-riveted joint, the unit being, as before, the diameter of the rivet. The

oblique distance from the centre of rivet in one row to the centre of rivet in the next row, may be equal to the pitch in single riveting, but in any case must not be less than $2\ d$. A butt joint would be similar, but would have four rows of rivets.

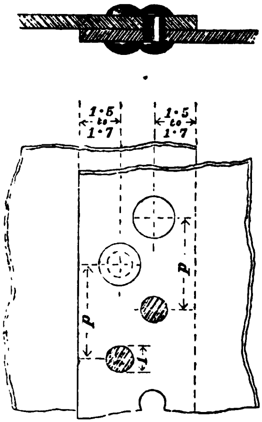


Fig. 36.

52. *Plates with thickened edges.*—To obviate the loss of strength at the riveted joints of boilers, plates with thickened edges have been used. Let t = thickness of plate, t_1 = thickness of edge, e = efficiency of joint. Then if $t_1 = \frac{t}{e}$, the joint will be as strong as the solid plate. The joint must be designed, as if the plate were t_1 inches thick. Only two edges of the plate are usually thickened, and these are placed, so as to form the longitudinal joint, which is subjected to the greatest strain.

53. *Junctions of more than two plates.*—In boiler work, and other cases where the seams are to be watertight, a difficulty arises, where the cross joints intersect the longitudinal joints, because there three plates overlap. At those joints the edges of one or more plates are thinned out by forging, so that the joint may be solid throughout. Fig. 37 shows a joint, at which three plates overlap. The middle plate, $a\ b\ c$, is thinned out. Fig. 38 shows a four-plate connection, where each of the two interior plates thins out at the corner. It will be seen

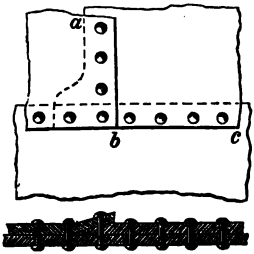


Fig. 37.

that the orged end is so lengthened as to be supported by an additional rivet in the thin part.

A somewhat similar case arises, when boilers are butt riveted, in dealing with the covering strips, at the points where the cross and longitudinal joints meet. In the best boiler work, the plates

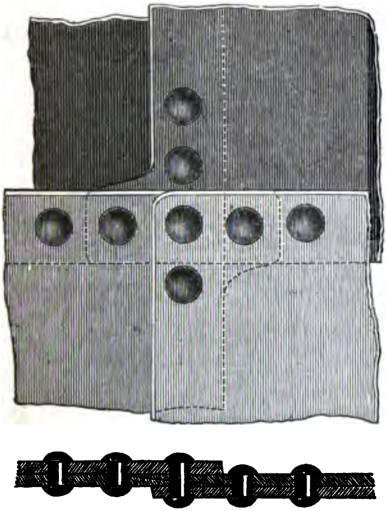


Fig. 38.

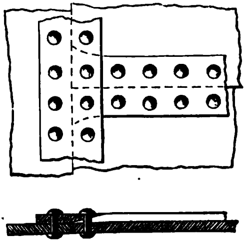


Fig. 39.

are planed at the edges, and fitted together accurately. The longitudinal joints are then riveted up, the covering strip being thinned out at the end of the joint. Lastly, the circumferential covering strip is welded into a hoop of the exact size, and shrunk over the cylinder formed by the plates. Fig. 39 shows the junction of the covering strips.

54. *Connection of plates not in one plane.*—This is commonly effected by the use of a kind of angular joint strip, called an *angle iron.* These angle irons are rolled of a great variety of sizes, and are of very great service in all descriptions of wrought iron work. Fig. 40 shows an angle iron joint. No very definite rule can be given for the size of angle iron to be used, but generally the mean thickness of the angle iron is about equal to, or a little greater than, that

of the plates to be connected. The width of each flange of the angle iron may be about four times the diameter of the

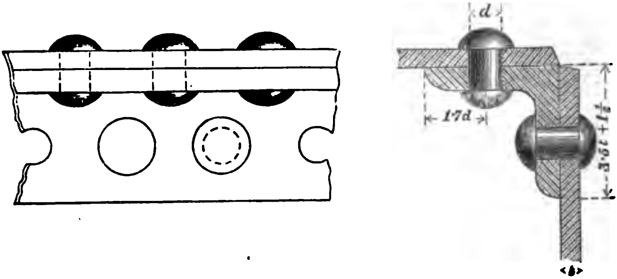


Fig. 40.

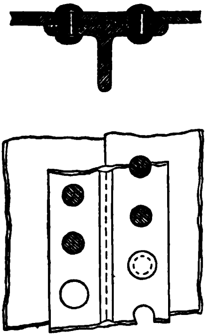


Fig. 41.

rivets used, or may be $3{\cdot}25\,t + 1{\cdot}5$, where t is the thickness of the plates. The angle iron usually tapers, so that it is rather thicker at the root than at the point. In bridge work, where the angle irons are used to confer stiffness, as well as strength, they are often heavier.

Fig. 41 shows a T iron joint, the object being to stiffen the plates against flexure.

Figs. 42, 43, 44, show methods of connecting plates by flanging the

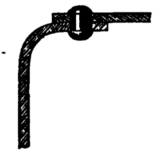


Fig. 42.

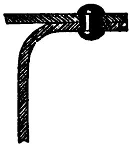


Fig. 43.

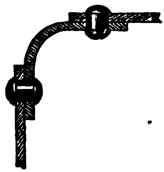


Fig. 44.

plates themselves, instead of using angle irons. This is more expensive, and is impracticable when the plates are

not of good quality. The curvature should not be too sharp. The inside radius may be, at least, four times the thickness of the plates. The width of overlap must be, at least, three times the diameter of the rivet.

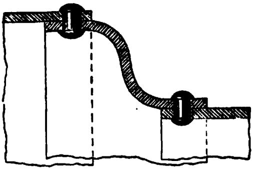


Fig. 45.

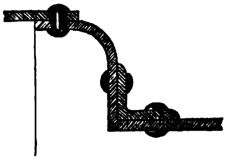


Fig. 46.

Figs. 45, 46, show joints used at the junction of the cylindrical barrel of locomotive boilers with the external fire-box.

55. *Connection of parallel plates.*—A case which frequently occurs, is where two plates, near together, require to be connected. For instance, at the bottom of the fire-box of locomotives, a connection has to be made between the inner and outer fire-box. The following sketches show how this may be effected.

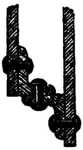


Fig. 47.

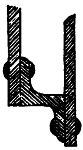


Fig. 48.

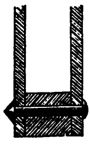


Fig. 49.

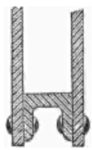


Fig. 50.

Fig. 51.

In fig. 47 there are two angle irons. This is rather complicated, and there are inside joints, which cannot be caulked. Fig. 48 is simpler, but has an inside joint, which cannot be caulked. Fig. 49 is an admirable joint, and is formed by what is termed a channel iron. But it is difficult to bend the channel iron round the corners of the fire-box.

Fig. 50 is simple, but forms a corner for the lodgment of sediment. Fig. 51 is the form most commonly used.? 49

Elliptical rivets.—Since the efficiency of the joint is the ratio $\frac{p-d}{p}$ of the distance between the holes to the pitch, we may increase the efficiency, by using rivets of elliptical section. With such rivets, placed with their least breadth in the line of fracture of the plates, the quantity $p-d$ would be greater, while the shearing section remained the same. Such rivets have been used by Mr. Webb.† By adopting the elliptical form, two variables, the axes of the ellipse, take the place of the single variable d, in the equations. It would thus be possible to satisfy the conditions of equal bearing, tearing, and shearing resistance, for rivets of any desired section. To do this, we should merely have to take, for one axis of the ellipse, the diameter of the rivet in the theoretical joint. The other axis would then be found for the given sectional area. Let d_1 be the diameter of rivet in the formula for the theoretical joint, and let a and b be the major and minor axes of an elliptical rivet, whose section $=\omega$. Then $b=d_1$, and $a=4\,\omega\div\pi\,b$.

56. *Position of rivets in tie bars and struts.*—When a bar, subjected to a longitudinal straining force, is attached at each end by a single rivet or pin, the rivets should be placed on the centre line of the bar. It is a fair assumption, and must be nearly true, that the straining force acts through the centre of the rivet. Hence, if the rivets are in the centre line of the bar, the resultant straining force passes through the axis of the bar, and the stress on each transverse section is uniform. If the rivets are not so placed, one side of the bar is more strained than the other, and gives way before the other has fully exerted its powers of resistance. When there are several rivets at each end of a bar, they should, for the same reason, be placed symmetrically on either side of the axis, and as uniformly distributed as possible over the area in which they are placed.

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If they cannot be placed symmetrically, an approximation is made to the best conditions, by arranging them, so that their common centre of gravity falls on the axis of the bar. In that case, if each rivet supports the same fraction of the load, the resultant force will still pass through the axis of the bar.

57. *Taper and curvature of boiler plates.*—When a boiler, boiler flue, or other cylindrical structure, is made up of slightly conical rings, which are slipped over each other to

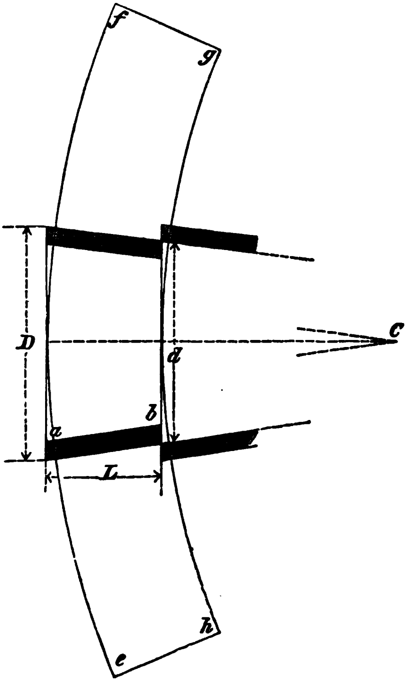


Fig. 52.

form the overlap, fig. 52, the joints being what are technically termed 'following' joints, the plates, instead of being rectangular, must be portions of the development of a cone.

Let D be the greater, and d the less, diameter of the conical frustum, and L its length; t, the thickness of the plates. Then $d=\text{D}-2t$ very nearly. The development of the frustum is an annular segment $efgh$, drawn with radii, $\text{R}=\text{c}a=\text{c}e$, and $r=\text{c}b=\text{c}h$, and whose lengths, measured along the arcs ef and hg, are πD and πd. Since the incli-

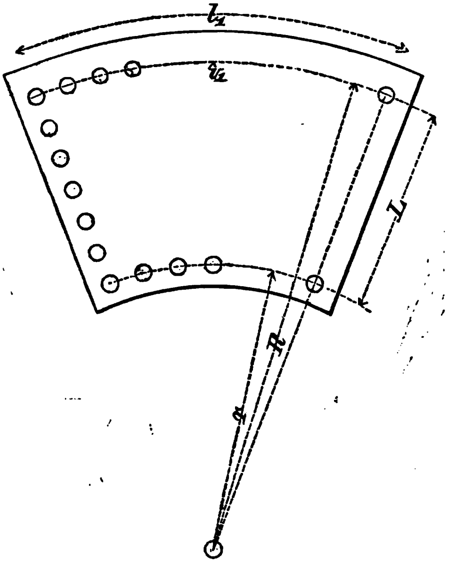


Fig. 53.

nation of the cone's sides is small, $\text{R}=\frac{\text{D L}}{\text{D}-d}=\frac{\text{D L}}{2t}$ nearly, and

$r=\frac{d\text{L}}{\text{D}-d}=\frac{d\text{L}}{2t}$ nearly.

For a boiler plate, let l_1, fig. 53, be the distance between longitudinal seams, measured at the larger end of cone, so that, if there are n plates in each ring, $l_1=\frac{\pi\text{D}}{n}$. Let L be the dis-

tance between the cross seams; v_1 and v_2, the versed sines of the arcs, formed by the rivets when developed.

$$R=\frac{D\,L}{2\,t} \qquad r=\frac{D\,L}{2\,t}-L$$

$$v_1=\frac{l_1^2\,t}{4\,D\,L}$$

$$v_2=\frac{l_1^2\,(D-2\,t)\,t}{4\,D^2\,L}$$

With these dimensions the centre lines of the rivets can be set out, and, if then, the width of overlap is added all round, the size of the plate is determined.

58. *Boiler stays* are fastenings analogous to rivets, which support flat surfaces. Those most commonly used, are of the form shown in fig. 54, which is a drawing of a copper stay for a locomotive fire-box. The stay is screwed through the two plates, which are connected, and then riveted over. Such stays are of wrought iron, of copper, or of steel. Copper has been most used, but iron is now often substituted for it. Such stays are rather liable to break across at the screw thread, inside the plates. To give warning of such a fracture, a small hole, $\frac{1}{8}$ in. or $\frac{3}{16}$ in. diameter, is sometimes drilled into the stay. If fracture occurs, warning is given by the leakage which ensues.

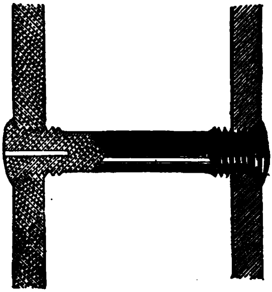


Fig. 54.

The stays are arranged equi-distant over the whole surface supported. If, therefore, D is the distance of the stays, centre to centre, and p the steam pressure in lbs. per sq. in., each stay supports D^2 sq. ins., and resists a pressure of $p\,D^2$ lbs. In locomotives, the stays are very

commonly $\frac{7}{8}$ in. diameter, and 4 to $4\frac{1}{2}$ ins. apart. The following rules are applicable to locomotives:

$$\left.\begin{array}{lr} \text{Thickness of plates, iron,} \quad t = \cdot 01\,D\sqrt{p} + 0 \cdot 1 & . \\ \quad " \qquad " \qquad \text{copper} = \cdot 012\,D\sqrt{p} + 0 \cdot 1 & . \end{array}\right\} \quad (25)$$

$$\left.\begin{array}{lr} \text{Diameter of stay, copper,} \quad \cdot 018\,D\sqrt{p} + \frac{1}{8} \quad . & . \\ \quad " \qquad " \qquad \text{iron,} \quad \cdot 006\,D\sqrt{p} + \frac{1}{8} \quad . & . \end{array}\right\} \quad (26)$$

The diameter is measured outside the screw thread.

CHAPTER V.

ON FASTENINGS.

Bolts, Nuts, Keys, and Cotters.

59. Bolts are chiefly used to resist straining forces, acting parallel to the axis of the bolt, and normal to the surfaces held together. When in shear, they are subject to the same rules as rivets. Sometimes screws are used, not as fastenings, but to transmit motion.

For manufacturing reasons, it is important that some common agreement should be arrived at, as to the form and dimensions of screws. Sir J. Whitworth first proposed a uniform system of screw threads, which is adopted universally in this country for all the more important parts of machines. For wrought-iron gas tubes, and for the cheaper kinds of metalwork, a finer pitched screw thread, known as the gas thread, is used. In America, Mr. Sellers has introduced a uniform system, very similar to Whitworth's.

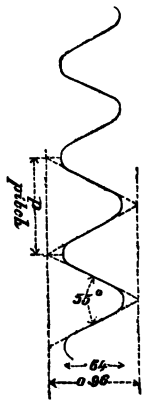


Fig. 55.

Pitch and form of screw threads.—Most commonly, screw threads are triangular in section, as shown in fig. 55, which represents the standard Whitworth thread. About $\frac{1}{6}$th of the depth of the thread is rounded off at both top and bottom, to facilitate the cutting of the screw, and to render it less liable to injury. Screws with rectangular threads, or so-called square threads, are sometimes adopted, fig. 56 *c*, especially when the screw is used to transmit motion. The

surface of the square thread is normal to the axis of the screw, and hence there is, with this form of thread, no oblique or bursting pressure on the nut, and the thread wears less. Two other forms of thread are occasionally used. Screws subjected to rough usage are of the form shown at *a*, fig. 56, the thread being similar to the square

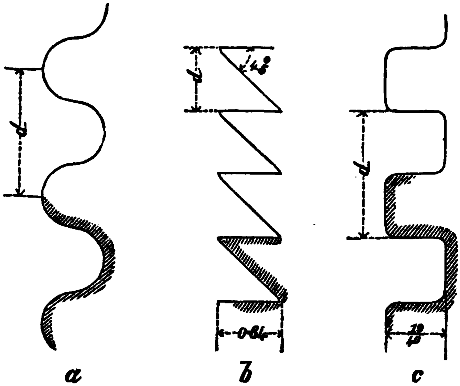


Fig. 56.

thread, with the angles much rounded. When a screw has to resist a pressure, acting always in one direction, the form shown at *b* is sometimes used. The face bearing the pressure is normal to the axis, as in the square thread; but the shearing resistance of the thread is twice as great as that of the square thread.

The pitch of screws is fixed by practical experience, so as to be suitable for cast and wrought iron. The pitch and number of threads per inch, as arranged by Whitworth for the different diameters of screws, are given in a table below.

The following formulæ give values nearly the same as those in the Tables :—

Whitworth Threads.

	Triangular Threads	Square Threads	
Pitch $= p =$	$0{\cdot}08\,d + 0{\cdot}04$	$0{\cdot}16\,d + 0{\cdot}08$	. . (1)

$$\text{Number of threads per inch} = \frac{1}{p},$$

Diameter at bottom of thread

$$\left.\begin{aligned} &= d_1 = d - \frac{1{\cdot}28}{n} = 0{\cdot}9\,d - 0{\cdot}05, \text{ for triangular threads} \quad . \\ &= d - \frac{3{\cdot}8}{n} = 0{\cdot}85\,d - 0{\cdot}075, \text{ for square threads} \quad . \end{aligned}\right\} \quad (2)$$

A square-threaded bolt is, therefore, slightly weaker, in tension, than a triangular-threaded bolt.

Fig. 55 shows the method of designing a Whitworth thread. Two parallel lines are drawn, $0{\cdot}96\,p$ apart. These are intersected by lines, inclined at 55°. Lastly, $\frac{1}{6}$th of the depth of the triangular spaces so obtained, is rounded off, both at top and at bottom. The square thread has usually twice the pitch of a triangular thread of the same diameter and the depth of the thread is $\frac{19}{40}$ of the pitch.

Whitworth Screws.

Diam. of Screw	Number of threads per in.	Pitch in inches	Diam. at bottom of thread	Safe working strength for wrought iron	Diam. of Screw	Number of threads per in.	Pitch in inches	Diam. at bottom of thread	Safe working strength
d.	n.	p.	d_1.	lbs.	d.	n.	p.	d_1.	lbs.
$\frac{3}{16}$	24	·041	·136	58	2	$4\frac{1}{2}$	·222	1·716	9280
$\frac{1}{4}$	20	·050	·186	109	$2\frac{1}{4}$	4	·250	1·966	12200
$\frac{5}{16}$	18	·056	·241	182	$2\frac{1}{2}$	4	·250	2·180	14920
$\frac{3}{8}$	16	·063	·295	274	$2\frac{3}{4}$	$3\frac{1}{2}$	·286	2·430	18560
$\frac{7}{16}$	14	·071	·347	378	3	$3\frac{1}{2}$	·286	2·634	21720
$\frac{1}{2}$	12	·083	·394	448	$3\frac{1}{4}$	$3\frac{1}{4}$	·308	2·884	26240
$\frac{5}{8}$	11	·091	·509	817	$3\frac{1}{2}$	$3\frac{1}{4}$	·308	3·106	30360
$\frac{3}{4}$	10	·100	·622	1208	$3\frac{3}{4}$	3	·333	3·356	35440
$\frac{7}{8}$	9	·111	·733	1674	4	3	·333	3·574	40040
1″	8	·125	·840	2217	$4\frac{1}{4}$	$2\frac{7}{8}$	·348	3·824	45840
$1\frac{1}{8}$	7	·143	·942	2776	$4\frac{1}{2}$	$2\frac{7}{8}$	·348	4·055	51640
$1\frac{1}{4}$	7	·143	1·067	3597	$4\frac{3}{4}$	$2\frac{3}{4}$	·364	4·305	58240
$1\frac{3}{8}$	6	·167	1·192	4448	5	$2\frac{3}{4}$	·364	4·534	60560
$1\frac{1}{2}$	6	·167	1·286	5200	$5\frac{1}{4}$	$2\frac{5}{8}$	·381	4·764	71280
$1\frac{5}{8}$	5	·200	1·411	6240	$5\frac{1}{2}$	$2\frac{5}{8}$	·381	5·014	78960
$1\frac{3}{4}$	5	·200	1·494	6960	$5\frac{3}{4}$	$2\frac{1}{2}$	·400	5·238	86640
$1\frac{7}{8}$	$4\frac{1}{2}$	·222	1·619	8240	6	$2\frac{1}{2}$	·400	5·488	94520

A Table of the number of threads for screws of intermediate decimal sizes will be found in Shelley's 'Workshop Appliances,' p. 103.

Gas threads.—For the wrought-iron tubes used for conveying gas, the Whitworth screw thread is not suitable. For these a special system of threads has been adopted, finer in pitch, and cutting less deeply into the metal of the tube, than the Whitworth thread would do.

Diameter in inches	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2
Pitch in inches	·036	·053	·053	·071	·071	·091	·091	·091
Nos. of threads per inch	28	19	19	14	14	11	11	11

Sellers's screw threads.—In the American, or Sellers's system, of screw threads, the sides of the thread are inclined at 60°, and a depth, equal to $\frac{1}{8}$ of the pitch, is cut square off at top and bottom of the thread. The pitch is

$$p = 0{\cdot}1\, d + {\cdot}025 \text{ nearly} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1\,a)$$

$$d_1 = d - \frac{1{\cdot}299}{n} = 0{\cdot}87\, d - {\cdot}03 \quad . \quad . \quad . \quad . \quad . \quad (2\,a)$$

60. *Strength of screw bolts.*—Putting P for the axial straining force acting on the bolt; d_1=diameter at bottom of thread; f=working stress per unit of area.

$$P = \frac{\pi}{4} d_1^2 f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and replacing d_1 by d, by means of equation (2),

$$\left.\begin{aligned} d &= {\cdot}055 + 1{\cdot}127\sqrt{\left(\frac{P}{f}\right)}, \text{ for triangular threads} \quad . \\ &= {\cdot}085 + 1{\cdot}32\sqrt{\left(\frac{P}{f}\right)}, \text{ for square threads} \quad . \end{aligned}\right\} (4)$$

This is a slightly improved form of the formula commonly used in determining the size of bolts. In order that it may be strictly applicable, the bolt ought to be subjected to an axial force only; but, in consequence of the obliquity of the bearing surface of the thread, the pressure between the bolt and nut has a component tending to twist the bolt; hence the stress on it is really a combined torsion and

tension, and not a simple tension. It is also probable, that at the section of the bolt at the bottom of a thread, the stress is not quite uniformly distributed. There is there a rapid change of section, and the stress is probably greater at the angle of the thread, and less in the interior of the bolt than the mean value. We may allow, roughly, for both the above causes of increased straining action, by taking for f a value less than that suitable for simple tension.

In many cases, however, there is an uncertainty in the determination of the load P. One part of the load, which may be termed the effective load, is ascertainable with tolerable accuracy. Another part, which is due to the force used in tightening the nut, and the amount of which depends on the skill and care of the workman, is less easily determined. If the nut is not screwed up before the load comes upon it, or, if it is screwed up, provided the connected pieces are not in actual contact, the effective load alone needs to be considered. On the other hand, if the bolt is screwed up, so as to develop a reaction between the connected pieces, this additional load must be estimated as forming part of the total load P. Practical experience shows that we may *roughly* allow for the difference of the action in these cases thus :—For press screws, and other bolts, which do not require to be tightened before the load comes upon them, f may be taken at 6,000 lbs. per sq. in. For accurately-fitted bolts, requiring to be tightened moderately, $f=4{,}000$. But for bolts which are used to draw joints steam-tight, and which must be severely tightened before the steam pressure begins to act, f ought not to exceed 1,600 or 2,000 lbs. per sq. in.

With these low values of f, P is the effective load only. The value of the working stress, in the Table above (p. 81), has been taken at 4,000 lbs., in calculating the strength of the bolts.

Strength of bolts, taking torsion into account.—Suppose that a load P is suspended from a square-threaded screw,

and that it is screwed up, without, however, bringing the parts connected by the screw into actual contact, so as to develop a reaction between them, additional to the load. In that case, the friction of the nut on its support does not affect the stress in the bolt, and there is a definite relation between the load P, and the twisting force Q, applied to the bolt. Let fig. 57 represent a screw thread at its mean

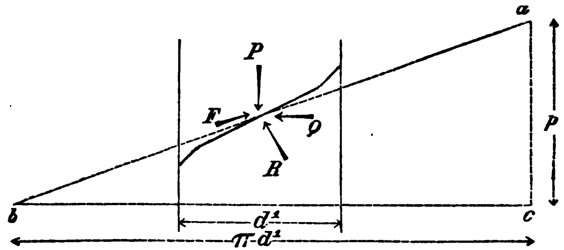


Fig. 57.

diameter d^1. The forces, acting between any two elements of the bolt and nut, are a vertical force P, due to the load; a horizontal force Q, due to the pull on the spanner; a reaction R, normal to the thread; and a friction F, parallel to the thread.

The obliquity of the thread is the same as that of its development $a\,b$ on a plane surface, which makes, with the horizontal, an angle whose tangent is $p \div \pi\, d^1$. Neglecting the small difference between the mean and outside diameters, d and d^1 of the thread, we get

$$Q = P\frac{p + \mu \pi d}{\pi d - \mu p} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where μ is the co-efficient of friction. If the thread is triangular instead of square, the normal reaction is greater, in the ratio of the slant length of a thread to its half thickness at the root. Hence, for triangular threads

$$Q = P\frac{p + 1{\cdot}15\, \pi \mu a}{\pi d - 1{\cdot}15\, \mu p} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The twisting moment of the force Q, acting at nearly $\frac{d}{2}$ from the axis, is Q $\frac{d}{2}$. Putting $\mu = 0 \cdot 15$, and using the previously-found values for the pitch, we get

Twisting moment $= M = 0 \cdot 2\ P\ d$, nearly, in either case. The greatest stress due to combined torsion and tension, is then—

$$f = \frac{4\ P}{\pi\ d_1^2}\left\{\tfrac{1}{3} + \tfrac{2}{3}\sqrt{\left(\frac{2\ M}{P\ d}\right)^2 + 1}\right\}$$

$$= 1 \cdot 339\ \frac{P}{d_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Putting for f, the safe working stress, and replacing d_1 by d, we get

$$\left.\begin{aligned} d &= \cdot 055 + 1 \cdot 285\sqrt{\frac{P}{f}}, \text{ for triangular threads} \\ &= \cdot 085 + 1 \cdot 361\sqrt{\frac{P}{f}}, \text{ for square threads} \end{aligned}\right\} \quad (8)$$

Comparing these with equations (4), in obtaining which the twisting moment was neglected, we see that the twisting moment adds about 15 per cent. to the diameter necessary for the bolt.

Strength of bolts when the initial tension, due to screwing up, is taken into account.—A nut is screwed up by means of a spanner, whose leverage is, on the average, 15 *d*. Suppose that a nut is screwed up by a force Q, applied to the spanner at radius R; the tension P_1, produced in the bolt, being expended in compressing the pieces connected by the bolt. The friction of the nut, on its seat, balances part of the force Q. That friction acts approximately at a radius $\frac{2}{3}\,d$, and its magnitude is $\mu_1\ P_1$ lbs. Hence

$$P_1 = \frac{Q}{\frac{2}{3}\mu_1 + \frac{1}{2}\frac{p + \mu\ \pi\ d}{\pi\ d - \mu\ p}} \cdot \frac{R}{d} \quad . \quad . \quad . \quad . \quad (9)$$

Taking $\mu_1 = \mu = 0{\cdot}15$, for a turned nut on a turned washer; $R = 15\,d$; $p = 0{\cdot}16\,d$, we get

$$P_1 = 82\,Q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9\,a)$$

The force applied by a workman in screwing up a bolt, will vary with the size of the bolt, and his experience teaches him in what cases a heavy pressure may be applied with safety. Taking the heaviest ordinary pull of the workman to be $Q = 30$ lbs.:

$$P_1 = 82 \times 30 = 2{,}460 \text{ lbs.}$$

a force sufficient to break a $\frac{3}{8}$-in. bolt, and to seriously injure a $\frac{1}{2}$-in. bolt. Hence, bolts of less than $\frac{3}{4}$-in. diameter are not used for joints requiring to be tightly screwed up.

In most cases, where a joint is made tight by bolting, the total load P is the effective load P_2, plus the additional load P_1, due to screwing up. Taking, then, the total load $= P = P_1 + P_2 = P_2 + 82\,Q$

$$\left.\begin{aligned} d &= 0{\cdot}055 + 1{\cdot}28\sqrt{\left(\frac{82\,Q + P_2}{f}\right)}, \text{ for triangular threads} \\ &= 0{\cdot}085 + 1{\cdot}35\sqrt{\left(\frac{82\,Q + P_2}{f}\right)}, \text{ for square threads} \end{aligned}\right\} (10)$$

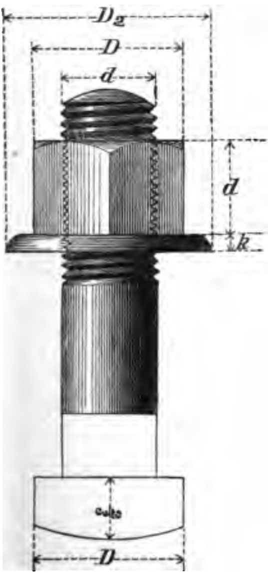
where, for Q, is to be put the assumed value of the force used in screwing up.

In the following Table, an empirical allowance is made for the stress due to screwing up, increasing with the size of the bolt. The Table should not supersede the exercise of judgment in particular cases. The real stress is taken at $f = 9{,}000$ lbs. The pull on the spanner is taken at about 10 lbs. for each 1,000 lbs. on the bolt, and the twisting moment is taken into account.

Strength of Bolts screwed up tightly.

Diameter of Bolt	Assumed load due to screwing up	Effective strength	Diameter of Bolt	Assumed load due to screwing up	Effective strength	Diameter of Bolt	Assumed load due to screwing up	Effective strength
d	82 Q	P_2	d	82 Q	P_2	d	82 Q	P_2
$\frac{1}{2}$	1000	43	$1\frac{1}{4}$	2700	4954	$2\frac{1}{2}$	5000	28570
$\frac{5}{8}$	1200	600	$1\frac{3}{8}$	3000	6552	$2\frac{3}{4}$	5000	36760
$\frac{3}{4}$	1500	1100	$1\frac{1}{2}$	3300	7810	3	5000	43870
$\frac{7}{8}$	1800	1812	$1\frac{3}{4}$	3600	11410	$3\frac{1}{4}$	5000	53590
1	2100	2643	2	4000	15790	$3\frac{1}{2}$	5000	63310
$1\frac{1}{8}$	2400	3567	$2\frac{1}{4}$	4500	21480	4	5000	85090

61. *Proportions of bolts and nuts.*—Fig. 58 shows the most ordinary type of bolt, nut and washer. The bolt has a square head, and a square neck, to prevent the rotation of the bolt, while the nut is being screwed up. The nut is hexagonal, and the washer circular. The washer is used when the bolt connects rough castings, and then forms a smooth seating, on which the nut turns. It



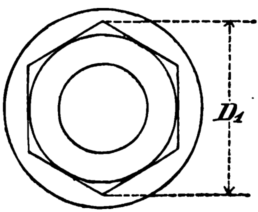


Fig. 58.

is sometimes used for appearance only. The following rules give good proportions :—

Hexagon Nuts.

Diameter across flats $= D = 1{\cdot}5\,d + 0{\cdot}18$ to $1{\cdot}5\,d + 0{\cdot}44$ (rough)

$1{\cdot}5\,d + 0{\cdot}06$ to $1{\cdot}5\,d + 0{\cdot}18$ (bright).

Diameter across angles $= D_1 = 1{\cdot}75\,d + 0{\cdot}16$ to $1{\cdot}75\,d + 0{\cdot}4$ (rough),

$1{\cdot}75\,d + 0{\cdot}07$ to $1{\cdot}75\,d + 0{\cdot}2$ (bright).

$D = 1{\cdot}5\,d + 0{\cdot}18$, and $D_1 = 1{\cdot}75\,d + 0{\cdot}16$, are very nearly Whitworth's standard sizes for finished nuts. In drawings, on a small scale, it is accurate enough to take $D_1 = 2\,d$.

Height of nut $= d$.

Height of lock nut $= \frac{d}{2}$.

Square Nuts.

Diameter across flats $= 1{\cdot}5\,d + 0{\cdot}18$ to $1{\cdot}5\,d + 0{\cdot}44$ (rough),

$= 1{\cdot}5\,d + 0{\cdot}06$ to $1{\cdot}5\,d + 0{\cdot}18$ (bright).

Diameter across angles $= 2{\cdot}12\,d + {\cdot}25$ to $2{\cdot}12\,d + 0{\cdot}6$ (rough),

$= 2{\cdot}12d + 0{\cdot}08$ to $2{\cdot}12d + 0{\cdot}25$ (bright).

The head of the bolt may be square, hexagonal, or circular. Its height is $\frac{2}{3}\,d$ to d.

Length of spanner $= 15\,d$ to $18\,d$.

Washers.

Thickness $0{\cdot}15\,d$; diameter $\frac{9}{8}\,D_1$.

Small washers are usually 14 B.W.G., or 0·083 in. thick.

Washers, for wood, may be $3\,d$ in diameter, and $0{\cdot}3\,d$ in thickness.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Size of hole before tapping	$\frac{3}{16}$	$\frac{21}{64}$	$\frac{29}{64}$	$\frac{17}{32}$	$\frac{43}{64}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{31}{32}$	$1\frac{7}{64}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{3}{4}$
D	$\frac{17}{32}$	$\frac{11}{16}$	$\frac{29}{32}$	$1\frac{1}{8}$	$1\frac{5}{16}$	$1\frac{17}{32}$	$1\frac{23}{32}$	$1\frac{5}{16}$	$2\frac{3}{32}$	$2\frac{7}{16}$	$2\frac{25}{32}$	$3\frac{9}{64}$
D_1	$\frac{1}{2}$	$\frac{25}{32}$	$1\frac{1}{32}$	$1\frac{9}{32}$	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{15}{16}$	$2\frac{1}{4}$	$2\frac{3}{8}$	$2\frac{13}{16}$	$3\frac{5}{32}$	$3\frac{5}{8}$

Weight of Bolts and Nuts in Pounds.

	Diameter of Bolt											
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
Weight per inch of bolt .	·017	·034	·057	·087	·124	·17	·22	·28	·34	·49	·68	·88
Weight of square head .	·017	·044	·088	·126	·260	·397	·544	·694	·943	1·667	2·535	3·571
Weight of nut and part of bolt included . .	·022	·0530	·110	·172	·324	·494	·681	·873	1·188	2·088	2·954	4·387
Weight of washer and part of bolt included . .	·011	·0154	·039	·048	·096	·132	·161	·178	·273	·489	·723	1·014

62. *Different forms of nuts.*—Ordinary nuts are chamfered off at an angle of 30° to 45°, as shown at *a*, fig. 59 ; or they are finished with a spherical bevil, struck with a radius of about 2 *d*, as shown at *b*. *Flange nuts*, *c*, are used when the hole, in which the bolt is placed, is considerably larger than the bolt itself. The flange covers and hides the hole. *Cap nuts*, *d*, are used where leakage along the screw thread

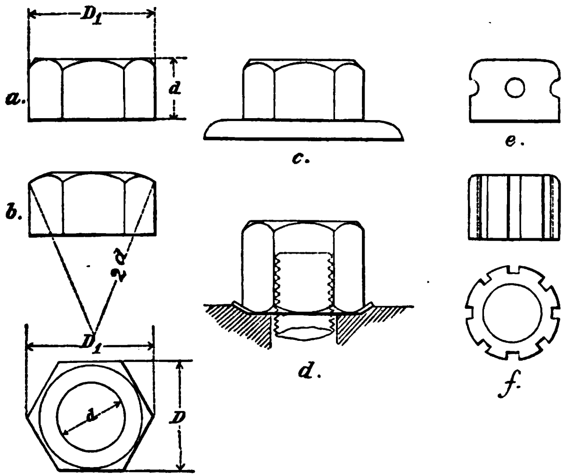


Fig. 59.

is feared. In the figure, a thin, soft copper washer is shown, which prevents leakage under the nut. *Circular nuts*, *e*, are occasionally used. They have holes, in which a bar, termed a 'Tommy,' is placed, for screwing them up. Sometimes grooves are cut, as shown at *f*. Steel nuts may be used, if great durability is required.

63. *Different forms of bolt heads.*—In fig. 60, *a* is a cup-shaped, *b* a countersunk, and *c* a square bolt-head. Rotation of the bolt is prevented in *a* by a square neck, in *b* by a set screw, in *c* by a snug forged on the bolt. Fig. 61 shows

a T-headed bolt in front and side elevation. Fig. 62 shows an eye bolt. Fig. 63 shows a spherical-headed bolt used,

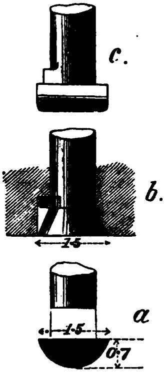


Fig. 60.

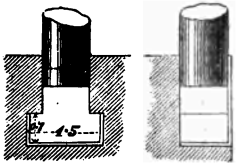


Fig. 61.

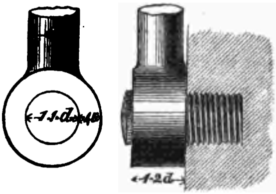


Fig. 62.

sometimes, for railway fastenings, with a square neck. The spherical head allows the bolt to take a fair bearing on the rail. The other figure shows a cup-head, with a snug forged

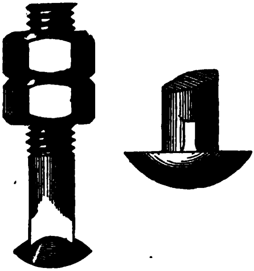


Fig. 63.

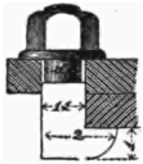


Fig. 64.

Fig. 65.

on the bolt, to prevent rotation when the bolt is screwed up. Proportional unit $= d$, in all these figures.

Fig. 64 is a *hook bolt*, which is used when one piece is too small to have a bolt hole through it, or when it is objectionable to weaken the piece by a bolt hole. Fig. 65

is a *stud*, which is screwed into one of the connected pieces, and remains in position when the nut is removed. Fig. 66 is a *set screw*, or bolt not requiring a nut. Fig. 67 shows a nut-headed bolt, or bolt having two loose nuts, instead of a

Fig. 66.

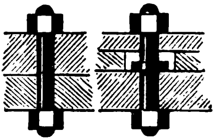


Fig. 67.

nut and head. The second figure is a similar bolt, with an intermediate head or flange.

Fig. 68 is a bolt leaded into stone work. The tail of the bolt is rectangular, with jagged edges.

Fig. 69 is a fang bolt used for attaching ironwork to wood, and especially for attaching rails to sleepers. The fangs of the broad triangular plate, which forms the nut, bite into the wood, while the bolt is rotated by the

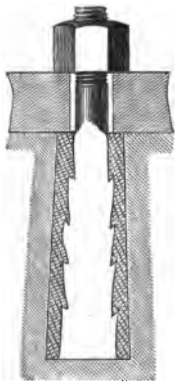


Fig. 68.

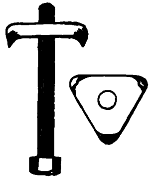
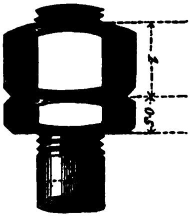


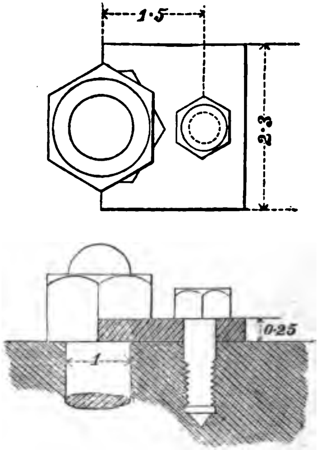
Fig. 69.

head, which bears on the ironwork. The large area of the nut prevents crushing of the wood.

64. *Locking arrangements for nuts* are intended to prevent the gradual unscrewing of nuts, subjected to vibration

and frequent changes of load. No nut accurately fits its bolt; a certain amount of play, however minute, always exists. When a nut, having play, is subjected to vibration, it gradually slacks back. This is, to a great extent, prevented by double nuts, shown in fig. 70. One of the nuts is termed a lock nut, and is usually half as thick as the ordinary nut. When there are two nuts, the whole load may be thrown on the outer nut. The outer nut ought, therefore, to be the thicker nut. It is common in practice to put the thinner nut outside, the reason being, that ordinary spanners are sometimes too thick to hold the thin nut, when screwed





Unit $= d$.

Fig. 70. Fig. 71.

home first. The more correct arrangement is that shown in the figure.

Another plan is to drill a hole through the top of the bolt above the nut, and drive a split pin or cotter through. The nut must always be in the same place when screwed up. A better plan is shown in fig. 71, a stop plate being used, fixed on one side of the nut. The set screw in the stop plate may have its diameter $=\frac{1}{4}d+\frac{1}{8}$.

A very neat arrangement is shown in fig. 72; the lower part of the nut is turned circular, and fits in a recess in the piece connected by the bolt. A set screw is tapped through,

and bears on the side of the nut. The diameter of the set screw may be $\frac{1}{8}d+\frac{1}{8}$. A stop ring is sometimes used, fig. 73, with a set screw tapped through it. The stop ring

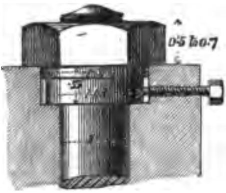


Fig. 72.

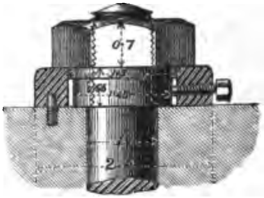


Fig. 73.

is of brass, or wrought iron, and it is prevented from turning by a stop pin of the same size as the set screw.

Elastic washers have been used as substitutes for lock nuts. Fig. 74 shows Grover's spring steel washer. When the nut is tightened up, the washer becomes nearly, but not quite, flat, and its elasticity neutralises the play of the nut on the bolt.

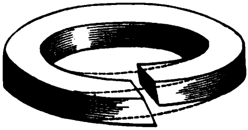


Fig. 74.

65. *Bolting of cast-iron plates.*—Cast-iron plates are united by bolts; flanges, to receive the bolts, are cast on the plates, and these may be external or internal. The flanges are of the same thickness as the plates, or a little thicker. The bolts are never less than $\frac{3}{4}$ in. in diameter, and the

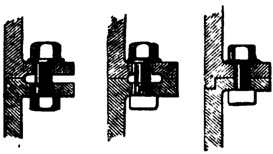


Fig. 75.

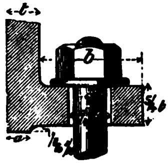


Fig. 76.

bolt diameter may be equal to the flange thickness. The fitting part of the flanges is often a narrow 'chipping strip,' which is faced by hand, or in the planing machine.

Fig. 75 shows three arrangements of the flanges and bolts. Fig. 76 gives the ordinary proportions of the bolt and flange.

Bolt diameter $= d = \frac{5}{4}t + \frac{1}{8}$ (but not less than $\frac{3}{4}$ in.).

Pitch of bolts about $6\,d$, or less, if necessary for strength.

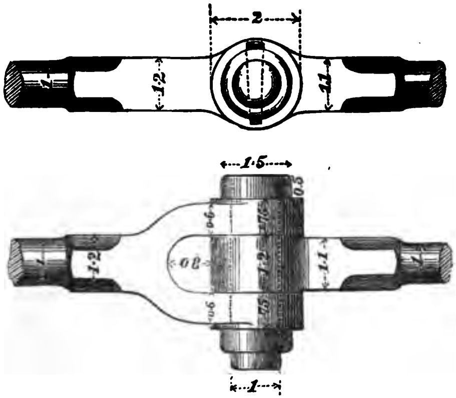
Width of chipping strip $= a = \frac{5}{4}t$.

Width of flange $= b = 2\,d + \frac{3}{4}$.

The open space between the flanges is sometimes filled with rust cement.

JOINT PINS. KNUCKLE JOINT.

66. A joint pin is a kind of bolt, so placed as to be in shear. Fig. 77 shows an arrangement known as a knuckle



Scale ¼

Unit = diam. of pin.

Fig. 77.

joint. The proportions are empirical. If the joint pin were subjected to simple shear at two sections, it would be strong enough, when its diameter was equal to 0·7 of the diameter

of the rods. But the pin wears, and is then subjected to bending, as well as shearing. When there is much motion at the joint, the width of the eyes of the rods, and the length of the pin, may be increased.

KEYS.

67. Keys are used for fixing wheels, pulleys, cranks and other pieces on shafts. They are tapered longitudinally, and are driven home very tightly into the recesses formed to receive them. The resistance to shearing $= b\,l\,f_s$, where f_s is the working shearing stress. The bearing surface of the key on the sides of the recess, is approximately $\frac{1}{2}\,t\,l\,f_c$, where f_c is the resistance to crushing. In order that the

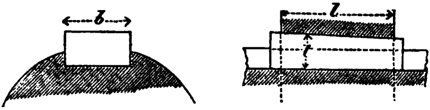


Fig. 78.

shearing and crushing resistance should be equal, when $f_c = 2f_s$, we must have $t = b$, or the key should be square in section. For practical reasons, however, the breadth of the key is increased, and its shearing resistance is usually in excess.

Saddle keys, fig. 79, *A*, are used for fixing light pulleys to shafts. In this case, the resistance to slipping is due to

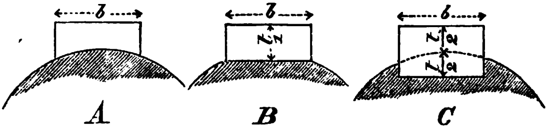


Fig. 79.

friction only. Hence, these keys are only used when the force transmitted is small. Keys on flats, fig. 79, *B*, are

also used in similar cases. A sunk key, fig. 79, *C*, is much safer, because slipping is prevented unless the key shears. A sunk key, with a saddle-key placed at right angles to it, is a very good arrangement. When a wheel nave is slightly larger than the boss on which it is keyed, it rocks, if it is held by a sunk key only. This rocking is entirely prevented by the saddle-key. When a key is so placed that it cannot be slacked or driven back from its smaller end, a gib end, or head, must be formed on it.

The keys for fixing wheels and pulleys on shafts, have often dimensions given by the following empirical rules :—

Diameter of eye of wheel $=d$

Width of key $=b=\frac{1}{4}d+\frac{1}{8}$ . . . . . (11)

Mean thickness of sunk key $=t=\frac{1}{10}d+\frac{1}{8}$. (12)

" key on flat $=t_1=\frac{1}{11}d+\frac{1}{16}$. (12*a*)

When wheels or pulleys, transmitting only a small amount of work, are keyed on large shafts, the dimensions above are excessive. In that case, let H. P. be the horses' power transmitted by the wheel or pulley, and N its revolutions per minute ; or, let P be the force in lbs., acting at its circumference, and R its radius, in ins. Then it is sufficient to take, in the expressions above,

$$d=\sqrt[3]{\frac{100 \text{ H. P.}}{\text{N}}} \text{ or } \sqrt[3]{\frac{\text{P R}}{630}} \quad . \quad . \quad . \quad (13)$$

Cone Keys. When a wheel has to be bored out, to pass over a shaft boss, cone keys are used to fix it on the shaft. These are of cast iron, and are cast in a single piece, with three parting plates, nearly, but not quite, dividing it into three pieces. The casting is bored and turned, afterwards split, and the rough edges chipped away. Thus are obtained three cast-iron, slightly-tapering keys, of the thickness necessary to fill the space between the eye of the pulley and the shaft.

COTTERS.

68. A cotter must have sufficient shearing strength and bearing surface, and at the same time should be of such a form as to diminish as little as possible the section of the parts connected. Fig. 80 shows a simple form of cottered

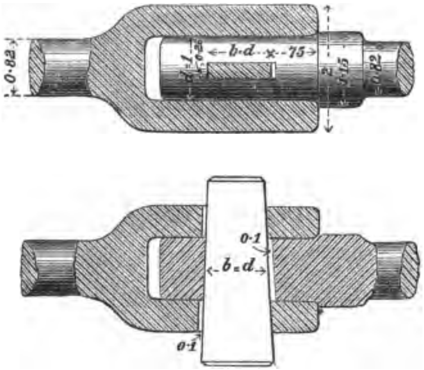


Fig. 80.

joint, between two wrought-iron bars transmitting a force in the direction of their axis. Taking the bearing resistance at double the shearing resistance, the bearing surface of the cotter on each bar should be half the area of the two sections in shear. If t = thickness of cotter, d = internal and D = external diameter of the socket,—

$$d\,t = (D - d)\,t = \tfrac{1}{2}\,(2\,b\,t)$$

which give

$$b = d$$

$$D = 2d.$$

Let f be the working compressive or tensile stress on the bars, f_s the working shearing stress on the cotters. The cotters will be as strong as the bars when

$$\left(\frac{\pi}{4}d^2 - d\,t\right) f = \left[\frac{\pi}{4}(D^2 - d^2) - (D - d)\,t\right] f = 2\,b\,t\,f_s$$

If both cotters and bars are of wrought iron, $f_s = f$; putting also $b = d$, we get

$$t = 0{\cdot}262\,d;\ D = 1{\cdot}332\,d \qquad . \quad . \quad . \quad . \quad (14)$$

The latter value gives too small a bearing surface for the cotter, and it is better to take $D = 2\,d$, as found above.

If the cotter is of wrought iron, and the rods of cast iron, we may take $f_s = 2\frac{1}{2}f$. Then

$$t = 0{\cdot}131\,d \text{ and } D = 2\,d \,. \qquad . \quad . \quad . \quad . \quad (15)$$

In practice it would often be convenient to make $t = 0{\cdot}26\,d$, and then $D = 1{\cdot}332\,d$ would be sufficient.

A steel cotter may be $\frac{3}{4}$ths the breadth of an iron one, the other dimensions remaining the same.

The unit for the proportions in fig. 80, is the diameter d of the enlarged part of the rod. The inner rod is slightly tapered, or has a collar, as shown. The cotter, when driven home, draws the two rods together, till the inner rod bears firmly on the outer rod. A clearance space is left in the cotter holes, to ensure the proper bearing of the rods. This clearance is termed the *draught.*

Fig. 81 shows a bolt cottered into a casting. The effective diameter of the bolt is the diameter at the bottom of the screw thread, and that is the value to be taken for d in proportioning the cotter. The proportions

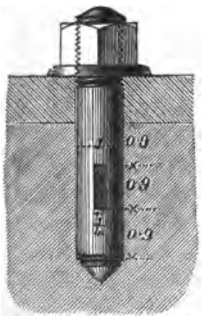


Fig. 81.

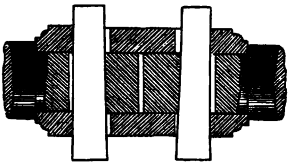


Fig. 82.

marked on the figure have been so modified that the unit is the gross diameter d of the bolt.

Fig. 82 shows a cottered coupling, suitable for rods transmitting a longitudinal force.

69. *Friction of Cotters.* Let P be the longitudinal force, transmitted through two rods, connected by a cotter, T the force applied at the end of the cotter, to maintain equilibrium; let one side of the cotter taper 1 in n, and let the coefficient of friction be $\frac{1}{m}$. Then, when the cotter is being driven

$$T = P\left(\frac{1}{m} + \frac{n+m}{mn-1}\right) = P\,\frac{2n+m}{nm} \text{ nearly} \quad . \quad (16)$$

And if the cotter is driven back or slacked,

$$T = P\left(\frac{1}{m} - \frac{m-n}{mn+1}\right) = P\,\frac{2n-m}{mn} \text{ nearly} \quad . \quad (17)$$

If $n = \frac{m^2-1}{2m}$, we get, in the latter case, $T = 0$, that is, the cotter will slack back under the action of the force P only. If $\frac{1}{m} = \frac{1}{15}$, the cotter will slip when the taper of the cotter is 1 in $7\frac{1}{2}$. The actual taper of cotters is much less than this, in order that there may be no danger of slacking.

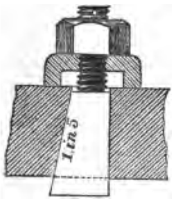


Fig. 83.

Unless special arrangements are made for preventing the slacking of the cotter, its taper is never more than 1 in 16, generally not more than 1 in 24, and often only 1 in 48.

If excessive taper must be given, to obtain sufficient draught, the end of the cotter is screwed, as shown in fig. 83, and a nut, bearing on a recessed washer or short tube, holds it in place. If the nut is tightened up when the pressure is acting on the cotter, we should have

$$\frac{\text{Net area of screw}}{\text{Shearing area of cotter}} = \frac{T}{P} = \frac{1}{m} + \frac{n+m}{mn-1} \quad . \quad . \quad (18)$$

In the cotter shown, the taper is $\frac{1}{n}=\frac{1}{5}$. Taking $\frac{1}{m}=\frac{1}{10}$, as a maximum,

$$\frac{\text{net area of screw}}{2\,b\,t}=0{\cdot}4$$

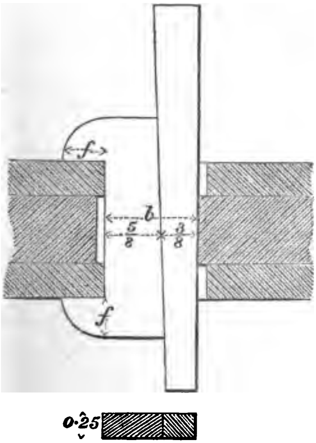
or net area of screw $=\frac{1}{5}$th of the cotter section. If the cotter is only tightened, when there is no strain on the rods, a much less section is sufficient. In that case

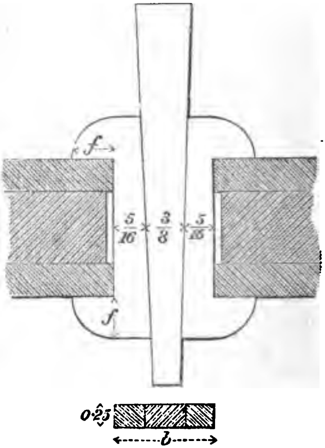
$$\frac{T}{P}=\frac{1}{m}-\frac{m-n}{m\,n+1} \quad . \quad . \quad . \quad (19)$$

but a minimum value should be taken for $\frac{1}{m}$.

Gib and Cotter.

70. When a cotter is used to connect strap-shaped parts to a more rigid rod, the cotter is divided into two parts, one





Unit $=b$.

Fig. 84. Fig. 85.

acting as an ordinary cotter, the other having hooked ends,

intended to prevent the spreading of the strap. It is convenient to make the outside of the gib parallel to the outside of the cotter, and to obtain the necessary draught, by inclining the division plane between them. The taper is usually 1 in 24 to 1 in 48 for simple cotters, and 1 in 8 to 1 in 16, when the slacking of the cotter is prevented by a screw. The total breadth b and thickness t of the gib and cotter, are the same as for a simple cotter.

Figs. 84, 85, show ordinary proportions of gibs and cotters. The unit for the proportions is the breadth b.

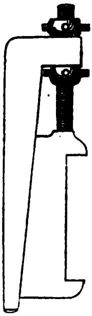


Fig. 86.

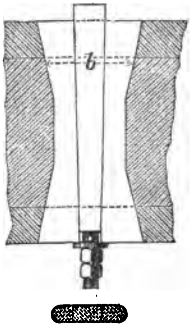


Fig. 87.

The simplest way of securing a cotter is by a screwed prolongation of the gib, as shown in fig. 86. A set screw, passed through one of the connected pieces, is sometimes used.

Fig. 87 shows another arrangement of gibs and cotter. In this case the space is restricted, and the draught required is very small. The cotter is secured by a screwed end, nut and washer.

CHAPTER VI.

ON PIPES AND CYLINDERS.

71. PIPES and cylinders, subjected to internal pressure, form parts of many machines. The proper proportions of these, and the modes of jointing them, will form the subject of the present chapter.

Thickness of cast-iron pipes used for water mains.—The ordinary rule for the thickness of a cylindrical vessel, subjected to an internal or bursting pressure, is given in § 26, eq. 2.

Let t = thickness of cylinder, in ins.

d = diameter of cylinder, in ins.

p = excess of internal over external pressure, in lbs. per sq. in.

f = safe limit of stress, in lbs. per sq. in.

$$t = \frac{p\,d}{2\,f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The average tenacity of the cast iron used for pipes, may be taken at 18,500 lbs. per sq. in. Taking the factor of safety at $3\frac{1}{3}$, the highest safe tension is 5,500 lbs. per sq. in. Allowance must, however, be made,—(a) for the irregular thickness of cast-iron pipes, which are often slightly thinner on one side than on the other; (b) for stresses due to hydraulic shock in the pipe, and to bending in consequence

of pressure of the earth above, or settlement of the earth beneath the pipe. A sufficient allowance will be made, if the pipe is calculated for three times the actual working pressure, or, what amounts to the same thing, if the limit of stress is taken at one-third the value given above. Hence, the apparent factor of safety for pipes is $3 \times 3\frac{1}{3} = 10$, and the greatest safe stress, due to the actual pressure in the pipe, is 1,850 lbs. per sq. in.

In the mains used for the conveyance of water, the external pressure is 1 atmosphere, or 33 ft. of water pressure, and the greatest internal pressure is generally less than 7 atmospheres, or 231 ft. of water. Hence, the excess of internal over external pressure may be taken at 6 atmospheres, or 90 lbs. per sq. in. Putting this value in the formula above, we get

$$t = \frac{90\,d}{2 \times 1850} = \cdot 0231\,d \quad . \quad . \quad . \quad . \quad (2)$$

Internal diameter of pipe in ins.

4	8	12	16	20	24	30	36	42

Thickness of pipe in ins.

0·0924	0·185	0·277	0·370	0·462	0·554	0·693	0·832	0·970

Thickness to nearest sixteenth of an inch.

$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	1

72. This table shows that some of the thicknesses given by the above rule, although ample margin of strength has been allowed, are so small that the pipes could not be cast with any certainty of success. Many years ago, the following rule for the thickness of water mains was given by Mr. Hawksley,

$$t = 0 \cdot 18\sqrt{d}$$

That rule represents, very fairly, the least thickness which it is desirable to attempt to cast. The following rule agrees still better with practical experience. Let $t_{\text{min.}}$ be the least thickness which should be adopted for a cylindrical pipe casting, of ordinary length and of diameter d, in order that there may be no special difficulty in getting it cast. Then

$$t_{\text{min.}} = 0{\cdot}11\sqrt{d} + 0{\cdot}1 \quad . \quad . \quad . \; (3)$$

Diameter of pipe in ins.

4	8	12	16	20	24	30	36	42	48	54	60

Least thickness of pipe in ins.

·320	·411	·481	·540	·592	·639	·703	·760	·813	·862	·908	·953

Thickness to nearest sixteenth of an inch.

$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{11}{16}$	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	1

Pipes of the thicknesses here given, will in general be safe for pressures not exceeding 6 atmospheres, or 90 lbs. per sq. in., when under 20 ins. diameter, and for 5 atmospheres, or 75 lbs. per sq. in., when under 60 ins. diameter. When pipes are subjected to greater pressure, it is desirable to use the more exact formula given in § 26 (eq. 2), in calculating the thickness. Putting in that formula $f = 1850$, it becomes

$$t = \frac{d}{2}\left\{\sqrt{\frac{2775+p}{2775-2p}} - 1\right\} \quad . \quad . \quad . \quad . \; (4)$$

From this formula the following table has been calculated.

Excess of Internal over External Pressure		Ratio of thickness to diameter of pipe
In lbs. per square inch	In feet of head of water	$\frac{t}{d}$
75	173	·021
90	208	·026
105	242	·030
120	277	·035
135	311	·039
150	346	·044
165	381	·048
180	415	·053
195	450	·058
210	484	·063
225	519	·068
250	577	·077
275	634	·085
300	692	·095
350	808	·114
400	923	·134
450	1039	·156
500	1154	·179
750	1731	·332
1000	2308	·603

73. In the following Table, the first column gives the least thickness of pipe which it is practicable to cast, which is the thickness to be adopted when equations (2) or (4) give a less value. The other columns give thicknesses calculated by equation (4). It should be remembered, that in obtaining these thicknesses an allowance has been made for bending stress, and hence somewhat less thicknesses may be adopted, in pipes so supported as to be protected from any bending. To convert feet of head of water into lbs. per sq. in., multiply by 0·4333.

Internal diameter of pipe in ins.	Least thickness of pipe by eq. (3)		Thickness necessary for strength, by eq. (4), for bursting pressures per sq. in. amounting to							
	Exact	Correct to nearest sixteenth	75 lbs.	90 lbs.	105 lbs.	120 lbs.	150 lbs.	180 lbs.	210 lbs.	250 lbs.
2	·256	$\frac{1}{4}$	...	...	...	...	...	...	...	...
3	·291	$\frac{5}{16}$	...	...	...	...	...	...	...	...
4	·320	,,	...	...	...	...	...	...	...	...
5	·346	$\frac{3}{8}$	...	...	...	...	...	...	...	·385
6	·369	,,	...	...	...	...	...	...	·378	·462
7	·391	,,	...	...	...	...	...	...	·441	·539
8	·411	$\frac{7}{16}$	...	...	...	...	...	·424	·504	·616
9	·430	,,	...	...	...	...	...	·477	·567	·693
10	·448	$\frac{1}{2}$	...	...	...	...	...	·530	·630	·770
12	·481	,,	...	...	...	...	·528	·636	·756	·924
14	·512	$\frac{9}{16}$	...	...	...	...	·616	·742	·882	1·08
16	·540	,,	...	...	...	·560	·704	·848	1·01	1·23
18	·567	,,	...	...	...	·630	·792	·954	1·13	1·39
20	·592	$\frac{5}{8}$	...	...	...	·700	·880	1·060	1·26	1·54
22	·616	,,	...	...	·66	·770	·968	1·166	1·39	1·69
24	·639	,,	...	...	·72	·840	1·056	1·272	1·51	1·85
30	·702	$\frac{11}{16}$	...	·780	·90	1·05	1·320	1·590	1·89	2·31
36	·760	$\frac{3}{4}$	...	·936	1·08	1·26	1·584	1·908	2·27	2·77
42	·813	$\frac{13}{16}$	·882	1·092	1·26	1·47	1·848	2·226	2·65	3·23
48	·862	$\frac{7}{8}$	1·008	1·248	1·44	1·68	2·112	2·544	3·02	3·70
54	·909	$\frac{15}{16}$	1·134	1·404	1·62	1·89	2·376	2·862	3·40	4·16
60	·952	,,	1·260	1·560	1·80	2·10	2·640	3·180	3·78	4·62
72	1·033	1	1·512	1·872	2·16	2·52	3·168	3·816	4·54	5·54
84	1·108	$1\frac{1}{8}$	1·764	2·184	2·52	2·94	3·696	4·452	5·29	6·47

74. *Thickness of steam pipes and steam cylinders of cast iron.*—This is determined in precisely the same way as the thickness of water mains. For steam pipes, the thicknesses given in the preceding Table will answer. For steam cylinders an allowance has to be made for re-boring. The cylinder thickness may be obtained from the following equation :—

$$t = \frac{p\,d}{3700} + c = \cdot 00027\,p\,d + c \quad . \quad . \quad . \quad . \quad (5)$$

where c ranges from 0·5 to 0·75 in carefully constructed engines, and is as much as 1·0 in some cases. The thickness of engine cylinders is determined almost entirely with respect to facility of casting, and is generally excessive as regards strength.

75. *Thickness of pipes of other materials :*

For lead pipes $t = \cdot 0025\,p\,d + \frac{3}{16}$

copper pipes $t = \cdot 00018\,p\,d + \frac{1}{16}$

wrought-iron pipes $t = \cdot 00006\,p\,d + \frac{1}{16}$ (if welded)

$= \cdot 00012\,p\,d + \frac{1}{16}$ (if riveted)

Copper steam, blow-off, or water pipes for engines are usually $\frac{1}{4}$ in. thick ; feed water pipes, $\frac{3}{16}$ in. ; and exhaust steam pipes, $\frac{1}{8}$ in. thick.

Pipe Joints.

75. Cast-iron pipes are connected by flange-joints, or by spigot and faucet joints. The former are stronger, easier to connect or disconnect, and are always used when the pipes are placed vertically. The latter are less costly, and are better for pipes laid in the ground, because they permit the pipes to adapt themselves to the inequalities of the ground while being laid, and the line of pipes retains a slight flexibility.

76. *Flanged Joints.*—The proportions of flanges have been to some extent given in § 65. Fig. 88 shows one

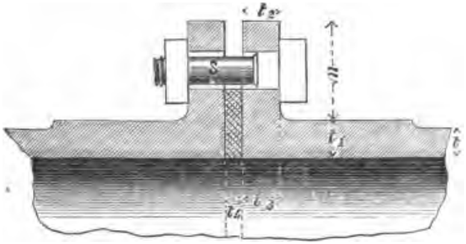


Fig. 88.

form of flanged joint for pipes, for which the following proportions may be used :—

$$\text{Thickness } t_1 = t_2 = \tfrac{4}{3}\, t$$

$$t_3 = \tfrac{3}{2}\, t$$

$$t_4 = \tfrac{3}{8} \text{ in.}$$

$$\text{Width} = w = 2\,\delta + 1$$

$$\text{Diam. of bolts} = \delta = 0{\cdot}1\, d \sqrt{\frac{p}{n}} + 0{\cdot}66$$

$$\text{Number of bolts} = n = 2 + \frac{d}{2}$$

$$\text{Diam. of bolt-hole} = \delta + \tfrac{1}{8}$$

The joint shown is made with a lead ring. The joint may be made by facing the flanges, and bringing them together with a string smeared with red-lead, or an india-rubber ring interposed. A rough joint is made with a ring of wrought iron, covered with tarred rope, the space between the flanges being filled up with rust cement.

Fig. 89 shows a pipe joint, where one flange is loose. The joint is made tight by a lead ring, well stemmed in.

Fig. 90 shows what is termed a lens joint, which is very easily made tight, and adapts itself to a slight flexure in the

direction of the pipe. The joint is made by a gun-metal ring, turned to spherical surfaces.

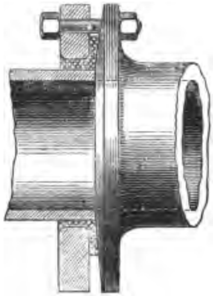


Fig. 89.

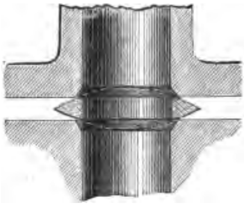


Fig. 90.

Fig. 91 shows the joint used by Sir W. Armstrong, for the pipes of his accumulator. These pipes are subjected to the enormous water-pressure of 800 lbs. per sq. in. The pipes are of the best remelted cast-iron, and are tested to 3,000 lbs. per sq. in. They are 5 ins. diameter, and 1 in. thick. Each end of the pipe has two strong elliptical flanges, $2\frac{3}{4}$ ins. thick, with two bolt-holes, $1\frac{5}{8}$ in. diameter. One pipe slightly enters into the other, forming a dove-tailed recess, in which is placed a gutta-percha ring, $\frac{1}{4}$ in. diameter.

Unit = *t.*

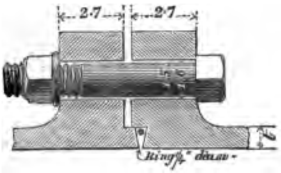


Fig. 91.

77. *Socket Joints.*—Socket pipes are jointed either with a gasket and lead joint, a rust joint, or a bored and turned joint. Fig. 92 shows an ordinary lead joint. When the pipes are in place, a few coils of gasket or tarred rope are driven into the socket. Clay is then put round the outside of the socket, and a lead ring is cast in it. The clay is removed and the lead stemmed tightly into the socket.

The proportions may be as follows: Let t=thickness, and d=diameter of pipe;

$$t_1 = 1{\cdot}07\,t + \tfrac{1}{16}$$
$$t_2 = 0{\cdot}025\,d + \tfrac{1}{4} \text{ to } 0{\cdot}025\,d + 0{\cdot}6$$
$$t_3 = 0{\cdot}045\,d + 0{\cdot}8$$
$$5 = 0{\cdot}01\,d + {\cdot}25 \text{ to } 0{\cdot}01\,d + {\cdot}375$$
$$b_1 = 0{\cdot}075\,d + 2\tfrac{1}{4}$$
$$b_2 = t_2$$
$$l = 0{\cdot}09\,d + 2\tfrac{3}{4} \text{ to } 0{\cdot}1\,d + 3$$
$$b_4 = b_3 = 0{\cdot}03\,d + 1$$

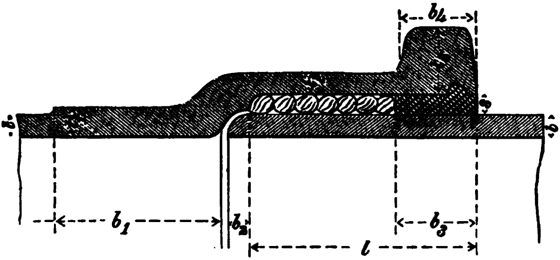


Fig. 92.

Fig. 93 shows a different form of socket. The proportions may be the same as those just given.

Figs. 94, 95 show two forms of bored and turned socket and spigot joints. When the bored and turned part is long, the pipes are rigid, and are liable to be broken by the earth pressure. Hence, the fitting part is now often only $\frac{5}{8}$ in. in length,

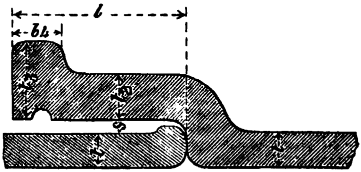


Fig. 93.

and has a very slight taper. The joint is made by painting over the faced part with red-lead, or with fresh and liquid

Portland cement. The pipe is then put in place, and driven home by a wooden mallet, or by swinging the next length of

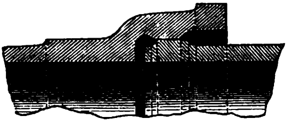


Fig. 94.

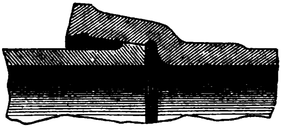


Fig. 95.

pipe. The socket is filled up with cement. Joints of this kind are more easily and quickly made than lead joints. Socket pipes should be cast with the socket downwards, and about a foot of length should be allowed at the spigot end, into which the scoriæ may rise, and which is broken off when the pipe is cast.

Fig. 96 shows two ways of making a socket joint for wrought-iron pipes. The sockets are of cast iron. Wrought iron is chiefly used for very large or very small pipes. From its thinness, it is liable to more injury from corrosion than cast iron, and hence it has been suggested that large wrought-iron mains should be lined with a thin coating of Portland cement.

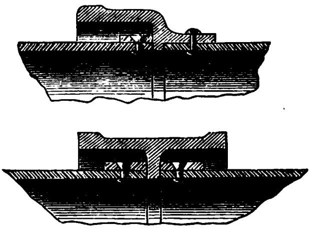


Fig. 96.

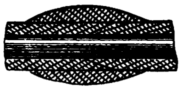


Fig. 97.

78. *Joints for lead pipes.*—Lead pipes are useful, because they are easily bent. They are manufactured by drawing them over a mandril by hydraulic pressure. They are sometimes lined with tin, when used to convey water which dissolves the lead. Joints may be made by flanging out the ends of the pipes, and compressing these flanges between two iron rings, with bolts. Commonly, the joint is made by soldering, and is termed a plumber's wiped joint, fig. 97.

CHAPTER VII.

JOURNALS, PIVOTS, AXLES AND SHAFTING.

JOURNALS.

79. JOURNALS are those parts of rotating pieces, which are supported by the frame of the machine. They are commonly cylindrical, but sometimes spherical or conical. Some journals run constantly, others support a piece which moves occasionally. In the latter case, the strength of the journal is chiefly to be considered; in the former, durability and freedom from liability to heat are as important as strength. Some journals are subjected to straining forces in the plane of their axis only, which produce bending and shearing stress. Others are subjected to bending and torsion, and are calculated by the rules for combined stress. Lastly, some journals are supported at one end only; others, which may be termed neck journals, are supported at both ends.

80. *Form of journals.*—The ordinary form of journals is shown in fig. 98. The bearing part of the journal is turned accurately cylindrical, and is terminated by raised parts or collars, which bear against the ends of the brass steps in which the journal revolves, and limit its end play.

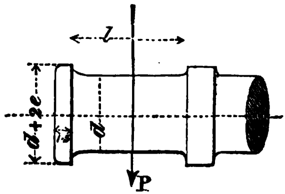


Fig 98.

The length of the brass step is, in some cases, 0·9 of the length of the journal. This permits a limited longitudinal motion, which ensures uniform wear of the step. In other cases, longitudinal play would interfere with the action of the machine, and it is made as small as possible. The proportional unit for fig. 98, is $e=\frac{d}{8}+\frac{1}{8}$ to $\frac{d}{10}+\frac{1}{8}$.

81. *Friction of Journals.*—It is not possible to estimate exactly the friction of journals, because the distribution of the pressure on the surface of the journal is dependent on the wear of the step. The frictional resistance to motion must be between the limits $\frac{\pi}{2}\mu\,P$ and $\frac{4}{\pi}\mu\,P$, where μ is the coefficient of friction, and P the load on the journal. It probably approaches the former limit when the journal is new, and the latter when it is worn. It is very commonly taken at $\mu\,P$ simply, which it would be if the journal touched the step at a line only. Taking this value, which is accurate enough for journals in their ordinary condition, the value of μ, according to Morin's experiments, is 0·05 to 0·07 for good journals, well lubricated. Experience with railway journals appears to show that the coefficient of friction is often much less than this. Experiments by Kirchweger gave, for the coefficient of friction of railway axles, $\mu=0{\cdot}009$ to 0·01 for wrought iron on white metal, and $\mu=0{\cdot}014$ for wrought iron on gun metal, when the journals were lubricated with oil. Experiments by Bokelberg and Welkner gave coefficients ranging from 0·0028 for small loads and low velocities, to 0·013 for great loads and high velocities, and in these experiments the coefficients were less for gun-metal than for white metal. The experiments are not accordant, but they show that the friction is sometimes less than it would be if calculated from Morin's values. The work expended in friction at N revs. per min. is

$$=T=\mu\,P\times\frac{\pi}{12}d\,N \text{ ft. lbs. per min.} \qquad (1)$$

82. *Length of journals.*—Common experience shows that for journals working at high speeds, a greater length is necessary than for journals running at low speeds. Journals running at 150 revs. per min., are often only one diameter long. Fan-shafts running at 1,500 revs. per min. have journals 6 or 8 diameters long.

If the journal works occasionally for short periods of time, it is desirable to make it short, because the less the length of the journal, the smaller its diameter may be for a given load, and consequently the less will be the friction. If the journal runs constantly, it requires a larger surface to ensure durability and coolness in working. The larger surface is better obtained by increasing the length than by increasing the diameter of the journal. In the former case, the friction remains the same, in the latter it is increased. Some increase of diameter is necessary to maintain equal strength, but the diameter should be increased only so much as is necessary for that purpose.

If a journal is strong enough, when of length l and diameter d, then if, for any reason, the length is increased to l_1, the diameter must be

$$d_1 = d\sqrt[3]{\frac{l_1}{l}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

to obtain the same strength as before.

The length of journals, to ensure durability and cool working, depends not only on the pressure to which the journal is subjected, but also on the material of the journal and steps; on the kind of motion, whether continuous rotation or oscillation; on the perfection of the lubricating arrangements, and on the accuracy of workmanship. It is not surprising, therefore, that, in different cases, journals, subjected to the same load and running at the same speed, should have different lengths.

The work T expended in friction produces a quantity of heat,

$$H = \frac{T}{J} = \mu P \ \frac{\pi d N}{12 J} \quad . \quad . \quad . \quad . \ (3)$$

where J is Joule's equivalent, or 772 ft. lbs. That heat should be dissipated as fast as it is generated, by conduction from the surface of the journal, and it appears to be a reasonable assumption that the rate of dissipation is proportional to the area of the surface through which the heat is conducted, or to the product $d\,l$. Let h units of heat be dissipated per minute, for each unit of bearing surface, $d\,l$. Then

$$h\,d\,l = \frac{\mu P \pi d N}{12 J}$$

$$l = \frac{\mu \pi}{12 J h} \times P N = \frac{P N}{\beta} \quad . \quad . \quad . \ (4)$$

where β is a constant, to be ascertained by experience in different cases. This formula may be put in a more convenient form for crank-pins. Let H. P. be the indicated horses' power transmitted to the crank-pin, R the crank radius in inches; then the mean pressure on the crank-pin is

$$P = \frac{33000 \times H.P. \times 12}{4 R N} \text{ nearly.}$$

Inserting this value in (4)

$$l = \gamma \ \frac{H.P.}{R} \quad . \quad . \quad . \quad . \quad . \quad . \ (5)$$

where $\gamma = \dfrac{99000}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \ (6)$

For marine engine crank-pins, β ranges from 250,000 to 300,000, and γ from 0·4 to 0·33.

For stationary engine crank-pins, the length given is usually greater. The older engines, which worked at low

pressures, give $\beta=66,000$; $\gamma=1\cdot5$. Some more modern engines give values of β ranging from 100,000 to 200,000, and of γ from $1\cdot0$ to $0\cdot45$.

The outside crank-pins of locomotives are necessarily contracted in length as much as possible. They are often of steel, and the workmanship is very good. Taking the average speed of the engine at 60 ft. per sec., and the steam pressure during the stroke at 100 lbs. per sq. in., we get $\beta=1,000,000$ to $1,500,000$, and $\gamma=0\cdot1$ to $0\cdot066$.

Railway-carriage axles are so largely used that they probably approach the minimum size consistent with durability. An axle $3\frac{1}{4}$ ins. in diameter, and 8 ins. long, would run cool under a load of $2\frac{1}{2}$ tons at 60 miles per hour. These data give $\beta=400,000$, and $\gamma=0\cdot25$.

83. If a journal of length l works satisfactorily at N revolutions under a load P, and l_1 N_1 P_1 are corresponding values for another journal, working in similar conditions, then, if the above theory is correct,

$$\frac{l_1}{l}=\frac{P_1 N_1}{P N} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For railway journals, running at the same speeds, the length should be simply proportional to the load. For engines of the same kind, running at the same speed, the crank-pin length should be proportional to the load on the piston. For locomotive engines, working at the same pressure, the crank-pin length should be proportional to the piston area.

It is obvious that the constants β and γ vary greatly in different cases. Possibly, the theory of the durability of journals is incomplete, but also, no doubt, the amount of durability of actual journals varies greatly.

84. The following table is calculated for $\beta=250,000$ $\gamma=0\cdot4$. For any other values of β or γ, the length is the tabular length $\times\frac{250,000}{\beta}$, or tabular length $\times\frac{\gamma}{0\cdot4}$.

Theoretical Journal Length in Inches.

Load on Journal in lbs.	Revolutions of Journal per minute					
	50	100	200	300	500	1,000
1,000	·2	·4	·8	1·2	2·0	4·0
1,500	·3	·6	1·2	1·8	3·0	6·0
2,000	·4	·8	1·6	2·4	4·0	8·0
3,000	·6	1·2	2·4	3·6	6·0	12·0
4,000	·8	1·6	3·2	4·8	8·0	16·0
5,000	1·0	2·0	4·0	6·0	10·0	20·0
10,000	2·0	4·0	8·0	12·0	20·0	40·0
15,000	3·0	6·0	12·0	18·0	30·0	...
20,000	4·0	8·0	16·0	24·0	40·0	...
30,000	6·0	12·0	24·0	36·0	...	...
40,000	8·0	16·0	32·0	...	...	...
50,000	10·0	20·0	40·0	...	...	...

85. The ordinary empirical mode of proportioning the length of journals, is to make the length proportional to the diameter, and to make the ratio of length to diameter increase with the speed. For wrought-iron journals,

$$\frac{l}{d} = \cdot 004\,\text{N} + 1 \quad . \quad . \quad . \quad . \quad (8)$$

N=50	100	150	200	250	500	1000
$\frac{l}{d}$=1·2	1·4	1·6	1·8	2·0	3·0	5·0

Cast-iron journals may have $\frac{l}{d} = \frac{9}{10}$, and steel journals may have $\frac{l}{d} = 1\frac{1}{4}$ of the above values.

86. *Strength of journals.*—In the simplest cases, journals are subjected exclusively to bending action, which may be considered to be uniformly distributed over the length of the journal.

Case I. Crank-pins, or overhung journals supported at one end. Let P be the uniformly-distributed load on the

journal in lbs.; d the diameter, and l the length of the journal in ins.; f the greatest safe stress per sq. in. for the material of the journal. The greatest bending moment at the fixed end of the journal is $\frac{1}{2}\text{P}\,l$. Equating this to the moment of resistance of a circular section, § 28,

$$\frac{\text{P}\,l}{2} = f\,\frac{\pi}{32}\,d^3 = \frac{f\,d^3}{10\cdot 2}$$

$$d = \sqrt[3]{\frac{5\cdot 1}{f}}\,\sqrt[3]{\text{P}\,l} = \sqrt{\frac{5\cdot 1}{f}}\,\sqrt{\left(\text{P}.\frac{l}{d}\right)} \quad . \quad . \quad . \ (9)$$

For wrought iron, f may be taken at 6,000 to 9,000 lbs. per sq. in., the stress on each fibre being alternately tensile and compressive (§ 25). For steel, f=13,500 to 9,000 lbs. for cast iron, f=3,000 to 4,500 lbs.

$\sqrt[3]{\frac{5\cdot 1}{f}}$ = 0·0947 to 0·0827 for wrought iron.
0·0827 „ 0·0723 „ steel.
0·1193 „ 0·1042 „ cast iron.

$\sqrt{\frac{5\cdot 1}{f}}$ = 0·0238 to 0·0291 for wrought iron.
0·0291 „ 0·0194 „ steel.
0·0413 „ 0·0337 „ cast iron.

This equation may be solved graphically. Take $a\,b = \text{P}\frac{l}{d}$, on any scale. Describe on $a\,b$ a semicircle. On the same scale take $a\,g$ = unity, and draw a perpendicular. Then $a\,c = \sqrt{\text{P}\,\frac{l}{d}}$

Take $a\,h = \sqrt{\frac{5\cdot 1}{f}}$, and draw a perpendicular to $a\,b$, cutting $a\,c$ produced in e. Then $a\,e = d$,

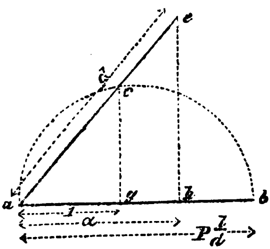


Fig. 99.

the diameter required. It will usually be convenient to

take $ab = \frac{P}{n^2} \cdot \frac{l}{d}$ and $ah = n\sqrt{\frac{5 \cdot 1}{f}}$, where n is some convenient multiplier. Then $ae = d$, as before. The multiplier may be chosen so as to make ab=about $1\frac{1}{2}$ to 3 times ac.

87. *Case II. Neck journals.*—The cross-head pins of engines are supported at each end, and loaded uniformly. The bending moment is then $P\frac{l}{8}$.

$$d = \sqrt[3]{\frac{1 \cdot 28}{f}}\sqrt{PI} = \sqrt{\frac{1 \cdot 28}{f}}\sqrt{\left(P\frac{l}{d}\right)} \quad . \quad . \quad (10)$$

The diameter of a neck journal may, therefore, be 0·63 of the diameter of the equivalent crank-pin journal, if it has the same length.

88. *Case III. Journals subjected to a transverse load and a twisting force.*—Crank-shaft journals. Let fig. 100 represent a side and end elevation of a crank and crank-shaft, and let it be required to determine the dimensions of

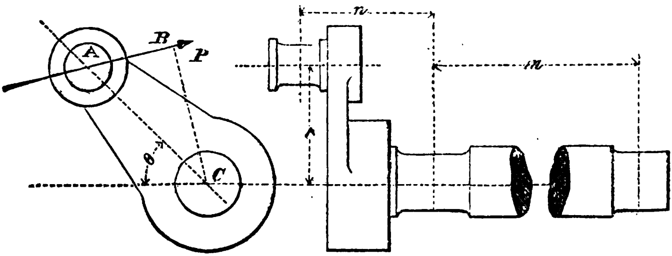


Fig. 100.

the shaft-journal nearest the crank. Let P, acting in the direction A B, be the pressure transmitted to the crank-pin by the connecting rod, and let Q and R be the reactions due to P at the shaft journals. These forces may be taken to act at the centres of the journals.

$$P\,n = Q\,m \text{ and } P + Q = R \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Hence,

$$Q = P\,\frac{n}{m}; \quad R = P\left(1 + \frac{n}{m}\right) \quad . \quad . \quad . \quad . \quad (12)$$

The force P produces a bending moment $P\,n$, at the centre of the shaft journal, causing bending in a plane parallel to its direction. At the same time, the journal is subjected to a twisting moment $P \times CB = P\,r \cos\theta$. At the dead point, where the direction of P passes through C, $r\cos\theta = 0$; and the twisting moment vanishes. If P is constant, the twisting moment is greatest when the connecting rod is at right angles to the crank, and is then equal to $P\,r$.

By the rules for combining bending and twisting action, § 42, the stress due to the combined moments is the same as that which would be produced by a simple bending moment.

$$M = \tfrac{1}{2}\,P\,n + \tfrac{1}{2}\sqrt{\left\{(P\,n)^2 + (P\,r)^2\right\}}$$

$$= \frac{P}{2}\left\{n + \sqrt{(l^2 + r^2)}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$= \frac{P}{2}\,(1{\cdot}84\,n + 0{\cdot}84\,r) \text{ nearly} \quad . \quad . \quad . \quad . \quad (13\,a)$$

Equating this to the moment of resistance of a circular section,

$$M = \frac{f\,d^3}{10{\cdot}2}$$

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{\left\{P\,(n + \sqrt{l^2 + r^2})\right\}} \quad . \quad . \quad . \quad (14)$$

$$= \sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{\left\{P\,(1{\cdot}84n + {\cdot}84r)\right\}} \text{ nearly} \quad . \quad . \quad (14a)$$

where $\sqrt[3]{\frac{5{\cdot}1}{f}}$ has the values given above.

These equations exaggerate a little the straining action,

because they neglect the distribution of the load on the surface of the journal. The length of the journal is to be calculated for the pressure R. Usually for crank-shafts $l = 1\frac{1}{2}$ to $1\frac{3}{4}\, d$, which allows a large margin for durability.

Diameter of Wrought-iron Crank-pin Journals in inches.

Load on Journal in lbs.	Ratio of Length to diameter						
	1·0	1·25	1·5	1·75	2	3	4
1,000	·84	·94	1·03	1·11	1·19	1·45	1·68
1,500	1·03	1·15	1·26	1·36	1·45	1·78	2·05
2,000	1·19	1·33	1·45	1·57	1·68	2·05	2·37
3,000	1·45	1·62	1·78	1·92	2·05	2·52	2·90
4,000	1·68	1·88	2·05	2·22	2·37	2·90	3·35
5,000	1·87	2·10	2·30	2·48	2·65	3·25	3·75
10,000	2·65	2·96	3·25	3·50	3·75	4·59	5·30
15,000	3·25	3·63	3·98	4·29	4·59	5·62	6·49
20,000	3·75	4·19	4·59	4·96	5·30	6·49	7·50
30,000	4·59	5·13	5·62	6·07	6·49	7·95	9·18
40,000	5·30	5·93	6·49	7·01	7·50	9·18	10·60
50,000	5·93	6·63	7·26	7·84	8·38	10·27	11·85

Pivot and Collar Bearings.

89. When a shaft is subjected to a force parallel to its axis, it requires a pivot, or collar, to prevent longitudinal displacement. A vertical shaft has usually a pivot at its foot, supporting the weight of the shaft and the gearing attached to it. The screw-shaft of a steam vessel is subjected to a longitudinal thrust, equal and opposite to the resistance of the ship. This thrust is usually transmitted to a collar-bearing, strongly connected to the framing of the ship.

90. *Friction of a pivot.*—Let d be the diameter of a flat pivot, P the load on it in lbs., μ the coefficient of friction; N the revolutions per minute. The work expended in friction lies between the limits

$$\tfrac{2}{3}\,\pi\,\mu\,\mathrm{P\,N}\,d \text{ and } \tfrac{1}{2}\,\pi\,\mu\,\mathrm{P\,N}\,d \text{ ft. lbs. per min.}$$

Taking the former estimate, the heat evolved is

$$\tfrac{1}{772} \cdot \tfrac{2}{3} \cdot \pi \mu \mathrm{P} \mathrm{N} d \text{ units per min.}$$

If h units are conducted away per sq. in. of bearing surface.

$$\frac{\pi}{4} d^2 h = \frac{1}{772} \cdot \frac{2}{3} \pi \mu \mathrm{P} \mathrm{N} d$$

$$\therefore d = c \mathrm{P} \mathrm{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where c is a constant determined by experience. According to Reuleaux, $c = 0{\cdot}0000272$.

91. When the speed is not great, it is desirable to limit the intensity of the pressure on the footstep, or there is danger of squeezing out the lubricant. Reuleaux gives the following values for the maximum intensity of the pressure on the pivot.

		$p=$
I.	Wrought-iron pivot on gun-metal . . .	700
II.	Cast-iron pivot on gun-metal	470
III.	Wrought-iron pivot on lignum vitæ . . .	1400

In order that the intensity of pressure may not exceed this limit, we must have

$$d = \sqrt{\frac{4}{\pi p}} \sqrt{\mathrm{P}} = k\sqrt{\mathrm{P}} \quad . \quad . \quad . \quad (16)$$

Allowing 10 per cent. of area for the oil-channels, $k = {\cdot}046$ in Case I.; $= {\cdot}057$ in Case II.; $= {\cdot}033$ in Case III.

Eq. 16 will give greater values of d, and is to be preferred when

$$\mathrm{N} < \frac{k}{c\sqrt{\mathrm{P}}}$$

$\frac{k}{c\sqrt{\mathrm{P}}}$	P=1000	2500	5000	7500	10,000	15,000
I.=	53	34	24	20	17	14
II.=	66	42	30	24	21	17
III.=	38	24	17	14	12	10

If the number of revolutions is less than is given in this Table, eq. 16 is to be taken; if greater, eq. 15.

92. *Collar Bearings*, fig. 101, are used when a vertical shaft has to be suspended from a framing, an arrangement used for turbine and centrifugal pump-shafts, in order to bring the bearing to a position where it can be easily lubricated. They are also used for screw-propeller shafts.

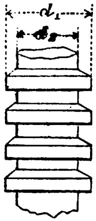


Fig. 101.

Let P be the load on the shaft (including its weight, if it is vertical); d_1 d_2 the outside and inside diameters of the collars; n, the number of collars; N, the number of revolutions per min. Then, by a similar process to that used for pivots, we get

$$n\,\frac{(d_1^{\,2}-d_2^{\,2})^2}{d_1^{\,3}-d_2^{\,3}}=c\,\mathrm{P}\,\mathrm{N} \quad . \quad . \quad . \quad . \quad (17)$$

$$n\,(d_1^{\,2}-d_2^{\,2})=k^2\,\mathrm{P} \quad . \quad . \quad . \quad . \quad (18)$$

where c and k have the same values as before. In using these equations, it is best to assume values for d_1 and d_2. Then n is easily found. The greater of the two values given by the equations is to be taken.

Axles and Shafts.

93. The terms *axle* and *shaft* are applied rather indiscriminately, to parts of machines which support rotating pieces, or which by their rotation convey and distribute motive power. They are usually cylindrical, but occasionally square or cross-shaped in section. They may be classified as follows:—

(1.) Axles loaded transversely, and subjected chiefly to bending action.

(2.) Transmissive shafting, subjected chiefly to torsion.

(3.) Crank-shafts and other shafts, subjected to combined torsion and bending.

94. *Axles loaded transversely.*—In designing axles of this kind, it is convenient to determine, first, the dimensions of the journals. If the axle is cylindrical, its diameter at any other point can be obtained from the journal diameter, if it is remembered that the diameters at any two points should be proportional to the cube roots of the bending moments at those points. If the section is not circular, it is still convenient to design a cylindrical axle, and then to replace the cylindrical sections by equivalent sections of any other form. If the axle rotates, the cylindrical form is the only one which is of equal strength in all positions. The mode of designing axles is best explained by examples.

95. *Example I.*—An axle is supported on two end

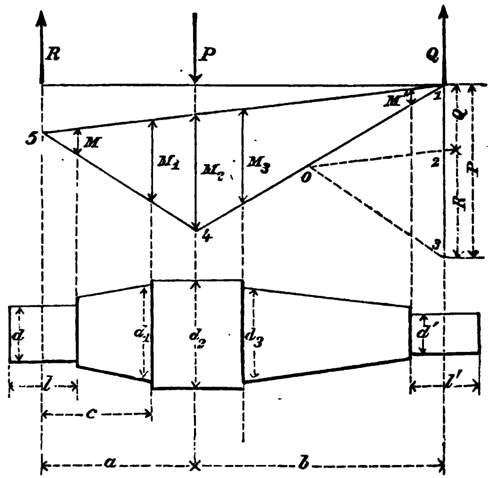


Fig. 102.

journals, and carries a load, P, at a point between the journals. Fig. 102 shows the axle. The load P is in equi-

librium with the reactions Q, R, acting at the centres of the journals.

$$Q = P \frac{a}{a+b}; \quad R = P \frac{b}{a+b}.$$

These are the loads for which the journals are to be calculated. From the rules in §§ 80–86 will be determined d, d', l, l, and the projections which limit the end-play. The axle diameters which it is most necessary to determine are those marked d_1, d_2, and d_3. The bending moments at those points are $M_1 = R\,c$; $M_2 = R\,a$; $M_3 = Q\,b$. The bending moments at the fixed ends of the journals are $M = R\,\frac{l}{2}$ and $M' = Q\,\frac{l'}{2}$.

Since at any section the diameter must be at least equal to $\sqrt[3]{\frac{5 \cdot 1}{f}}\sqrt[3]{M}$,

$$\left.\begin{aligned} \frac{d_1}{d} &= \sqrt[3]{\frac{M_1}{M}} = \sqrt[3]{\frac{2\,c}{l}} \\ \frac{d_2}{d} &= \sqrt[3]{\frac{M_2}{M}} = \sqrt[3]{\frac{2a}{l}} \\ \frac{d_3}{d'} &= \sqrt[3]{\frac{M_3}{M'}} = \sqrt[3]{\frac{2\,b}{l'}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (19)$$

The smallest values of the diameters consistent with the requirements of strength are, therefore, easily obtained from the journal diameters.

It is often convenient to measure the bending moments from the bending moment curve, which is easily drawn thus :—Take 13 on the direction of Q produced, and = P, on any scale ; choose any pole O, and draw 1 O 4, meeting the direction of P produced in 4. Join 3O and draw 45 parallel to 3O, meeting the direction of R in 5. Join 51, and draw O 2 parallel to it. Then 12, 23 are the values of Q and R on the scale assumed for P. The vertical ordinates of the

triangle 1 4 5, are proportional to the bending moments at the corresponding points of the axle. The values of M, M_1, M_2, etc., measured on the diagram, may be used in the preceding equations, in determining the diameters of the axle.

The boss at the loaded part of the shaft, is intended for keying on the wheel, or other part supported by the axle. Its projection must therefore be sufficient for cutting a key-way, § 67, even if it is then larger than is necessary for strength. If at any part the axle is not circular, it is only necessary to equate the modulus of a section of the required form, to the modulus of the circular section previously determined. Thus, if the section is to be square, the equation

$$0{\cdot}118\, s^3 = {\cdot}0982\, d^3$$

will give the side of the square, the values of the moduli having been taken from Table IV. p. 35. If the axle rotates, the value of the modulus must be that which corresponds to the position in which it is weakest.

96. *Example II.*—The axle supports two parallel loads between the journals. The bending moment curve is drawn thus: Let *a b* be the centres of the journals, *c d* the points at which the loads P Q are applied. At the points *a* and *b* the reactions R, S are produced by the action of P and Q. On the direction of S set off 12, 23, equal to Q and P on any scale. Choose a pole, O; join O 1, intersecting the direction of Q in 4. Join O 2 and draw 45 parallel to it, intersecting the direction of P in 5. Join O 3, and draw 5 6 parallel to it, intersecting the direction of R in 6. If, now, O 7 is drawn parallel to the closing

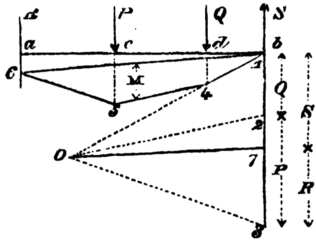


Fig. 103.

line 6 1, 3 7 and 7 1 will be equal to the reactions R and S. Also, 6 1 4 5 is the bending moment polygon, the breadth of which at any point, measured parallel to the forces, is proportional to the bending moment at the corresponding point of the axle. Having, therefore, the bending moments, the diameters of the axle may be obtained from the journal diameters, as before.

When $P = Q$ and $ac = db$, the case is one which occurs very commonly in practice, in which it will be found that the bending moment is uniform from c to d. A railway carriage axle is in this position when the carriage is at rest, or moving along a straight portion of line. In passing round curves, however, it is subjected to torsion as well as bending; and in consequence of the pressure of the wheel flange against the rail, the forces are no longer parallel. Then the bending action between c and d is no longer uniform. It is for this reason that railway axles are tapered a little towards the centre.

The bending moment curve for forces not parallel, is drawn in the same way as before, the only difference being that the lines 1 2, 2 3, 3 7, 7 1, parallel to the forces, no longer fall on a single line, but form a closed polygon.

97. *Shafts transmitting power, and subjected to torsion only.*—Rotating shafts are very extensively used, in transmitting the energy of prime-movers to the various parts of the factory or workshop in which it is applied to useful purposes. Such shafting was at one time of timber, then cast iron was adopted, and later still, wrought iron has almost entirely superseded cast iron, except in a few cases, where the shafting is not subjected to much impulsive action. In transmitting power shafts are subjected to torsion, but they are also subjected to bending action, due to their own weight, the weight of the wheels and pulleys they support, to the thrust of the gearing and the tension of the belting connected with them, and to other causes. This bending action is, to a great extent, indeterminate; it will, therefore, be

convenient to consider, first, the torsion due to the power transmitted, and then to examine how an allowance can be made for the other straining actions.

Let H. P. be the indicated horses' power transmitted.
- N the number of revolutions of the shaft per minute.
- P the twisting force in lbs., acting on a shaft at a radius R in ins.
- f the greatest safe stress for the material of the shaft, in lbs. per sq. in.
- d the diameter of the shaft in ins.

The twisting moment is in statical inch lbs.,—

$$T = P R = 63{,}024 \frac{\text{H. P.}}{N} \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

The moment of resistance of a circular section with respect to torsion is (§ 36) 0·196 $d^3 f$. Hence,

$$d = \sqrt[3]{\frac{5{\cdot}1}{f}} \sqrt[3]{P R} = \alpha \sqrt[3]{P R} \quad . \quad . \quad . \quad . \qquad (21)$$

$$= \sqrt[3]{\frac{63{,}024}{0{\cdot}196 f}} \sqrt[3]{\frac{\text{H. P.}}{N}} = \beta \sqrt[3]{\frac{\text{H. P.}}{N}} \quad . \quad . \quad . \qquad (22)$$

Taking $f = 9{,}000$ for wrought iron; $= 4{,}500$ for cast iron; and $= 13{,}500$ for steel; we get,

	$\alpha =$	$\beta =$
For wrought iron	·08275	3·294
For cast iron	·1042	4·150
For steel	·0723	2·877

98. *Shafts subjected to torsion and bending.*—Let M_t be the twisting moment, and M_b the bending moment, at any section of a shaft. Then, the combined straining action is equivalent to that which would be produced by a twisting moment T, given by the following equation, which is eq. 27 of § 42, in a slightly different form.

$$T = M_b^2 + \sqrt{(M_b^2 + M_t^2)} \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

Let $M_b = k\,M_t$ in any given case, so that k is a known fraction. Then

$$T = (k + \sqrt{k^2 + 1})\,M_t \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$= (1{\cdot}83\,k + {\cdot}83)\,M_t \text{ approximately} \quad . \quad . \quad . \quad (24\,a)$$

Then the preceding formulæ may be used in designing the shaft, if the equivalent twisting moment is substituted for the actual twisting moment P R in eq. 21. Hence,

$$d = \sqrt[3]{(k + \sqrt{k^2 + 1})}\sqrt[3]{\frac{5{\cdot}1}{f}}\sqrt[3]{M_t} \quad . \quad . \quad . \quad . \quad (25)$$

Or, if d is the proper diameter of the shaft, calculated for the combined bending and twisting action, and d' is the diameter calculated for the twisting action alone, by eq. 21 or 22; then

$$d = n\,d' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where n is equal to $\sqrt[3]{(k + \sqrt{k^2 + 1})}$, or to $\sqrt[3]{(1{\cdot}83\,k + 0{\cdot}83)}$ nearly. The following Table gives some values of n for given values of k.

k=0·25	0·50	0·75	1·0	1·25	1·50	1·75	2·0	3·0
n=1·09	1·17	1·26	1·34	1·42	1·49	1·56	1·62	1·83

It appears, from some calculations of Prof. Rankine, that for such cases as the propeller shafts of steam-vessels, where the straining action, additional to the torsion transmitted, is chiefly due to the weight of the shaft itself, k=0·25 to 0·5, and the diameter of the shaft should be 1·09 to 1·17 times the diameter, calculated from the torsion alone. For line shafting in mills, the bending action is often much greater, and the twisting moment is not constant, but rises above the mean value, calculated from the power transmitted. Practical experience appears to show, that for ordinary light shafting, k is 0·75 to 1, and the diameter of the shafting is 1·26 to 1·34 times the diameter, calculated from the mean

torsion alone. For crank-shafts and heavy shafting subjected to shocks, $k=1$ to 1·5, and the diameter is 1·34 to 1·49 times that calculated from the torsion alone. Cases occur in which still greater allowance must be made.

Diameters of Wrought-iron Shafts for given Twisting Moments in inch lbs., calculated by Eq. 21.

Twisting moment PR	Diameter d	Twisting moment PR	Diameter d
250	·52	60,000	3·24
500	·66	70,000	3·41
750	·75	80,000	3·57
1,000	·83	90,000	3·71
1,500	·95	100,000	3·84
2,000	1·04	110,000	3·97
2,500	1·12	120,000	4·08
3,000	1·19	130,000	4·19
4,000	1·31	140,000	4·30
5,000	1·42	150,000	4·40
6,000	1·50	175,000	4·63
7,500	1·62	200,000	4·84
10,000	1·78	250,000	5·21
12,500	1·92	300,000	5·54
15,000	2·04	400,000	6·10
17,500	2·15	500,000	6·57
20,000	2·25	600,000	6·98
25,000	2·42	750,000	7·53
30,000	2·57	1,000,000	8·28
35,000	2·70	1,250,000	8·92
40,000	2·83	1,500,000	9·47
45,000	2·94	1,750,000	9·97
50,000	3·05	2,000,000	10·42

For cast-iron shafts, multiply the diameters in the table by 1·26. For steel shafts, multiply by 0·874. To allow for bending action, amounting to k times the twisting action, multiply also by the value of n corresponding to k, given in the short Table above.

Diameters of Shafts, when the Horses' Power and Revolutions per minute are given, calculated by Eq. 22.

Divide the H.P. by the number of revolutions. The diameter will be found opposite the nearest number to the quotient in the following Table :—

$\frac{\text{H.P.}}{\text{N}}$	Diameter d	$\frac{\text{H.P.}}{\text{N}}$	Diameter d
·012	0·753	4·5	5·44
·025	0·963	4·75	5·54
·050	1·213	5·0	5·63
·075	1·389	5·5	5·82
·1	1·529	6·0	5·99
·15	1·750	6·5	6·15
·2	1·926	7·0	6·30
·25	2·075	7·5	6·45
·3	2·205	8·0	6·59
·35	2·321	9	6·85
·4	2·428	10	7·10
·45	2·524	11	7·33
·5	2·614	12	7·53
·6	2·777	13	7·75
·7	2·925	14	7·94
·8	3·06	15	8·12
·9	3·18	16	8·30
1·0	3·29	17	8·47
1·25	3·55	18	8·63
1·5	3·77	19	8·79
1·75	3·97	20	8·93
2·0	4·15	21	9·08
2·25	4·32	22	9·23
2·5	4·47	23	9·38
2·75	4·61	24	9·51
3·0	4·75	25	9·66
3·25	4·88	26	9·76
3·5	5·00	27	9·88
3·75	5·12	28	10·00
4·00	5·23	29	10·12
4·25	5·34	30	10·23

If the shaft is of cast iron or steel, or if bending action is to be allowed for, proceed as indicated at the foot of the preceding Table.

99. Usually in line shafting the power is taken off at

various points in the length of the shaft. Hence, the shaft may be gradually reduced in diameter, and thus material is economised and the friction diminished. A long shaft consists of lengths of shafting, each of uniform diameter, the reduction of diameter being made in passing from one length to the next. When a shaft is calculated very closely in size, its ends must have bosses, to receive the keyways for fixing the couplings. The plan of graduating the size of the shaft to the work transmitted, has some serious disadvantages. The bearings which support the shaft and the couplings, wheels and pulleys fixed to it, are not interchangeable in position, if the diameter of the shaft is variable. This gives rise to much trouble and expense, if the machinery requires to be re-arranged. Shafts of uniform diameter are now often adopted, and for such shafts the forging of bosses on the ends is unnecessary, because at most points the shaft has surplus strength.

Small shafts often give trouble from insufficient stiffness, although they have ample strength. For such shafts, $\frac{3}{8}$ in. to $\frac{5}{8}$ in. may be added to the diameter, which is sufficient for strength, in order to secure stiffness and freedom from vibration. For long shafts, and when $\frac{\text{H. P}}{\text{N}}$ is less than 1, the diameter may be calculated by Redtenbacher's rule, which makes the angle of torsion a fixed proportion of the length of the shaft. Then,

$$d=\beta\sqrt[4]{\frac{\text{H. P.}}{\text{N}}} \quad . \quad . \quad . \quad . \quad (27)$$

where β has the same value as before, and the diameter is to be multiplied by the same values of n, to allow for bending action.

The span between the bearings of shafting should be so arranged as to limit the deflection of the shaft to a fixed proportion of its length. Let L be the span between the bearings, in inches. Then

$$L = \gamma \sqrt[3]{d^2} \quad . \quad . \quad . \quad . \quad (28)$$

where $\gamma = 60$ to 75, for shafting supporting its own weight only, and $\gamma = 54$ to 60, for shafting carrying the ordinary proportion of pulleys or gearing.

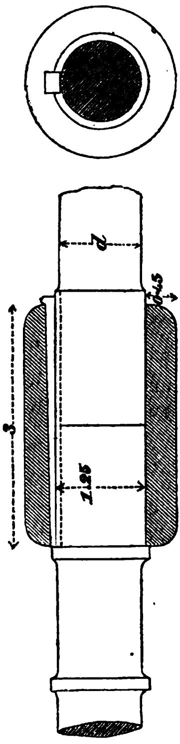


Fig. 104.

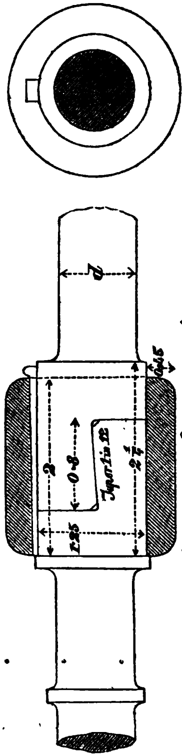


Fig. 105.

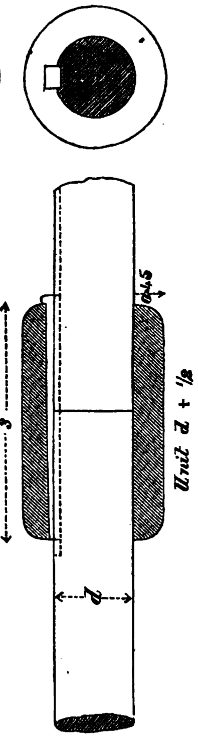


Fig. 106.

Couplings.

100. It is inconvenient to make shafting in long lengths, and hence, lines of shafting are composed of separate lengths,

coupled together. The coupling may be a permanent, or *fast coupling*, which can only be removed by unscrewing bolts or slacking keys, or a disengaging, or *loose coupling*, which is provided with arrangements for throwing part of the shafting out of gear as often as necessary. In some loose couplings, the connection is a frictional one only, and if any sudden strain comes on the machinery, the coupling slips. Special forms of coupling are used when the connected shafts are not in one line.

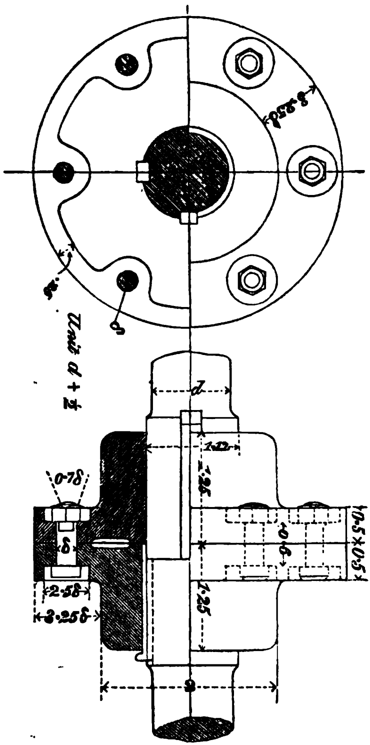


Fig. 107.

101. *Fast couplings.*—Figs. 104, 105, and 106, show three forms of coupling, known as 'box' or 'muff' couplings. In these a cast-iron box, or hollow cylinder, is fitted over the ends of the shafts. In figs. 104 and 106, the coupling is termed a butt coupling, and relative movement of the shafts is prevented by a wrought-iron key, which lies in a key-way, cut half into the box and half into the shaft ends. Fig. 105, is a half lap coupling, the shaft-ends overlapping, so as to prevent relative motion independently of the key, whose chief function is to fix the box rigidly in place. The keys are proportioned according to the rules in § 67. In fig. 105,

the key is a saddle-key. The other dimensions may be obtained from the proportional numbers. The half-lap coupling is an excellent coupling for shafts not exceeding 5 ins. diameter, but is somewhat expensive. The butt coupling is cheaper, but less secure. Both forms are free from projections likely to catch the clothes of a workman.

102. Fig. 107 shows a flange, or face-plate coupling. It consists of two parts of cast iron, firmly fixed by keys to the two shaft ends. The face of each coupling is turned after it has been keyed on the shaft, so that it is accurately perpendicular to the axis of the shaft. The couplings being brought together are fixed by bolts, which prevent relative movement by their resistance to shearing. In the coupling shown, the bolt-heads are sunk in the substance of the flange, for safety. The bolt-holes must be drilled, and the bolts carefully fitted. The number of bolts may be

$$n=3+\frac{d}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

the nearest even number being usually taken.

Let R be the radius of the bolt circle. Then, when the shaft is strained to its limit of elasticity, eq. 21 gives, for the shearing force on the bolts,

$$P=\frac{f\,d^3}{5{\cdot}1\,R}$$

The resistance of the bolts to shearing is $\frac{\pi}{4}n\delta^2 f_s$. Equating this to the shearing force, we get, for the diameter of the bolts,

$$\delta=0{\cdot}577\sqrt{\left(\frac{d^3}{n\,R}\right)} \quad . \quad . \quad . \quad . \quad (30)$$

In practice the bolts are often a little larger, and may be

$$\delta=\frac{d}{n}+\tfrac{1}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30a)$$

To keep the shafts in line, the end of one shaft may enter into the coupling on the other $\frac{1}{4}$ to $\frac{1}{2}$ inch.

103. *Sellers's Double Cone Vice Coupling.*—With box couplings it is generally necessary to forge bosses on the shaft-ends, to receive the couplings. This prevents pulleys and wheels being put on the shafts from the ends. The face-plate coupling depends for its solidity on a taper key, and cannot be often loosened without danger of impairing its accurate adjustment relatively to the axis of the shaft. Mr. Sellers has introduced a coupling which obviates these difficulties, and which does not require such perfect fitting. Fig. 108 shows this coupling in longitudinal section, and end elevation and cross section. It consists of an outer cylindrical muff, or barrel, enclosing the ends of the shafts. The inside of this is turned to a double conical form. Between the barrel and the shaft are two sleeves, the outsides of which are conical, and fit the box, and the insides are cylindrical, and fit the shaft. These sleeves are pressed together by three screw-bolts, parallel to the shaft. The

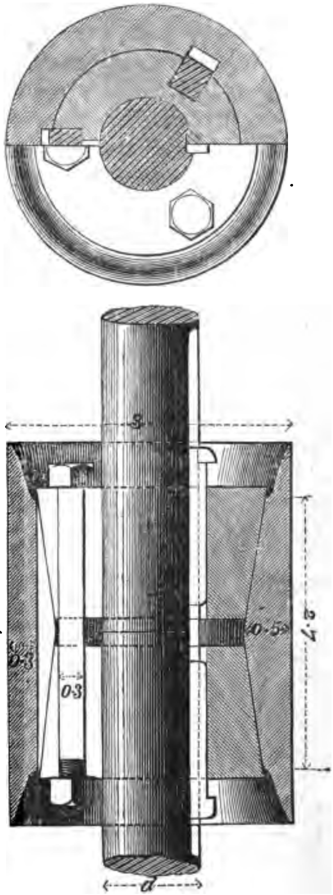


Fig. 108.

bolts are square in section, and rest in slots cut into the sleeves and the barrel. To give elasticity to the sleeves, they are completely cut through on one side, at the bottom of one of the bolt slots. Each sleeve is drawn inwards with equal force, and grasps the shaft with equal tightness. A key is driven into each shaft-end, as an additional precaution, but these keys should fit sideways only, and not at top and bottom. They do not then exercise any bursting force on the coupling. Absolute equality of size of

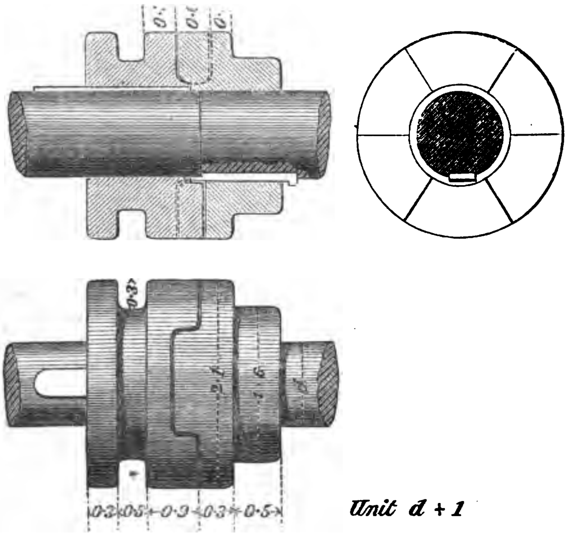


Fig. 109.

the shafts is unnecessary. When two shafts of unequal size are connected, the larger is turned down, at the end, to the size of the smaller. These couplings are sometimes difficult to disconnect. To obviate this, it has been proposed to tap a set screw through the barrel, having a conical end pressing against the two inner sleeves. When this set screw is turned, it separates the sleeves. If the parts are well oiled before they are put together, there is no great difficulty in

disconnecting. The bolts are taken out, and the coupling struck with a wooden mallet, or a wedge is driven into the split through the sleeves.

104. *Claw coupling.*—For very large, slowly-rotating shafts, it is desirable that the coupling should have a slight amount of play, so that when the shafts are a little out of line, the coupling accommodates itself to the obliquity, without straining the shafts. The claw coupling may then be used,

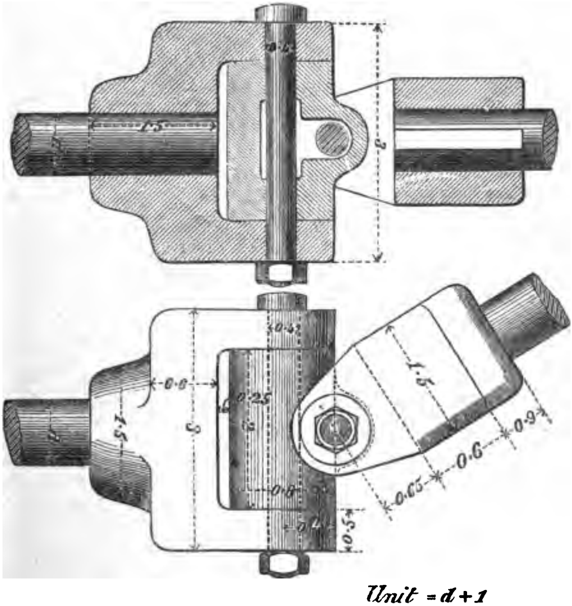


Fig. 110.

either as a fast coupling, or so arranged that one half can be slid back, and the shafts thrown out of gear. Fig. 109 shows this coupling, arranged for disengaging. It consists of two parts, like the face-plate coupling; but each part has projections, which fit in recesses in the opposite coupling.

In the coupling shown, the left-hand part is prolonged, and has a groove cut round it. In this fit the jaws of a lever, for sliding it back. The right-hand part is firmly keyed on its shaft. The left-hand part slides on a fixed key, or feather, which is not tapered. The claws of one coupling fit a little loosely in the recesses of the other, so as to permit a small amount of play.

105. *Universal coupling.*—When the axes of two shafts which are not in line intersect, they may be connected by a Hooke's joint, or universal coupling, shown in fig. 110. The velocity ratio of the shafts is then variable, but if their directions make a small angle, the variation is not great, and is generally unimportant. The proportional unit for the dimensions is $d+\frac{1}{2}$ or $d+1$.

NOTE.—*Planished wrought-iron shafting.* The round bars which come from the rolling-mill are rough and slightly crooked. Shafting made from such bars must be turned from end to end in the lathe, to obtain uniformity of diameter and smoothness of surface. A process has, however, lately been introduced which promises to supersede turning in many cases. By passing the bar while still hot between rapidly revolving bevilled rollers, the scale is cleaned off and the bar rendered so straight and regular that it may be used for shafting, after having been merely polished with a file and emery stick, either in the lathe or in place.

necessary, to make the bearing permanently independent of wear. That is, to line the bearing with two brass or gunmetal steps, which can be removed and replaced by new ones, when so much worn that the cap adjustment is insufficient to keep the journal steady. The brass steps tend also to preserve the journal from irregular wear.

Fig. 112 shows a typical pedestal, with the foundation, or wall plate, on which it is fixed. This wall plate spreads the pressure of the pedestal over a larger area, and affords a levelled surface, on which the pedestal can be adjusted with less trouble than on the rough masonry of a wall. The steps are shown externally of hexagonal form, the shape most convenient for hand fitting. They are often cylindrical, and are then turned in the lathe, and the pedestal is bored out to receive them. The steps have flanges, to prevent lateral movement. The under surface of the pedestal, and the upper surface of the wall plate, have narrow chipping strips, to facilitate the adjustment of level. The bolt holes in the wall plate and pedestal base are oblong, so that the pedestal can be shifted laterally in either direction. When adjusted to its true position, it is fixed by hard wood wedges, driven between the ends of the pedestal and jaws cast on the wall plate.

The diameter and length of the steps are the same as those of the journal. The other dimensions may be obtained from the proportional figures, the unit for which is $d + \frac{1}{2}$.

107. Fig. 113 shows sections of three ordinary forms of brass or gunmetal steps, and a half plan, half longitudinal, section of a step. The step is fitted to the pedestal for a portion of its width only, at each end, the intermediate part being recessed, and left rough. When the step is turned, instead of being fitted by hand, it sometimes bears on the pedestal over its whole width. The hexagonal step cannot turn in its seat. The cylindrical step requires a snug, to prevent turning. The composition of the gunmetal, white

brass and phosphor bronze, used for steps, is given in Chapter I. When antifriction metal is used, it is usually applied as a lining to a gunmetal step, and is cast in shallow recesses formed to receive it.

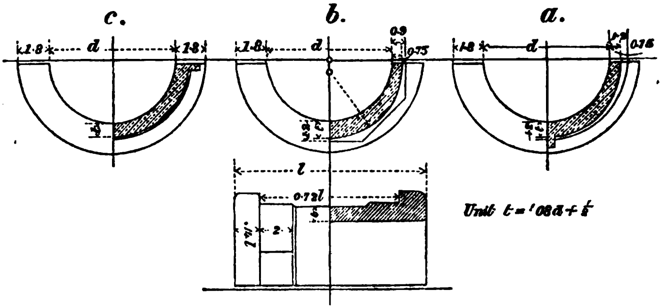


Fig. 113.

The thickness of the steps at the bottom, where the wear is greatest, may be

$$t=0{\cdot}07d+\tfrac{1}{8} \text{ to } 0{\cdot}1d+\tfrac{1}{8}.$$

At the sides the thickness may be $\frac{3}{4}t$. The proportional unit for the dimensions of the steps is t.

108. *Weight of pedestals.*—The approximate weight of the cast iron in pedestals is given approximately by the following equation :—

$$w=1{\cdot}1d^3+ 18 \text{ lbs.}$$

and the weight of a pair of steps is

$$w=0{\cdot}23d^3+ 6 \text{ lbs.}$$

Table of Pedestal Proportions.

Diameter of journal, in ins.	Length of bearing, in ins.	Height to centre	Diameter of bolts	Size of bolt holes	Length of base	Centres of cap bolts	Centres of base bolts	Thickness of step at bottom
$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{8}$	$\frac{1}{2}$	$\frac{5}{8} \times 1$	$8\frac{7}{8}$	$3\frac{1}{2}$	$7\frac{1}{4}$	$\frac{1}{4}$ to $\frac{5}{16}$
2	3	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{3}{4} \times 1\frac{1}{4}$	11	$4\frac{3}{8}$	9	$\frac{5}{16}$ to $\frac{3}{8}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{7}{8} \times 1\frac{1}{2}$	$13\frac{1}{4}$	$5\frac{1}{4}$	$10\frac{7}{8}$	$\frac{5}{16}$ to $\frac{7}{16}$
3	4	$3\frac{3}{4}$	$\frac{7}{8}$	$1 \times 1\frac{5}{8}$	$15\frac{1}{2}$	$6\frac{1}{8}$	$12\frac{5}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{5}{16}$	1	$1\frac{1}{8} \times 1\frac{3}{4}$	$17\frac{1}{2}$	7	$14\frac{3}{8}$	$\frac{3}{8}$ to $\frac{1}{2}$
4	5	$4\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{1}{4} \times 2$	20	$7\frac{7}{8}$	$16\frac{1}{4}$	$\frac{7}{16}$ to $\frac{9}{16}$
5	6	6	$1\frac{3}{8}$	$1\frac{3}{8} \times 2\frac{1}{4}$	24	$9\frac{5}{8}$	$19\frac{7}{8}$	$\frac{1}{2}$ to $\frac{3}{4}$
6	7	7	$1\frac{5}{8}$	$1\frac{7}{8} \times 2\frac{1}{2}$	$28\frac{1}{2}$	$11\frac{3}{8}$	$23\frac{3}{8}$	$\frac{9}{16}$ to $\frac{13}{16}$
7	8	$8\frac{1}{8}$	Two $1\frac{1}{4}$	$1\frac{3}{8} \times 2\frac{1}{4}$	...	$12\frac{1}{4}$	...	$\frac{5}{8}$ to $\frac{7}{8}$
8	9	$9\frac{1}{8}$	,, $1\frac{1}{2}$	$1\frac{3}{4} \times 2\frac{1}{2}$	...	14	...	$\frac{11}{16}$ to 1
9	10	$10\frac{1}{4}$	,, $1\frac{5}{8}$	$1\frac{7}{8} \times 2\frac{1}{2}$	...	$15\frac{3}{4}$	...	$\frac{3}{4}$ to 1
10	11	$11\frac{1}{2}$	,, $1\frac{3}{4}$	$2 \times 2\frac{3}{4}$	...	$17\frac{1}{2}$	...	$\frac{7}{8}$ to $1\frac{1}{8}$
12	13	$13\frac{1}{2}$	,, $2\frac{1}{8}$	$2\frac{3}{8} \times 3\frac{1}{8}$	...	21	...	1 to $1\frac{1}{8}$

From 7″ upwards, the pedestals have two bolts on each side, both in cap and base plate.

109. When a pedestal requires to be elevated above its support, the form shown in fig. 114 is used. The propor-

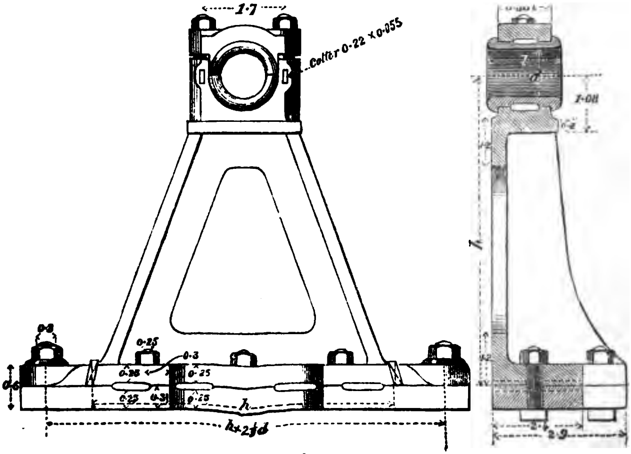


Fig. 114.

tions of the steps, cap and cap bolts, are the same as for an ordinary pedestal. The other dimensions are given on the figure, the proportional unit being, as before, $d+\frac{1}{2}$.

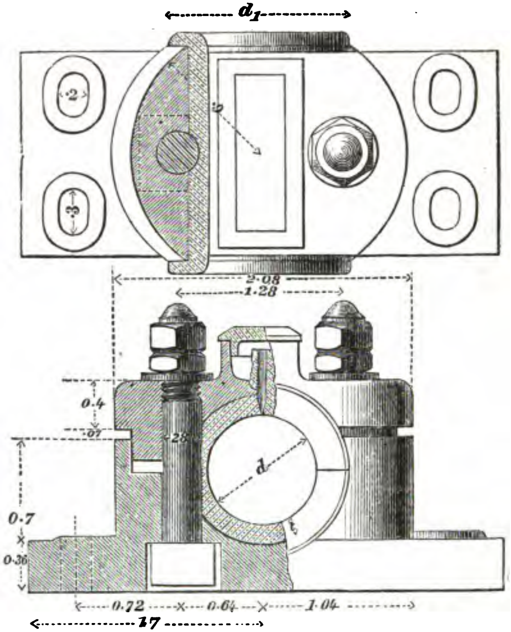


Fig. 115.

At times a pedestal requires to be contracted in dimensions. Fig. 115 shows a very neat and compact pedestal, designed by Mr. Arthur Rigg, C.E. The cap fits in a cylindrical recess in the body, which can be turned out in the lathe. The bolt holes are bored out, and the recess for the steps also. The pedestal may be still further contracted, by making the cap bolts double-ended, and using them both for attaching the cap to the body, and the body to its support. The base is then absent.

Sometimes a pedestal has to be fixed to a wall. Then the bracket pedestal shown in fig. 116 is used. The unit

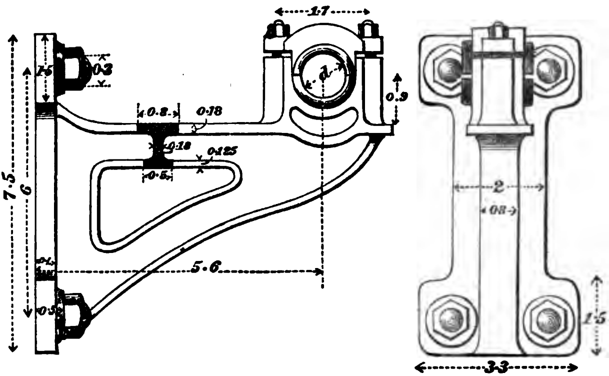


Fig. 116.

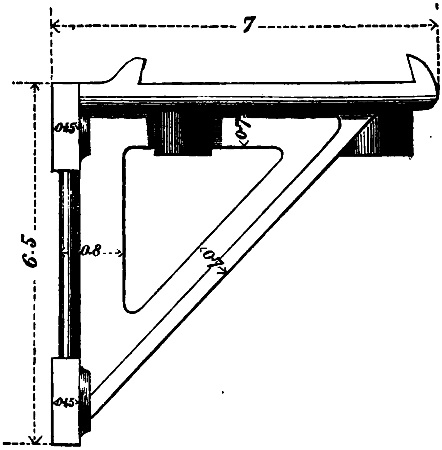


Fig. 117.

for the proportions is $d+\frac{1}{2}$. An ordinary pedestal may be used, fixed on a bracket such as that shown in fig. 117. The

recess under the bearing in fig. 116 serves to receive a tin dish, which catches the oil drippings. When a pedestal is fixed in a wall, a wall box is used. These wall boxes (fig.

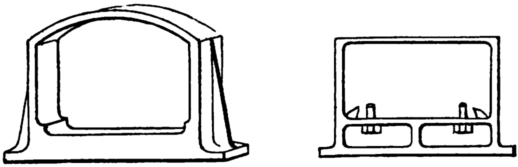


Fig. 118.

118) not only give a firm and level support to the pedestal, but they carry the wall over the opening, and give a regular form to the opening.

110. *Long bearings for high speed shafts.*—When a shaft runs at a high speed, the bearings must be long, to secure durability. The steps are then often of cast iron, which answers well as a support for wrought iron, if sufficient bearing surface is given. But the longer the bearings are, the more important it becomes that they should be exactly concentric, and in line. For long shafts, it is then desirable to give the steps a spherical seat, so that they may, to some extent, adjust themselves to the position of the shaft. In America fast running shafts, supported on cast-iron bearings four diameters long, have been extensively used, and for carrying these shafts,

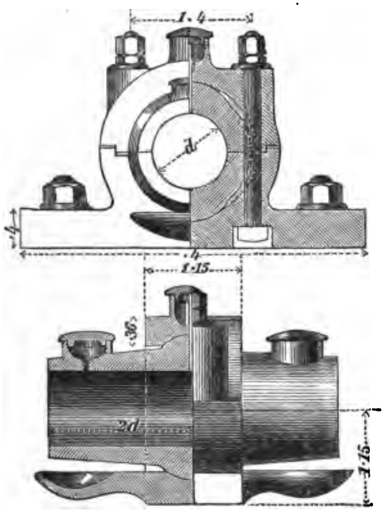


Fig. 119.

Mr. Sellers has introduced the pedestal shown in fig. 119. The steps are supported on the spherical parts, and can rotate slightly, either horizontally or vertically. The spherical parts ought, in strictness, to be struck from the same centre on the axis of the shaft. In the drawings in Mr. Sellers' paper ('Journal of Franklin Institute,' 1872, and 'Engineering,' vol. xv. p. 17), the spherical surfaces seem to be struck with a rather shorter radius. The lubrication of these pedestals is peculiar. The ordinary lubrication is at the centre of the pedestal; in addition to this, two cup-shaped hollows are formed near the ends of the top step. These are filled with a mixture of tallow and oil, which is solid at ordinary temperatures, and melts at about 100° F. If the step heats from failure of the ordinary lubrication, the tallow melts, and prevents injury to the shaft. A drip cup is provided under each end of the pedestal.

111. *Self-lubricating pedestals.*—Many pedestals have

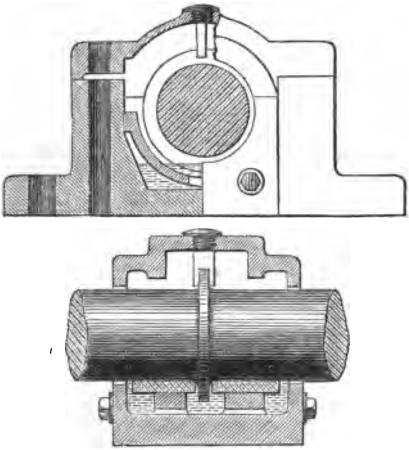


Fig. 120.

been designed with oil reservoirs, which enable the pedestal to run six months, without additional lubrication. Fig. 120

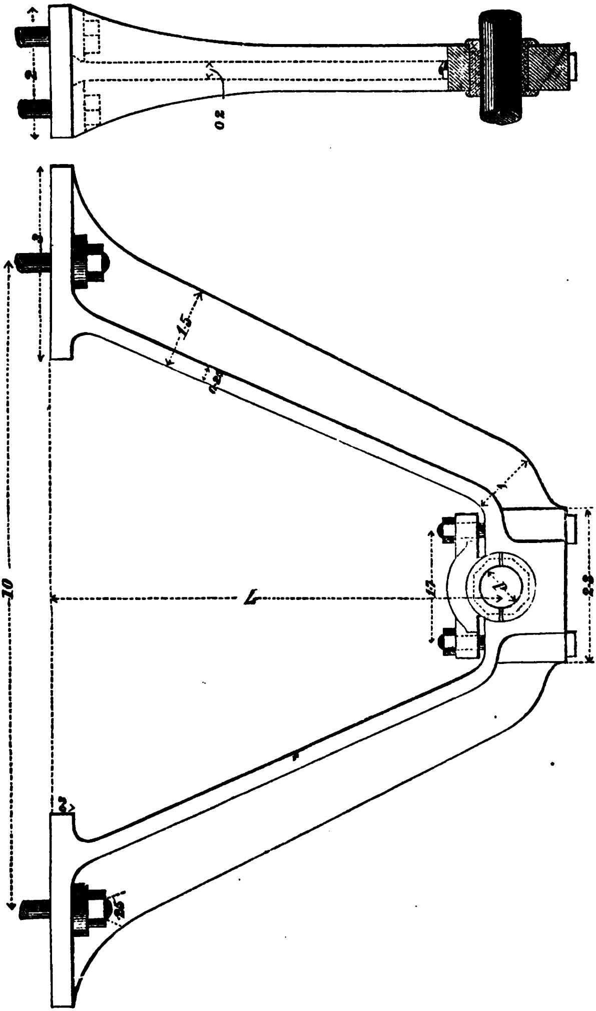


Fig 121

shows Möhler's pedestal. This has a lower brass, only divided into two portions by a collar on the shaft. The lower part of the pedestal is hollow, and forms a reservoir, into which the collar dips. As the shaft revolves, the collar

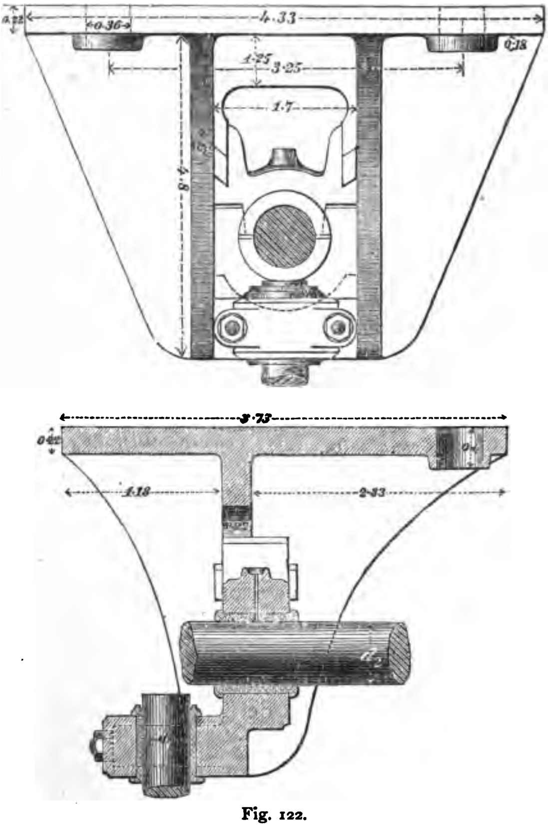


Fig. 122.

lifts the oil, and distributes it to the shaft on either side. The surplus oil flows back into the reservoir. The objection to these pedestals is that they require a large supply of

oil at first, which gradually becomes viscid by absorption of oxygen, and is then useless.

112. *Hangers.*—When a shaft is supported from the ceiling girders, the pedestal is modified in form, and is termed a hanger. Two forms are used; in one, fig. 121, the pedestal base is bent round and upwards, and attached to the ceiling on both sides; in the other, the pedestal is supported on one side only. The latter arrangement facilitates the dismounting of the shaft, but requires more metal in the hanger.

113. Fig. 122 shows a hanger for two shafts, whose directions intersect. This happens when one shaft drives another by bevil gearing. The cap of the upper pedestal is kept in place by keys. The steps, bolts, and caps of this hanger may be designed as for ordinary pedestals. The proportional unit for the remainder of the pedestal is

$$d_2 + 0{\cdot}4\, d_1 + \tfrac{1}{2},$$

where d_2 is the diameter of the greater, and d_1 that of the smaller, of the two shafts.

AXLE BOXES.

114. Axle-boxes are peculiarly formed journal-bearings, by which the weight of locomotive engines and railway carriages is transmitted to the axles. The axle-boxes of carriages are of very various forms, and will not be treated here. The axle-boxes of locomotive engines are more simple, and may be briefly considered, as illustrations of modified pedestals. In a good axle-box the lubrication should be constant, and not wasteful; the journal should be protected from grit; and should fit easily in the step, with a moderate amount of end play. Axle-boxes consist of an outer casing, a step of gunmetal or other alloy, and a hollow shell, closing the under side of the box, and receiving the surplus oil. The outer casing is accurately faced on

both sides, to fit the space prepared for it in the horn plates, between which it slides vertically, and it is provided with flanges, which permit a small amount of lateral play. The upper part is formed into an oil box, from which copper tubes conduct the oil to the journal. It has also usually a socket, to receive the end of the spindle, through which the

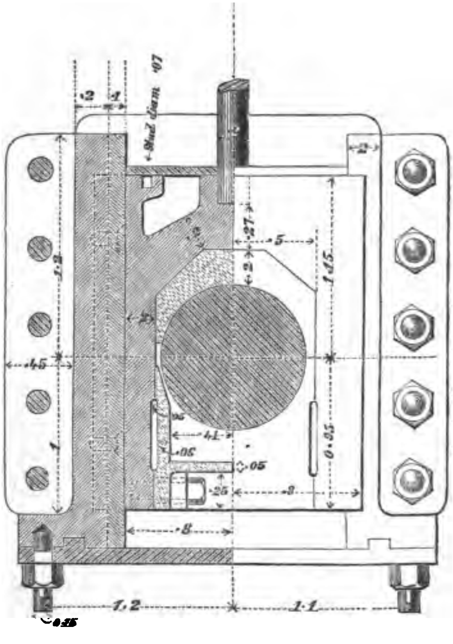


Fig. 123.

weight of the engine is transmitted to the box. This rod bears on a plate spring at its upper end. External axle-boxes are closed in front, and being of rather complex form, are of cast iron. Axle-boxes for internal framings are simpler, and are of forged wrought iron, or of cast steel. In the older axle-boxes, wedges were provided on each side, to com-

pensate for wear, but these are now generally omitted. The step is like that of an ordinary pedestal, and is sometimes lined with soft metal. For rolling stock the pressure does

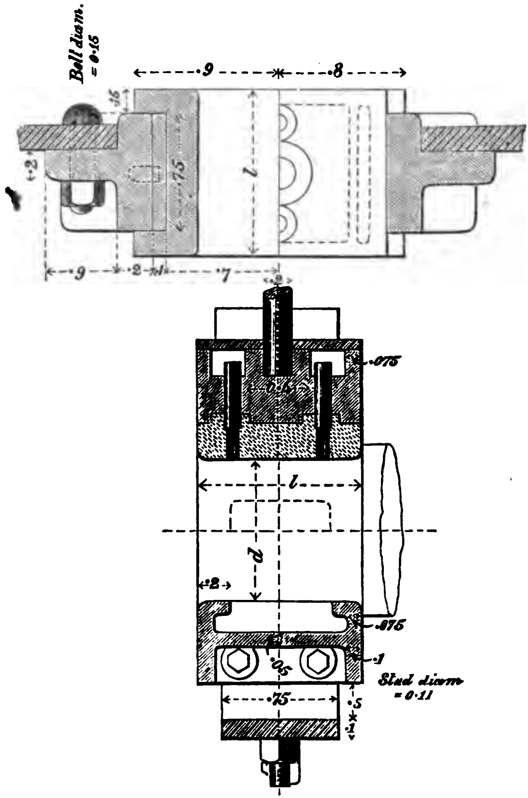


Fig. 124.

not generally exceed 300 lbs. per sq. in. of bearing surface, and for engines it rarely exceeds 224 lbs. The bearing surface is the product *dl* of the diameter and length of the journal.

The axle-box shown in figs. 123, 124, is a trailing axle box of cast iron. Let D be the diameter of the cylinder. The bearing surface on each side may be $0{\cdot}4\ D^2$ The thickness of the step is $\frac{d}{5}$ to $\frac{d}{6}$, where d is the journal diameter. Lengthways, the step may be $\frac{1}{16}$th shorter than the journal, to allow a little end play. The unit for the proportional figures is $d+\frac{1}{2}$.

FOOTSTEP BEARINGS.

115. When a shaft is vertical its lower end rests in a kind of pedestal, termed a footstep. The ordinary arrangement is shown in fig. 124. Lateral motion is prevented by

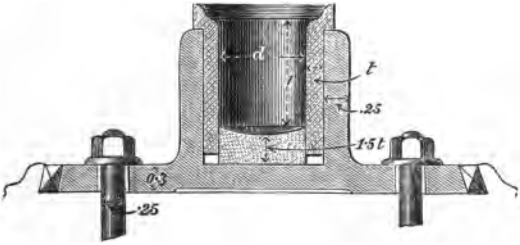


Fig. 125.

a brass bush, fitting in a cast-iron fixing, and end movement, by a brass or steel slightly cup-shaped disc, on which the shaft pivot revolves. The thickness of the brass may be $t=0{\cdot}07\,d+\frac{1}{8}$. The unit for the other dimensions is $d+\frac{1}{2}$.

When exact vertical and lateral adjustment of the footstep is necessary, the arrangement in fig. 126 is adopted. The lateral adjustment is effected by four set screws, and the vertical adjustment by a single set screw. The horizontal screws are tapped into the casting, and fixed by lock nuts. The vertical screw has two nuts. Unit $d+\frac{1}{2}$.

When a footstep works under water, there is difficulty in ensuring proper lubrication of the pivot. Fig. 127 shows a

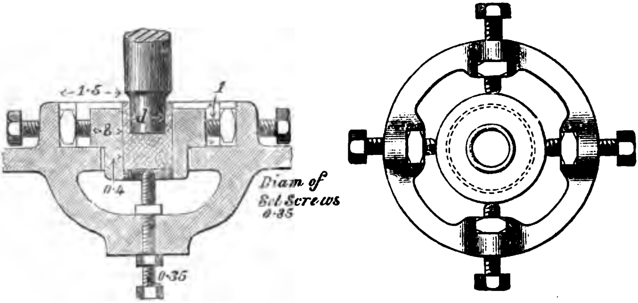


Fig. 126.

turbine pivot, enclosed in an oil casing, through which a flow of oil can be ensured. Two small copper pipes are connected with the casing, and these are conducted to points

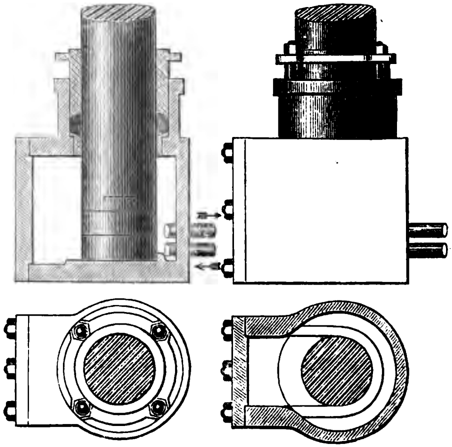


Fig. 127.

above the water level. The shaft passes into the casing through a stuffing box. The end of the shaft is provided

with a steel disc, working on a similar disc fitted in the casing.

Another arrangement which is very effective, is to dispense with the ordinary metal pivot, and replace it with a pivot of lignum vitæ. For metal on wood, water is an excellent lubricant, and such bearings work with very little

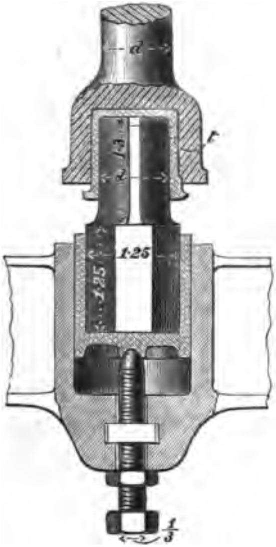


Fig. 128.

wear under great pressures. The pivot, fig. 128, is inverted, so that grit is less likely to enter. The pivot is adjusted vertically by a set screw. The end of the shaft is enlarged, bored out, and fitted with a brass step. A groove is cut round the pivot, which being always filled with water, ensures proper lubrication.

CHAPTER IX.

TOOTHED GEARING.

116. GEARING is a general term for the means of transmitting motion, but it is especially employed to denote the wheels by which motion is transmitted from one shaft to another. The wheels employed for transmitting motion are almost always toothed wheels, but it is convenient to study first the action of toothless rollers, because each kind of toothed wheel is equivalent cinematically to a toothless roller.

Constant Velocity Ratio.

Parallel shafts.—Let two accurately-turned cylindrical rollers be keyed on the shafts, of such a size that they are in contact. Then, if one shaft revolves, the other must revolve also, provided the resistance to motion is not greater than the frictional resistance of the rollers to slipping. If there is no slipping, the velocities of the rollers at the point of contact must be equal.

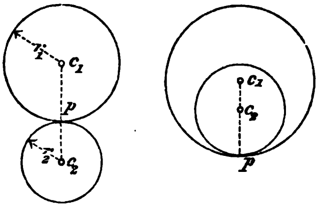


Fig. 129.

Let N_1 N_2 be the number of revolutions of the shafts per minute, and r_1, r_2 the corresponding radii of the wheels. The velocities at the point

of contact are $2\pi r_1 N_1$, and $2\pi r_2 N_2$. Since these are equal,

$$\frac{N_1}{N_2} = \frac{r_2}{r_1}$$

and the velocity ratio is constant. Toothed wheels corresponding to rollers of this kind, are called spur wheels. The surfaces of the rollers are termed *pitch surfaces*. Planes normal to the shafts cut the pitch surfaces in circles termed *pitch lines*. The point of contact, p, is the *pitch point*.

Shafts the directions of which intersect.

Let two conical rollers be placed on the shafts, then one will drive the other with uniform velocity ratio, as in the last case. Toothed wheels corresponding to these rollers are

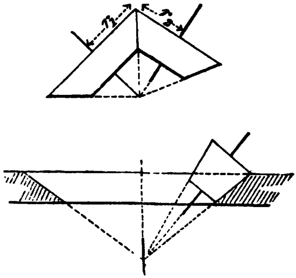


Fig. 130.

termed bevil wheels. In practice, the shafts are in most cases at right angles. The radius, or diameter, of bevil wheels is conventionally measured at the larger end.

Shafts the directions of which are not parallel and do not intersect.

Place on the shafts frusta of hyperboloids, generated by the revolution of a straight line about an axis to which it is not parallel. Then one shaft will drive the other with constant velocity ratio, though there will be sliding in the direc-

tion of the line of contact. Toothed wheels corresponding to these rollers are termed skew bevil wheels.

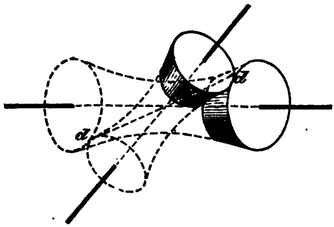


Fig. 131.

VELOCITY RATIO VARIABLE.

In all the preceding cases the sections of the rollers by planes perpendicular to the shafts are circular, and the ratio $\frac{r_2}{r_1}$, which is also the velocity ratio, is constant. If the rollers are of elliptical or of some other form of section, the ratio $\frac{r_2}{r_1}$ and the velocity ratio are variable. In any case the condition $r_1+r_2=$ constant must be fulfilled, or the contact of the rollers during rotation will cease.

117. With toothless rollers it is difficult to transmit much force, without causing a slipping of the rollers. The rollers may be covered with india-rubber or leather, which compresses a little, and thus neutralises the effect of inaccuracy of form, but such arrangements are rarely used. A better plan is to cut circumferential wedge-shaped grooves in the rollers, and to place them so that the projections on one roller fit the recesses of the other. Gearing of this kind is termed frictional gearing. The resistance to slipping is greater than with smooth rollers, but slipping is not altogether prevented. In some circumstances this is an advantage. A much more generally adopted modification of smooth rollers is to form teeth upon them; the teeth of

one wheel fit the spaces in the other, and slipping is prevented if the teeth do not break.

If the teeth are properly formed, the motion of the wheels is identical with that of two imaginary smooth rollers, whose surfaces form what are termed the pitch surfaces of the wheels. Hence, if r_1 r_2 are the radii of the wheels, measured to the pitch surfaces,

$$\frac{N_1}{N_2}=\frac{r_2}{r_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

as before. The importance of forming the teeth so that the velocity ratio is constant, is very great. If the velocity of the driven wheel varies a little during the contact of each pair of teeth, an irregular and injurious motion is imparted to the machinery. In addition, since the inertia of the driven machinery resists alteration of velocity, the tooth of the driven wheel will be carried forward relatively to the driving tooth, and will strike against the next tooth in front. The wheels then work with noise and vibration. This action is termed back lash.

118. *Material of gearing.*—Ordinary gearing is made of cast iron, and two methods are adopted in moulding wheels. The older plan is to construct an accurate pattern in wood of the entire wheel; from this pattern any number of wheels can be moulded. The other plan is to mould the rim of the wheel, from a pattern of two or three teeth only, in a wheel-moulding machine. The pattern of the teeth is fixed to a radial arm, which can be revolved very accurately, through any required fraction of the circumference. The arms of the wheel are formed by dry sand-cores, moulded in core boxes. Very small wheels are cast with a blank rim, and the teeth cut out in a wheel-cutting machine.

Pattern-moulded wheels are less accurate than machine-moulded wheels, partly because the wood pattern is liable to warp, and partly because in moulding the wheel, it is necessary to give slight taper, or draught, to the teeth.

The wheels should be placed in gear with the draught of one wheel in the opposite direction to the draught of its fellow, but this is not always attended to. The result is that the teeth bear on each other chiefly on one side. Machine-cut wheels are expensive, and the teeth are generally inferior in form to those of moulded wheels.

119. *Mortice wheels.*—When wheels are run at high velocities, the teeth of one of each pair are of wood, and are termed cogs. These cogs are morticed into an iron rim, and shaped by hand. The iron wheel, which works with a mortice wheel, is usually 'pitched and trimmed,' that is, the rough surface of the teeth is chipped away, and the teeth are filed perfectly smooth. This ensures greater accuracy in the form of the teeth, and prevents the destruction of the wood cogs by the rough surface of the casting. Mr. Heys states that, with machine-moulded wheels, it is only necessary to clear off the sand from the surfaces of the teeth, and to file them smooth. No chipping is necessary.

When wheels are subjected to vibration and shock, gun-metal or phosphor-bronze is preferable to cast iron. In a few cases, wheels have been made of wrought iron or cast steel. Mr. Heys prefers malleable cast iron to any other material in such cases.

120. *Relation between the number of teeth and the radius of the wheel.*—The distance measured along the pitch line, from the centre of one tooth to the centre of the next, is termed the *pitch* of the wheel. Ordinarily the pitch ranges from $1\frac{1}{2}$ in. to 4 ins., but sometimes it passes these limits. Let $p =$ the pitch, $d =$ the diameter, $n =$ the number of teeth in a wheel. For reasons which will appear presently, the pitch for any one wheel is made uniform all round the circumference. Then, np is equal to the circumference of the wheel,

$$\left.\begin{aligned} d &= \frac{p}{\pi} n \\ n &= \frac{\pi}{p} d \end{aligned}\right\} \quad \cdots \cdots \quad (2)$$

To lay off the pitch on the pitch line.—The followin struction is convenient, when the wheel is so large is impossible to find the exact pitch, by stepping rou pitch-line. Let the circle, fig. 132, be the pitch-line. At any point, *a*, draw the tangent *a b*. Make *a b* = the pitch. Take *a c* = $\frac{1}{4}$ *a b*. With centre *c*, and radius *c b*, draw the arc *b d*. Then the arc *a d* is = *a b*, and is the pitch laid off on the pitch-line. When the wheel has many teeth the arc *a d* sen coincides with its chord, but, if it has few teeth, there appreciable error in taking the chord *a d* equal to the p

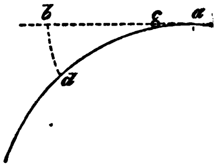


Fig. 132.

Pitch in inches	$\frac{\pi}{p}$	$\frac{p}{\pi}$
$\frac{1}{2}$	6·2832	0·1592
$\frac{3}{4}$	4·1888	0·2387
1	3·1416	0·3183
$1\frac{1}{8}$	2·7926	0·3581
$1\frac{1}{4}$	2·5132	0·3979
$1\frac{3}{8}$	2·2848	0·4377
$1\frac{1}{2}$	2·0944	0·4775
$1\frac{3}{4}$	1·7952	0·5570
2	1·5708	0·6366
$2\frac{1}{4}$	1·3963	0·7162
$2\frac{1}{2}$	1·2566	0·7958
$2\frac{3}{4}$	1·1424	0·8754
3	1·0472	0·9549
$3\frac{1}{2}$	0·8976	1·1141
4	0·7854	1·2732
$4\frac{1}{2}$	0·6981	1·4324
5	0·6283	1·5916
$5\frac{1}{2}$	0·5711	1·7507
6	0·5236	1·9099
$6\frac{1}{2}$	0·4833	2·0691
7	0·4488	2·2283
$7\frac{1}{2}$	0·4188	2·3875
8	0·3927	2·5465
9	0·3491	2·8647
10	0·3142	3·1829

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121. *Parts and proportions of teeth.*—Fig. 133 shows the general form of wheel teeth, drawn, for convenience, on a straight pitch line. *b d* is the thickness of a tooth ; *d e*, the side clearance ; *e b*, is the width of a space ; *b a* is the face, and *b c*, the flank of a tooth ; *g h* is the bottom clearance.

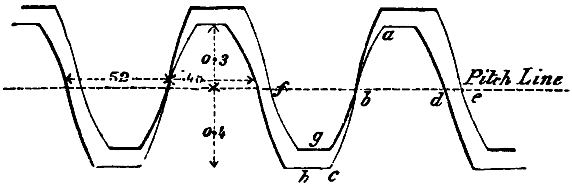


Fig. 133.

In working, the faces of the teeth of one wheel come into contact with the flanks of the teeth of the other wheel. Also, while the point of contact is approaching the line of centres, the flank of the driving acts on the face of the driven tooth ; while the point of contact is receding from the line of centres, the face of the driving acts on the flank of the driven tooth. The arc of the pitch-circle, through which the wheel turns during contact, is called the arc of contact, and the portions into which it is divided by the pitch point, are the arcs of approach and recess. The length of the arc of approach depends on the length of face of the driven tooth, and the arc of recess on the length of the face of the driving tooth.

In ordinary gearing, the teeth have the following proportions :—

Pitch $= p$

Thickness of tooth $= bd = \cdot 48\,p$ to $\cdot 485\,p$

Width of space $= be = \cdot 52\,p$ to $\cdot 515\,p$

Height of tooth $= \cdot 6\,p$ to $\cdot 75\,p$

Height above pitch line $= \cdot 25\,p$ to $\cdot 33\,p$

Depth inside pitch line $= \cdot 35\,p$ to $\cdot 42\,p$

The following proportions are good :—

Thickness of tooth	$= 0{\cdot}48\ p - {\cdot}03$,	for pattern-moulded wheels.
"	$= 0{\cdot}485 p - {\cdot}03$,	for machine-moulded wheels.
Width of space	$= 0{\cdot}52\ p + {\cdot}03$,	for pattern-moulded wheels.
"	$= 0{\cdot}515 p + {\cdot}03$,	for machine-moulded wheels.
Height above pitch line	$= 0{\cdot}3 p$	
Depth below "	$= 0{\cdot}35 p + 0{\cdot}08$	

Mortice wheels usually have the wooden cogs of one wheel thicker than the iron teeth of the other wheel, and the teeth are sometimes a little shorter than those of ordinary gearing. The clearance also may be very much reduced. The following proportions are good :

Thickness of iron teeth . . .	$0{\cdot}395 p$
" wood cogs . . .	$0{\cdot}595 p$
Height outside pitch line . .	$0{\cdot}25 p$
Depth inside pitch line . . .	$0{\cdot}29 p + {\cdot}08$

The width of the face of the wheel varies much. Increase of width does not increase the strength of teeth, but increases their durability. The average practice is to make the width of face 2 to $2\frac{1}{2}$ times the pitch.

Conditions determining the Form of Teeth.

122. *Condition of continuous contact of a pair of teeth.*—Let A and B be two spur wheels rotating at any moment with angular velocities $+\omega_1$ and $-\omega_2$. Nothing will be changed in the relative motion of the two wheels, if a rotation $-\omega_1$ is impressed on each. Then A's angular velocity of rotation will be $\omega_1 - \omega_1 = 0$, that is, it will be at rest. The centre of B will rotate about A with the velocity $-\omega_1$, and B

will rotate about its own centre with the velocity $-(\omega_1+\omega_2)$. Hence, the motion of B will be the same as if it were at the moment rotating about an axis, placed at a point dividing the line of centres in the ratio $\omega_1 : \omega_2$, or it will roll on the pitch line of A. Now let a tooth be fixed to B. In order that that tooth may remain continuously in contact with a tooth on A, the form of the latter must be the envelope of the successive positions of the tooth on B, as it moves round A. The form of the tooth on B is not arbitrary. Only certain forms give to the envelope shapes which are practically realisable as wheel teeth.

If two solids, such as two wheel teeth, move in contact, they must have equal velocities in the direction of their common normal. For if the velocities were not equal, one tooth would be penetrating into the space occupied by the other. But in the case above one tooth is at rest. Therefore the other can have no velocity in the direction of the normal to its surface at the point of contact. Hence the point of contact must always fall in such a position that the normal to the tooth at that point passes through the axis of rotation at the moment, that is, the point which divides the line of centres in the ratio $\omega_1 : \omega_2$, which coincides with what is commonly termed the pitch point.

Condition that the teeth may act simultaneously.—In order that all the pairs of teeth may have the same work to do, they must come into action at the same point, and remain in action while the wheel turns through the same angle. All the teeth should therefore be similar in form. In actual wheels, two pairs of teeth (sometimes more) are simultaneously in contact. Let α and β be two positions of the simultaneous points of contact of two teeth *a* and *b*. Then, under the conditions assumed above, when A is reduced to rest and B moves round it, the two envelopes passing through α and β are simultaneously described. Therefore ω_1, ω_2 must be the same for all the teeth, both in the position α and the position β. Since a corresponding relation must hold for

all pairs of contact points, ω_1 and ω_2 must be constant for the whole period of contact, that is, the velocity ratios of the wheels must be constant.[1]

Condition of constancy of velocity ratio.—To a certain extent this is secured by having the teeth numerous and small. But in addition, the single and sufficient condition which ensures the constancy of the velocity ratio, during the action of each pair of teeth, is this:—The common normal to two teeth at the point of contact, must always pass through the pitch point,[2] a condition which is fulfilled if one tooth is the envelope of the relative positions of the other.

Influence of the form of the tooth on its strength.—It will be seen presently, that the teeth tend to break across at the root. The teeth are stronger the shorter they are, and the thicker they are at the root. They cannot be shortened without reducing the arc of contact, and their length should be such as to ensure a sufficient, but not excessive, arc of contact. The thickness at the root depends on the form selected for the teeth. Involute teeth are generally stronger than cycloidal teeth: with cycloidal teeth, the teeth are stronger the smaller the diameter of the describing circle used for the flanks. In no case should the flanks be described with a rolling circle the diameter of which is greater than half the diameter of the pitch line, inside which it is rolled.

Conditions of durability.—The rolling of the teeth over each other, so as to spread the contact over a considerable length of tooth, is advantageous, but the sliding of the teeth against each other is injurious. The amount of sliding action is the difference of the length of the face of one tooth and the acting part of the flank of its fellow. To ensure durability these two should be nearly equal. The wider the face of the wheel, the more area there is to resist

[1] This was pointed out to the Author by Professor Huët of Delft.

[2] If the pitch line is not circular, and the common normal to the teeth always passes through the pitch point, the wheels have the same varying velocity ratio as smooth rollers, coinciding with the pitch lines.

the pressure. According to Willis, the friction is more injurious during the approach of the teeth to the pitch point, than when receding from it. For this reason it would be advantageous to shorten the faces of the teeth of the driven wheel, even at the cost of having to lengthen those of the driving wheel.

123. *Arc of contact.*—If from the points where the contact of a pair of teeth begins and ends, radii are drawn to the centre of the pitch circle, they will cut off an arc of the pitch circle, termed the arc of contact. This is divided by the pitch point into two parts, termed the arcs of approach and recess. To ensure continuous working, one pair of teeth must always be in gear, and hence the arc of contact must be at least equal to the pitch. Wheels are sometimes made so that contact occurs only during the receding of the teeth from the line of centres. Then the arc of recess must be at least equal to the pitch. Ordinary gearing is so constructed that the wheels are reciprocal, that is, either wheel of a pair may be the driver. In that case, the arcs of approach and recess are equal. In ordinary gearing the arc of contact varies from 1·6 to 2 times the pitch.

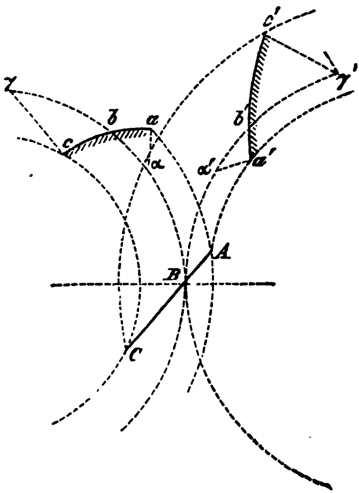


Fig. 134.

124. *Given the form of tooth of one wheel, to find the proper form of the tooth of another wheel, to gear with it.*—Let *a b c* be the given tooth; *b* B, *b′* B the pitch lines of the wheels; B, the

pitch point, or point of contact of the pitch lines. From any points, *a c*, draw normals, $a\,\alpha$, $c\,\gamma$, to the curve of the given tooth, cutting the pitch line in α, γ. Then the points *a b c* should be points of contact, when α, b, γ, are at the pitch point. From B set off $BA = a\,\alpha$, $BC = c\,\gamma$. Then A is the point where *a* is in contact; B the point where *b* is in contact, and C the point where *c* is in contact, and some line, passing through A B C, is the path of contact. Set off arc $B\alpha'$=arc $B\alpha$; also $\alpha'\,a' = \alpha\,a$. Then a' is a point in the tooth of the second wheel, which will come in contact with *a* at A, and will have a common normal, passing through the pitch point. Set-off arc $B\,b'$ = arc $B\,b$; then b' will come in contact with *b* at B. Also, set-off arc $B\gamma'$=arc $B\gamma$, and $\gamma'\,c' = \gamma\,c$; then c' will come in contact with *c* at C. A curve $a'\,b'\,c'$ through the points so found, will be the required tooth. In certain cases the construction becomes impossible, and the given tooth is of unsuitable form. Forms should be avoided which make a tooth entirely concave.

Cycloidal Teeth.

125. If a circle rolls on the circumference of another circle, a point in its circumference describes a curve, termed an epicycloid. If the rolling circle rolls inside the base circle, a point in its circumference describes a curve, termed a hypocycloid. It is shown, in treatises on applied mechanics, that, if the faces of the teeth of each wheel, and the flanks of the teeth of the other, are respectively epicycloids and hypocycloids, generated by the same rolling circle, rolling outside and inside the given pitch lines, then the teeth will move so that the common normal at the point of contact will pass through the pitch point, and the condition of uniform velocity ratio will be fulfilled. With teeth of this kind, the point of contact moves over an arc of a circle of radius equal to that of the rolling circle, and having its centre on the line of centres.

External Cycloidal Teeth.—Let A_1 A_2, fig. 135, be the pitch circles, p the pitch point, $a_1 c_1$ the tooth of A_1, and $a_2 c_2$ that of A_2. The face $p a_1$ and the flank $p c_2$ work together, and are described by the rolling circle R, rolled outside A_1 and inside A_2. The face $p a_2$ and the flank $p c_1$ work together, and are described by the rolling circle R′, rolled outside A_2 and inside A_1. The rolling circles in the figure are so drawn that their centres are on the line of centres. Draw arcs $a_2 x$, $a_1 y$ concentric with the pitch circles. Then, xpy is the path of contact. Suppose the wheels turn in the direction of the arrow, and that the lower one is the driver. During approach, the flank of the driver is in contact with the face of the driven tooth, and contact begins at y. During recess, the face of the driver acts on the flank of the driven tooth, and contact ends at x. Radii drawn through x and y will mark off on the pitch lines the arcs of approach and recess. Conversely, if the arc of approach or recess is marked off on the pitch line, and radii drawn cutting the rolling circles, the points x and y will be found, which define the height of the teeth for given arcs of contact.

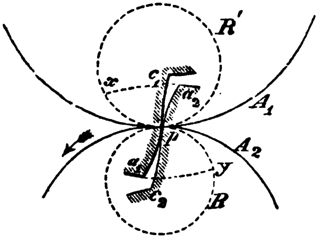


Fig. 135.

Internal cycloidal teeth.—Let A_1 A_2, fig. 136, be the pitch lines, and p the pitch point; a_1 c_1 the tooth belonging to A_1 and a_2 c_2, the tooth belonging to A_2. The flank $p c_1$ works on the face $p a_2$; both are epicycloids described by the rolling circle R, rolling

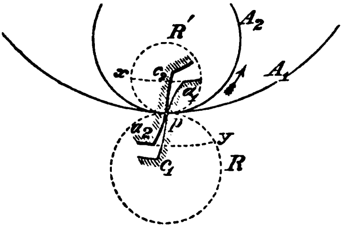


Fig. 136.

outside A_1 and A_2. The face $p\ a_1$, and the flank $p\ c_2$ work together, and are hypocycloids described by R' rolling inside A_1 and A_2. Through a_1 draw an arc $a_1\,x$ concentric with A_1, and through a_2, an arc $a_2\,y$ concentric with A_2, cutting the rolling circles in x and y, then $x\,p\,y$ is the path of contact. Radii drawn from the centres of A_1 A_2 to x and y, will mark off the arcs of approach and recess.

The diameter of the rolling circle must never be greater than the radius of the pitch circle, inside which it is rolled. When it is equal to the radius, the flanks of the teeth described by it, become radial straight lines. Teeth with radial flanks, were at one time much used. They are now not much used, except in certain cases, for mortice wheels. When a pair of wheels only are required, the rolling circle, for both faces and flanks of both wheels, may often be made, with convenience, equal in diameter to the radius of the smaller wheel.

When a set of wheels are to be constructed, any two of which will gear together, the same rolling circle must be taken for the faces and flanks of all the wheels of the set. It is then best to take, for the diameter of the rolling circle, the radius of the smallest wheel of the set. Let p be the pitch, d the diameter of the rolling circle, thus determined, n the number of teeth of the smallest wheel of the set.

$$d=\frac{n\,p}{2\,\pi} \qquad (3)$$

$n=$	$d=$
11	$1{\cdot}751\,p$
12	$1{\cdot}910\,p$
13	$2{\cdot}068\,p$
14	$2{\cdot}228\,p$
15	$2{\cdot}387\,p$
16	$2{\cdot}546\,p$
20	$3{\cdot}183\,p$
25	$3{\cdot}981\,p$

Since the introduction of machine-moulding, it is less an object than it used to be to make wheels in sets.

126. *Methods of drawing cycloidal teeth.*—The curves of the teeth may be found by rolling a templet of the size of the rolling circle, inside and outside templets of the size of the pitch circle. A pencil held in contact with the rolling templet, describes the required curve. The curves may also be obtained by the ordinary rules for describing cycloidal curves. When they have been drawn, it is usually convenient to replace the cycloidal curves by circular arcs, sensibly coinciding with them, and which can be used by the pattern-maker more conveniently than the true curves. In proceeding thus, two sources of error are introduced. It is not easy to draw small cycloidal arcs very exactly, and in fitting circular arcs to them, a new source of error is introduced. To obviate these objections, it was proposed by Professor Willis, to find directly the centres of circular arcs which would approximate to the cycloidal arcs. The method of Professor Willis, however, does not give a very good approximation, the teeth being too thin at the points, and too thick at the roots.

The following method, founded on Rankine's rules for rectifying circular arcs, gives a much nearer approximation to the true curves; in fact, with teeth of ordinary size, there is no appreciable difference between the cycloidal and circular arcs. The method is also much easier in practice than that of drawing first the true curves. The method is based on this principle. For each cycloidal arc a circular curve is found, which coincides with it at the pitch line, and at $\frac{2}{3}$ its length from the pitch line, and which has at the latter point a common normal with it.

In fig. 137, the strongly marked circle $b\,p\,b'$ is the pitch circle, and p the pitch point. The complete dotted circles are the rolling circles. The height of the tooth outside the pitch line is $p\,r$, and its depth within it is $p\,s$, so that the circles through r and s, are the addendum and root circles.

The arcs $v\,p\,w$ mark the path of contact. As the rolling circle R rolls to the position R′, the tracing point moves from p to m, marking out the epicycloid $p\,m$, which forms the face of the tooth.

Method I.—Take $p\,c = \frac{2}{3}\,p\,r$, and draw the arc $c\,e$ concentric with the pitch line. Step off arc $p\,b$ = arc $p\,e$. Take the chord $p\,e$ in the compasses, and with centre b mark off $b\,m = p\,e$; then m is a point of the true epicycloid, and $m\,b$

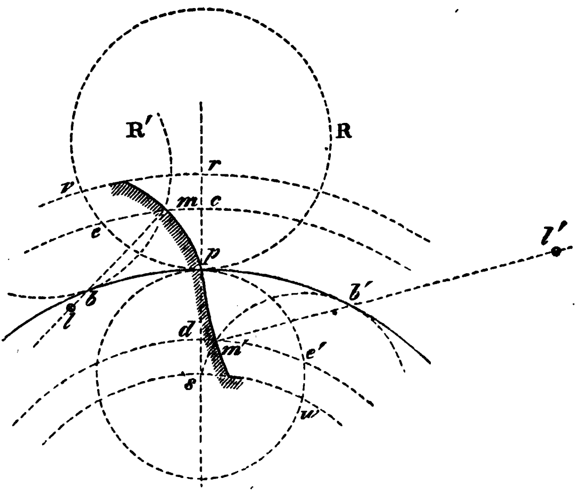


Fig. 137.

is the normal to the curve at m. It is then easy to find in $m\,b$, by trial, a centre l for a circular arc, which will pass through m and p. That circular arc will be the required approximation to the epicycloid.

For the flank of the tooth, make $p\,d = \frac{2}{3}\,p\,s$, and draw the arc $d\,e'$. Step off with the compasses, the arc $p\,b'$ = arc $p\,e'$. With centre b' and radius = chord $p\,e'$, cut $d\,e'$ in m'. Then m' is a point in the hypocycloid, and $m'\,b'$ is the normal to the

curve at m'. Find, by trial, a centre, l' on $m' b'$, for a circular arc, passing through m' and p. That arc is the required approximation to the hypocycloid.

Method 2.—The following method (fig. 138) is the same as the last, except that all the points are found by construction, instead of by trial. Take, as in the last method, $p\,c = \frac{2}{3}$ of the height of the tooth, outside the pitch line, and $p\,d = \frac{2}{3}$ the depth within the pitch line. Draw the arcs

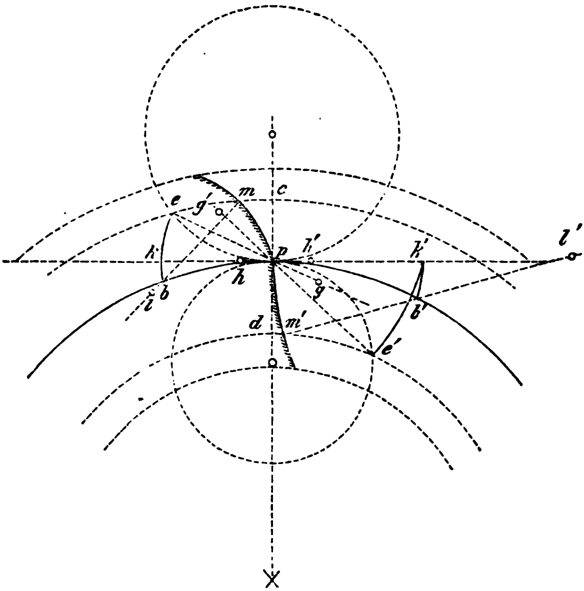


Fig. 138.

$c\,e$, $d\,e'$ concentric with the pitch line. Through the pitch point p, draw a tangent to the pitch circle. Join $e\,p$, produce it, and make $p\,g = \frac{1}{2}\,p\,e$. With centre g, and radius $g\,e$, describe an arc $e\,k$, cutting the tangent at the pitch point

in k. Then, $p\,k=$ arc $p\,e$ In $p\,k$ take $p\,h=\frac{1}{4}\,p\,k$. From centre h, with radius $h\,k$, describe an arc $k\,b$, cutting the pitch line in b. Then, arc $p\,b=$arc $p\,e$. With centre b and radius=chord $p\,e$ cut $c\,e$ in m. Join $m\,b$, and in $m\,b$ find a centre l of a circular arc, passing through m and p. That point l may be found by joining $m\,p$, and drawing a line, bisecting $m\,p$ at right angles. The line so drawn will intersect $m\,b$, produced in l. Then the arc $p\,m$, drawn with centre l and radius $l\,m$, is the required approximation to the epicycloid. The same construction gives the flank of the tooth, and the same description is applicable, if accented letters are substituted for unaccented letters.

127. *Method* 3. *Mr. Heys' method.*—The following method, adopted by Mr. Heys of Manchester, gives very good approximations, so far as the author has tested it. Let O A be the line of centres, P the pitch line, and x the pitch point. At x, draw the tangent $x\,g$, and make $x\,g=0{\cdot}571$ of the diameter of the rolling circle. Through g, parallel to O A, draw B C. Make g B$=g\,x$, and gC=diameter of rolling circle. Join O B, O C, and produce O C. Draw $a\,y$, $b\,z$, parallel to $x\,g$, and at a distance from it$=\frac{1}{8}\,x\,g$. Take $b\,z'=b\,z$. Then, y is the centre for a circular arc, approximating to the hypocloidal flank of the teeth, and $y\,x$ is its radius. Also, z' is the centre for a circular arc, approximating to the face of the

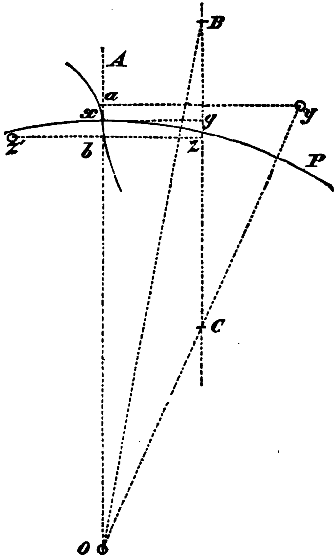


Fig. 139.

tooth, and $z'x$ is its radius. In practice, it is accurate enough to take, if $p =$ pitch,

$$x\,g = g\,\mathrm{B} = 1{\cdot}125\,p$$

$$\mathrm{B\,C} = 3\,p$$

$$x\,a = x\,b = \tfrac{1}{7}\,p$$

Involute Teeth.

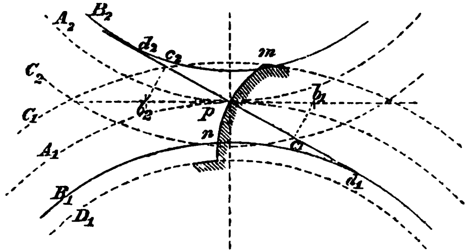
128. When the path of contact is a straight line, inclined to the line of centres, the form of the teeth is an involute of a base circle, concentric with the pitch circle, and having the path of contact for a tangent. The pressure between the teeth is in the direction of their common normal very nearly, and this normal coincides in involute teeth with the path of contact. Hence, if the path of contact deviates much from the tangent to the pitch circle at the pitch point, the pressure is correspondingly oblique to the direction of motion, and considerable stress is thrown on the supports of the wheel. Usually, the path of contact makes an angle of $74\frac{1}{2}°$ with the line of centres, and $15\frac{1}{2}°$ with the tangent to the pitch circle. Then the diameter of the base circle from which the involute is described, is $\frac{63}{65}$ths of the diameter of the pitch circle.

In fig. 140, let A_1 A_2 be the pitch circles of the wheels, and p the pitch point. Draw the path of contact $d_1\,p\,d_2$, making the desired angle with the line of centres. The base circles B_1 B_2 are concentric with A_1 A_2, and touch the path of contact at d_1 d_2. The greatest possible length of the path of contact is $d_1\,d_2$. If the length of the path of contact is given, draw the tangent $b_1\,p\,b_2$. Take $b_1\,p$, $p\,b_2 =$ the given arcs of approach and recess. Drop perpendiculars $b_1\,c_1$, $b_2\,c_2$, on the path of contact. Then $c_1\,c_2$ is the length of the actual path of contact. A circle, C_1, concentric with A_1 and passing through c_2, will mark off the length of the teeth of A_1, and a circle C_2 concentric with A_2, passing through

c_1, will mark off the length of the teeth of A_2. The root circle D_1 must be drawn, so as to fall below C_2 at the line of centres, by a distance equal to the bottom clearance, which is 0·08 to 0·1 of the pitch.

The involute is not difficult to describe, but the following method gives a very accurate circular approximation.

Fig. 140.



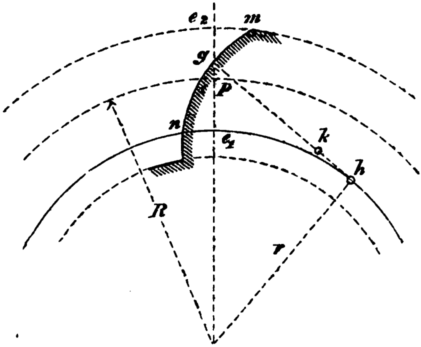


Fig. 141.

Let $e_1\, e_2$, fig. 141, be the working height of the teeth, or the distance between circles $C_1\ C_2$, in fig. 140 measured along the line of centres. Take $e_1\, g = \frac{2}{3}\, c_1\, e_2$. Draw a tangent $g\,h$ to the base circle. Take $h\,k = \frac{1}{4}\, h\,g$. Then, a circle $m\,n$, struck from k with radius $k\,g$, will be the required approximation to the involute. It will coincide with the involute at n and

g, and will have the same normal at *g*. The part of the tooth below the base circle may be a tangent to the arc at *n*. This part does not come in contact with the teeth of the other wheel.

Involute teeth have two remarkable properties. All involute wheels, whose teeth have the same pitch and the same obliquity of the line of contact, work well together. A pair of involute wheels may be drawn a little further apart, without the accuracy of action of the teeth being impaired, though the arc of contact is diminished. Involute wheels cannot be made with very long teeth, because then the obliquity of the line of contact must be great. Hence, the centres cannot be moved much further apart than their normal distance, without too much reducing the arc of contact. But this property of involute wheels is a valuable one, as it neutralises the injurious effect of wear of the supports of the wheels. With the angle of obliquity given above, the smallest number of teeth in an involute wheel should be twenty-five. With fewer teeth the arc of contact is too small. The obliquity of action is ordinarily alleged as a serious objection to involute wheels. Its importance has perhaps been overrated.

TEETH OF BEVIL WHEELS.

129. The teeth of bevil wheels may be cycloidal or involute, and are described in the same way as the teeth of spur wheels, upon a development of the conical surfaces which limit their length. Let fig. 142 represent the section of a bevil wheel rim. O *a* is the intersection of the conical pitch surface with the plane of the paper, O *d* the axis of the wheel. Let *a c* be the width of face of the wheel. Draw *a d*, *c e*, perpendicular to O *a*, cutting the axis of the wheel in *e* and *d*. Then the teeth are limited in length by the conical surfaces, whose intersections with the paper are *e c*, *d a*, and which have O *d* as axis. With centre *d* and radius *d a*, draw a circle. That circle is the virtual pitch-line of the

ends of the teeth, and the teeth are described on that circle as if it were the actual pitch line.

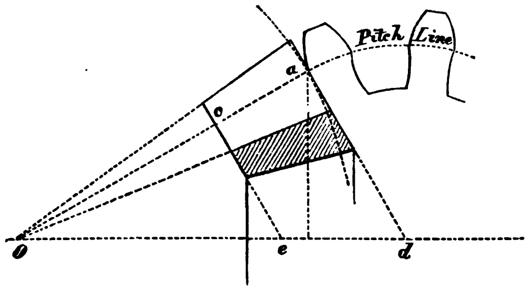


Fig. 142.

STRENGTH OF WHEEL TEETH.

130. In determining the strength of wheel teeth, it is not usually necessary to take into account their curved form. It is sufficiently accurate to treat the tooth as a rectangular cantilever (fig. 143), of thickness 36, uniform and equal to the thickness of the actual tooth at the pitch-line. Usually at least two pairs of teeth are simultaneously in contact. The pressure transmitted is therefore shared by two or more pairs of teeth. The wheels cannot be made accurately enough to ensure an equal distribution of the pressure. Hence, if P is the whole pressure transmitted, the greatest pressure on one pair of teeth is $n\,P$, where n is a fraction lying between $\frac{1}{2}$ and 1. The teeth are in contact at a line which, in spur wheels, is parallel to the axis of rotation. The line of contact varies in position during the action of the teeth, and either at the beginning or end of contact coincides with the extreme edge of the tooth. Ordinarily, in teeth which have worn a little by mutual friction, the pressure will be distributed with approximate uniformity along the edge of the tooth, and will tend to break the tooth across at its root along its whole breadth. An-

other contingency less favourable to the strength of the tooth is possible. From inaccurate form in the teeth or inaccurate fixing of the wheels, the pressure may be restricted to a small portion of the edge of the tooth. In that case, to ensure safety, the tooth must be strong enough to sustain the pressure n P applied at one corner, as shown in fig. 143.

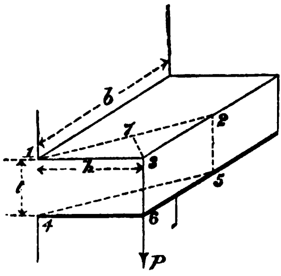


Fig. 143.

Let n P be the whole pressure on the tooth; H the number of horses' power transmitted by the wheel; N the number of revolutions of the wheel per minute; R its radius in inches; V the velocity of the pitch line in ft. per sec.

$$\mathrm{V} = \frac{2\,\pi\,\mathrm{R\,N}}{12 \times 60} = \cdot 00873\,\mathrm{R\,N} \quad . \quad . \quad . \qquad (4)$$

$$\mathrm{P} = \frac{550\,\mathrm{H}}{\mathrm{V}} = 63{,}020\,\frac{\mathrm{H}}{\mathrm{R\,N}} \quad . \quad . \quad . \qquad (5)$$

Let the height of the tooth 13, fig. 143, $= h$; its thickness $14 = t$; the width of face $= b$. Then, if the pressure n P is applied at a corner, it tends to break off a triangular prism, bounded by a plane 1 2 5 4, which passes through the root of the tooth. Draw 37 perpendicular to that plane, and let the angle 2 1 $3 = \theta$; then,

$$\overline{37} = \overline{13} \sin \theta = h \sin \theta.$$

$$\overline{12} = \overline{13} \sec. \theta = h \sec. \theta.$$

The bending moment of n P, with respect to the section 1 2 5 4, is $n\,\mathrm{P}\,h \sin \theta$. The moment of resistance of that section to bending is $\frac{1}{6} f h t^2 \sec. \theta$. Equating the bending

moment and moment of resistance, we get for the greatest stress due to bending,

$$f = \frac{3\,n\,P}{t^2} \sin 2\,\theta$$

which will be a maximum when $\theta = 45°$ and $\sin 2\,\theta = 1$. Then,

$$f = \frac{3\,n\,P}{t^2}$$

If f is the greatest safe stress,

$$\left.\begin{aligned} P &= \tfrac{1}{3}\,\frac{f\,t^2}{n} \\ t &= \sqrt{\left(\frac{3\,n\,P}{f}\right)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

131. It is convenient to express t in terms of the pitch p. For unworn teeth $t = 0{\cdot}48$ for cast-iron teeth. Since, however, the teeth must be strong enough when worn, we may take $t = 0{\cdot}36\,p$. For mortice teeth of hard wood, $t = 0{\cdot}595\,p$ when the teeth are new, and we may take $t = 0{\cdot}45\,p$ for worn teeth. Then, introducing these values in eq. 6,

$$\left.\begin{aligned} p &= 4{\cdot}8\,\sqrt{\frac{n\,P}{f}} \text{ for iron teeth} \\ &= 3{\cdot}85\sqrt{\frac{n\,P}{f}} \text{ for wood teeth} \end{aligned}\right\} \quad . \quad . \quad (7)$$

These formulæ cease to be applicable, if $b < h$, but this does not occur in wheels of ordinary proportions.

In obtaining the above formulæ, some assumptions are made, and the value of n is undetermined. For different wheels p is simply proportional to $\sqrt{P}$, and we may therefore write,

$$p = K\,\sqrt{P} \quad . \quad . \quad . \quad . \quad (8)$$

and determine K from existing wheels. K will be found to vary considerably in different cases. In slowly-moving

gearing, especially in gearing worked by hand and not subjected to much vibration or shock, $\text{K}=\cdot04$ for iron wheels. In ordinary mill-gearing, running at a greater speed and subjected to considerable vibration, $\text{K}=\cdot05$, and in wheels subjected to excessive vibration and shock, as in the gearing which drives machine tools, $\text{K}=0\cdot06$. For mortice gearing $\text{K}=\cdot06$.

If, now, it is assumed that $n=\frac{2}{3}$, which cannot be very far from the truth, we get for the values of f for cast iron, corresponding to the three cases above, 9,600, 6,100, and 4,300 lbs. per sq. in.; values which are quite consistent with ordinary practice in the use of cast iron, to resist transverse straining actions. For hard wood, when $\text{K}=\cdot06, f=2,740$.

132. *Strength of teeth when the influence of the width of face is taken into account.*—In the foregoing investigation, the pressure is assumed to be concentrated at the corner of the tooth, and consequently the strength is independent of the width of the tooth. For well-constructed and carefully erected mill-gearing this is a very improbable condition. If the pressure is distributed along the edge of the tooth, the bending moment at its root is $n\,\text{P}\,h$. The moment of resistance of the section of the tooth is $\frac{1}{6}f\,b\,t^2$. Equating these,

$$\text{P}=\frac{1}{6}\;\frac{b\,t^2}{n\,h}f$$

Let $t=0\cdot36p$ for iron, and $0\cdot45p$ for wood teeth; $h=0\cdot7p$ for iron, and $0\cdot6p$ for wood teeth; $n=\frac{2}{3}$.

$$\left.\begin{aligned}\text{P}&=\cdot046\,b\,p\,f \text{ for iron teeth}\\ &=\cdot084\,b\,p\,f \text{ for wood teeth}\end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

The following is the most convenient form of these equations:

$$p=\text{K}_1\sqrt{\frac{p}{b}}\sqrt{\text{P}} \quad . \quad . \quad . \quad . \quad (10)$$

Where K_1 in practice is about $0\cdot0707$ for iron wheels, and

0·0848 for mortice wheels, when the breadth of face is not less than twice the pitch. These values give for the stress on the teeth 4,400 lbs. per sq. in. for iron teeth, and 1,650 lbs. per sq. in. for wood. These are low enough values to allow for some inequality in the distribution of the pressure along the edge of the tooth. The following table facilitates the use of this equation :

$$\frac{p}{b} = \quad 2 \quad 2\tfrac{1}{4} \quad 2\tfrac{1}{2} \quad 3 \quad 3\tfrac{1}{2} \quad 4$$

$$K_1\sqrt{\frac{p}{b}} = \cdot 0500 \quad \cdot 0475 \quad \cdot 0447 \quad \cdot 0408 \quad \cdot 0378 \quad \cdot 0354 \quad \text{Iron teeth.}$$

$$= \cdot 0600 \quad \cdot 0565 \quad \cdot 0536 \quad \cdot 0490 \quad \cdot 0453 \quad \cdot 0424 \quad \text{Wood teeth.}$$

No specific rule can be given to decide between the cases in which eq. 8 and eq. 10 should be used. It is really a question of the degree of security against accident which is desired.

Safe Pressure on Wheel Teeth from Equation 8.

Pitch in ins.	Safe pressure on teeth in lbs.			
	Iron teeth, Little shock	Iron teeth, Moderate shock	Iron teeth, Excessive shock	
1	625	400	277	The pressures in the last column are applicable to mortice teeth calculated by eq. 8.
$1\frac{1}{4}$	975	624	432	
$1\frac{1}{2}$	1,406	900	623	
$1\frac{3}{4}$	1,912	1,224	848	
2	2,500	1,600	1,108	
$2\frac{1}{4}$	3,162	2,024	1,402	
$2\frac{1}{2}$	3,906	2,500	1,732	
$2\frac{3}{4}$	4,726	3,024	2,094	
3	5,625	3,600	2,493	
$3\frac{1}{4}$	6,600	4,224	2,926	
$3\frac{1}{2}$	7,658	4,900	3,393	
$3\frac{3}{4}$	8,787	5,624	3,895	
4	10,000	6,400	4,432	
$4\frac{1}{2}$	12,656	8,100	5,608	
5	15,625	10,000	6,924	
$5\frac{1}{2}$	18,906	12,100	8,379	
6	22,500	14,400	9,972	

Safe Pressure on Teeth of Ordinary Cast-Iron Gearing by Equation 10.

Pitch in ins.	Pressure on teeth in lbs. when $\frac{b}{p}=$ 2	$2\frac{1}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4
1	400	450	500	600	700	800
$1\frac{1}{4}$	624	702	780	936	1,092	1,248
$1\frac{1}{2}$	900	1,012	1,150	1,400	1,550	1,800
$1\frac{3}{4}$	1,224	1,377	1,530	1,836	2,142	2,448
2	1,600	1,800	2,000	2,400	2,800	3,200
$2\frac{1}{4}$	2,024	2,277	2,530	3,036	3,542	4,048
$2\frac{1}{2}$	2,500	2,812	3,125	3,750	4,375	5,000
$2\frac{1}{4}$	3,024	3,402	3,780	4,536	5,292	6,048
3	3,600	4,050	4,500	5,400	6,300	7,200
$3\frac{1}{4}$	4,224	4,752	5,280	6,336	7,392	8,448
$3\frac{1}{2}$	4,900	5,512	6,125	7,350	8,575	9,800
$3\frac{3}{4}$	5,624	6,327	7,030	8,436	9,842	11,248
4	6,400	7,200	8,000	9,600	11,200	12,800
$4\frac{1}{2}$	8,100	9,112	10,125	12,150	14,175	16,200
5	10,000	11,250	12,500	15,000	17,500	20,000
$5\frac{1}{2}$	12,100	13,612	15,125	18,150	21,175	24,200
6	14,400	16,200	18,000	21,600	25,200	28,800

The pressures for mortice wheels may be taken at $\frac{7}{10}$ths of those for iron wheels.

133. *Wheels for high speeds.*—At high speeds the influence of shocks and vibrations becomes more serious. Reuleaux has proposed to allow for this by making the value of the stress f decrease inversely as the cube root of the velocity of the pitch line. Then for cast-iron

$$f=\frac{10{,}000}{\sqrt[3]{V}} \quad . \quad . \quad . \quad . \quad (11)$$

134. *Strength of bevil wheels.*—In stating the size of bevil wheels, the pitch at the outer circumference of the wheel is always given, but in estimating their strength the pitch at the inner circumference of the rim should be taken.

Let p_i, p_o be the pitches at the inner and outer circumferences, r_i and r_o the corresponding radii of the smaller, and R_i, R_o those of the larger wheel. Width of face $= b$. Let $R_o + 0{\cdot}4\, r_o = m$. Then,

$$p_i = p_o \frac{r_i}{r_o} = p_o \frac{m-b}{m} \text{ nearly.}$$

It is p_i, not p_o, which should be taken in estimating the strength of the wheel. For all other purposes p_o is used.

135. *Shrouded wheels.*—The teeth of wheels are sometimes united at the ends by annular rings cast with the wheel, and the wheel is then said to be shrouded. The shrouding may extend the whole depth of the teeth of the pinion of a pair of wheels. In that case the shrouding has the effect of neutralising the weakness of the teeth, which in very small wheels are of a weak form. With the pinion shrouded, it is stronger than the wheel, but it wears more rapidly than the wheel, so that the shrouding may be regarded as a provision against the failure of the pinion in consequence of wear. If both wheel and pinion are shrouded to half the depth of the teeth, the strength of the pair of wheels is increased, perhaps 50 per cent. But this arrangement is seldom adopted.

136. *Width of face of wheel.*—The durability of wheels is increased by making the wheels wider. In practice, b is rarely less than $1\frac{1}{2}\, p$ in wheels used to transmit power, and that width answers well for wheels moving slowly or intermittently. For ordinary mill-gearing $b = 2\,p$ to $4\,p$.

Wear of wheels.—No exact data of the wear of wheels in given circumstances have yet been recorded. The following theory may be useful as a guide when there is a doubt as to the width to be given to a pair of wheels.

Let R_1, R_2 be the radii of a pair of wheels, making N_1, N_2 revolutions per minute, and transmitting H horses' power,

Let p be the pitch, and b the width of face of the wheels. Then the work lost in friction is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H \text{ ft. lbs. per min.} \qquad . \qquad . \ (12)$$

The wearing surface of a tooth is proportional to bp, and the whole wearing surface of the pinion is proportional to $R_1 b$. Supposing the total wear to be proportional to the work expended, the depth worn away in the unit of time is proportional to

$$p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H \div R_1 b$$

Suppose the wheel to be worn out when the depth worn away is γp, where γ is some fraction varying in different circumstances, but constant for wheels in similar conditions. Then, for equal durability,

$$\gamma p \div \frac{p\left(\frac{1}{R_1}+\frac{1}{R_2}\right) H}{R_1 b} = \text{constant}$$

or

$$b = k_1 \frac{H}{R_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \qquad . \qquad . \ (13)$$

where k_1 is a constant to be determined by experience. Since $P R_1 N_1$ is proportional to H, we have also

$$b = k_2 P N_1 \left(\frac{1}{R_1}+\frac{1}{R_2}\right) . \qquad . \ (13a)$$

or when the pinion is small compared with the wheel,

$$b = k_1 \frac{H}{R^2_1} = k_2 P \frac{N_1}{R_1} \text{ nearly} . \qquad . \qquad . \ (13b)$$

Average values for these constants would be of little service, because the conditions in which wheels are employed are so variable. If k_1 or k_2 is deduced from a pair of wheels known to have worked well in given conditions,

the value so obtained may be applied to determine the minimum width of another pair of wheels which are to work in similar conditions.

Construction and Proportions of Wheels.

137. *Rim of wheel.*—In iron wheels the teeth are cast on, and in mortice wheels they are tenoned into, a continuous

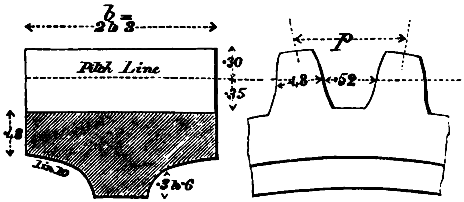


Fig. 144.

rim. Fig. 144 shows the section of a spur-wheel rim, and fig. 145 that of a bevil-wheel rim. The unit for the proportional figures is the pitch. The proportional figures for the

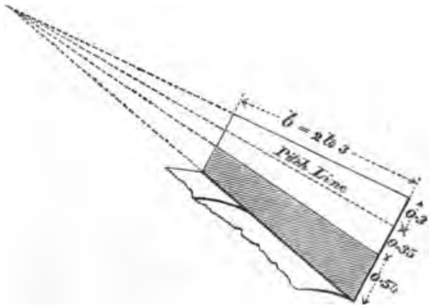


Fig. 145.

teeth are approximate only, more exact proportions having been already given in § 121.

Fig. 146 shows the section of a mortice spur-wheel rim, the

end elevations indicating two ways of forming the tenons. The mortice teeth are either fixed by wood keys, or by

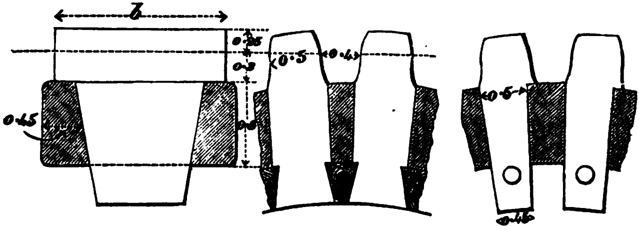


Fig. 146.

round iron pins driven in behind the rim of the wheel. Both methods are shown in fig. 146. In fig. 147 the cogs are fixed by bolts, iron plates about 2 ft. long being fitted to the inside of the rim of the wheel. Fig. 148 shows a mortice bevil wheel.

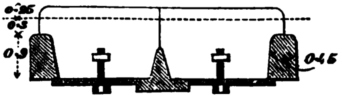


Fig. 147.

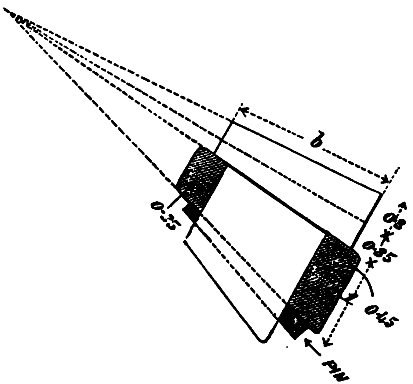


Fig. 148.

The radiating lines in the figures of bevil wheels meet at the intersection of the shafts on which the wheels are placed.

138. *Arms of wheels.*—The arms of wheels are most commonly cross-shaped in section for spur-wheels, and T-shaped for bevil-wheels. For machine-moulded wheels, the arms are often I-shaped, the spaces between the arms being cored out in casting with loam cores. The number of arms in wheels is fixed very arbitrarily. Usually there are four arms for wheels not exceeding four feet diameter; six arms for wheels of from four feet to eight feet; and eight arms for wheels from eight feet to sixteen feet diameter.

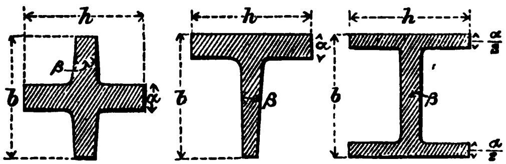


Fig. 149.

The arms are subjected to bending, and when the arms and rim are cast in one piece, they are fixed at both ends. If the arms are attached to the rim by bolts, they are free at the rim, and fixed at the nave. It will be assumed that the arms are equally loaded, and that they may in all cases be treated as if they were fixed at one end and free at the other. This will give a slight excess of strength when the arms are cast in one with the rim, but such arms are at the same time weakened by contraction in cooling.

Let ν be the number of arms,

R, the radius of the wheel,

P, the total pressure transmitted (§ 130).

Then the bending moment on each arm is $PR \div \nu$ nearly. The strength of the arm is almost entirely due to that part which is parallel to the plane of rotation. The ribs or feathers at right angles to this part add very little

to the resistance to the force acting on the wheel. They are necessary to give lateral strength and rigidity, and to resist accidental straining actions at right angles to the plane of rotation. Let h be the width, and a the thickness of the arm, exclusive of the feathers. The moment of resistance of that section is $\frac{1}{6} a h^2 f$. Equating this to the bending moment

$$a h^2 = \frac{6 \text{ P R}}{\nu f} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

In proceeding to design the arm either of the three following methods may be followed.

(1.) Given the ratio $\frac{h}{a}$, and the limiting stress on the arm.

If $\frac{h}{a} = 5$, we get from eq. 14,

$$h = \sqrt{\frac{30}{f}} \; \sqrt[3]{\frac{\text{P R}}{\nu}}.$$

The limiting stress must be taken at a low value, partly to allow for unequal distribution of load on the arms, and partly because of the initial stresses due to contraction in cooling. If $f = 3{,}000$ lbs. per sq. in.,

$$h = \frac{0{\cdot}2154}{\sqrt[3]{\nu}} \sqrt[3]{\text{P R}} \quad . \quad . \quad . \quad . \quad (15)$$

$\nu =$	3	4	6	8	10	12
$\frac{0{\cdot}2154}{\sqrt[3]{\nu}} =$	1·49	1·36	1·19	1·08	1·00	0·94

(2.) Since the arm must be of equal strength with the teeth, we may replace P by its value in terms of the pitch in eq. 9, namely,

$$\text{P} = 0{\cdot}046 \, b p f$$

Introducing this in eq. 14,

$$a h^2 = 0{\cdot}276 \frac{b p \text{ R}}{\nu} \quad . \quad . \quad . \quad . \quad (16)$$

Let $a = 0{\cdot}2\,h$

$$h = \frac{1{\cdot}113}{\sqrt[3]{\nu}} \sqrt[3]{(b p \mathrm{R})} \quad . \quad . \quad . \ (17)$$

$\nu=$	3	4	6	8	10	12
$\frac{1{\cdot}113}{\sqrt[3]{\nu}}=$	·772	·701	·613	·557	·517	·486

A comparison of some existing wheels shows that the arms are sometimes one-fifth wider than is given by this rule, this additional width being required to meet the stresses due to contraction.

(3.) Given the thickness of the arm. It is desirable to make the different parts of the wheel nearly uniform in thickness to secure regularity in cooling and contraction. Let $a = 0{\cdot}48\,p$, so that the arm is the same thickness as the teeth. Introducing this in eq. 16,

$$h = \frac{0{\cdot}758}{\sqrt{\nu}} \sqrt{b \mathrm{R}} \quad . \quad . \quad . \quad . \ (18)$$

$\nu=$	3	4	6	8	10	12
$\frac{0{\cdot}758}{\sqrt{\nu}}=$	·438	·379	·309	·268	·240	219

One-fifth may be added to the dimensions thus obtained to allow a margin against contraction, and for the unequal loading of the arms.

The dimensions given by the foregoing rules apply to the section of the arm produced to the centre of the wheel. Towards the rim the arm is usually tapered, the amount of taper being $\frac{1}{4}$ in. per foot of length on each side. The thickness of the arm a is constant.

The width of the cross feathers (marked b in fig. 149) may be b to $1\frac{1}{4}\,b$ at the centre, and $\frac{3}{4}\,b$ to $\frac{15}{16}\,b$ at the rim.[1] The thickness of the feathers may be $\beta = 0{\cdot}3\,p$. The feathers must be slightly tapered at right angles to their length, so as to draw easily from the sand.

[1] Except in fig. 149, b is the width of the face of the wheel.

139. *Nave of the wheel.*—The thickness δ of the nave of the wheel may be taken at $0{\cdot}4\ \sqrt[3]{(p^2\,\text{R})}+\frac{1}{2}$, and its length may be at least three times its thickness. Generally the nave length is not less than $b+0{\cdot}06\,\text{R}$ in iron wheels, and $b+p+0{\cdot}06\ \text{R}$ in mortice wheels, so that it may project a little beyond the rim. The key for fixing the wheel on the shaft may be $0{\cdot}4\,\delta$ wide and $0{\cdot}2\,\delta$ thick.

Mr. Heys uses the following rule for the nave thickness:—

$$\delta=\tfrac{5}{4}\sqrt[3]{\text{R}}+0{\cdot}7p+0{\cdot}1\,d-1$$

Where d is the actual diameter of the eye of the wheel.

Screw Gearing.

140. In screw gearing the wheels have cylindrical pitch surfaces, like those of spur wheels, but the teeth are not parallel to the axes. The line in which the pitch surface intersects the face of a tooth is part of a helix drawn on the pitch surface. A screw wheel may have one or any number of teeth. A one-toothed wheel corresponds to a one-threaded screw; a many-toothed wheel to a many-threaded screw. In screw gearing the axes may be placed at any angle.

141. *Screw-gearing with parallel axes.*—Gearing of this kind was invented by Dr. Hooke. Let an ordinary spur wheel be cut into n slices by planes perpendicular to the axis. Let the slices be so arranged that, for example, in passing from left to right across the face of the wheel, each successive slice is $\frac{1}{n}$th of the pitch behind the previous one. Such a wheel is termed a stepped spur wheel. Two such wheels will work together, and they have the advantage compared with ordinary wheels that one or other of the pairs of slices are always in contact at a distance not exceeding $\frac{1}{n}$th of the pitch from the pitch point.

As the slices come successively into gear, the motion of the wheels is very regular. Such wheels were at one time used for driving planing machine tables, and in other cases where regularity of motion was important. If the slices are infinitely numerous, then the front of the tooth intersects the pitch cylinder in a helical line, and we get the screw wheels shown in fig. 150. In two wheels of this kind which gear together the pitch measured circularly is equal; the obliquity is equal, but in opposite directions, and the velocity ratio is inversely as the radii of the wheels.

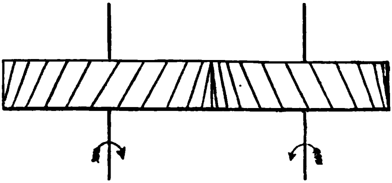


Fig. 150.

142. *Screw gearing when the axes are not parallel.*—When the axes are not parallel the pitch cylinders touch at a single point, which may be termed the pitch point. Draw through that point a tangent to the pitch surfaces. If helices are traced on the pitch cylinders touching that tangent, they define the fronts of teeth which will drive each other.

The common tangent to the pitch surfaces and the teeth is termed the line of contact. It is shown at $a\,b$, fig. 152; the angles θ_1 θ_2 it makes with the axes are termed the angles of inclination of the teeth. The number of threads ν, in a screw wheel, is equal to the number of helices which intersect any plane perpendicular to the axis. Let fig. 151 represent a series of helices (in this case four), intended to mark out the teeth of a screw wheel. The same screw thread intersects a line $a\,b$, parallel to the axis at a and b. Then $a\,b$ is the *axial pitch* of the screw, and the distance $ac = p = \frac{ab}{\nu}$ is the *divided axial pitch*. Let a plane $d\,e$ perpendicular to

the axis intersect two successive threads in d and e. Then $d e$ is the circumferential pitch c, and is equal to $\frac{2\pi r}{\nu}$ where r is the radius of the pitch cylinder. Draw $e f$ perpendicularly to the threads. Then the distance $e f$ is the divided *normal pitch*, n. Let the tangent $g h$ to a thread make the angle $g h a = \theta$ with the axis of the wheel. Then from the properties of helices the following relations obtain :—

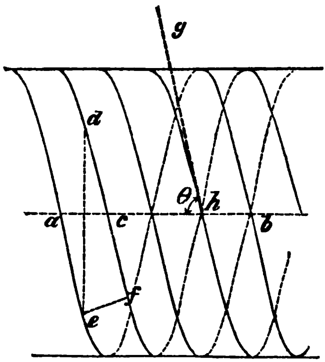


Fig. 151.

$$\left.\begin{aligned} \tan\theta &= \frac{2\pi r}{\nu p} \\ p : c : n &:: 2\pi r \cot\theta : 2\pi r : 2\pi r\cos\theta \\ &:: \cot\theta \quad : \quad 1 : \cos\theta \end{aligned}\right\} \qquad (19)$$

Let fig. 152 represent two screw wheels projected on the common tangent plane to the two pitch cylinders. Let the angle between the axes $=i$, and let the tangent to the teeth $a b$ make with the axes the angles θ_1, θ_2, so that

$$\theta_1 + \theta_2 = i$$

Let α_1 α_2 be the angular velocities of the wheels; r_1 r_2 their radii; and ν_1 ν_2 the number of threads of each. Let c_1 c_2 be the circumferential, n_1 n_2 the divided normal, and p_1 p_2 the divided axial pitches. Then,

$$\frac{\alpha_1}{\alpha_2} = \frac{\nu_2}{\nu_1} \quad . \quad . \quad . \qquad (20)$$

Let ν_1 and ν_2 be decided upon. The surface velocities of the wheels are $\alpha_1 r_1$ and $\alpha_2 r_2$, and these are proportional to the circumferential pitches, because each wheel rotates a

distance equal to the circumferential pitch in the same time. Hence,

$$\frac{c_1}{c_2}=\frac{a_1 r_1}{a_2 r_2} \quad . \quad . \quad . \quad (21)$$

If the circumferential pitches are chosen so as to satisfy this relation, then the axial pitch and inclination of the

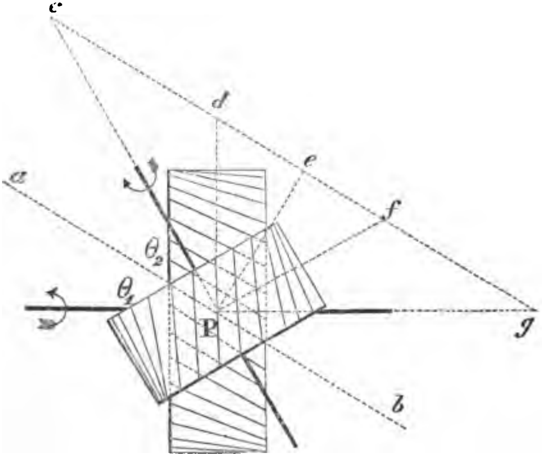


Fig. 152.

threads are determined by the condition that in two wheels which work together the normal pitches must be equal. Hence,

$$n_1=n_2$$

and using the proportions in eq. 19

$$c_1 \cos \theta_1=c_2 \cos \theta_2$$

$$\left.\begin{aligned}\therefore \cos \theta_1 &= \frac{c_2 \sin i}{\sqrt{(c_1^2-2c_1 c_2 \cos i+c_2^2)}}\\ \cos \theta_2 &= \frac{c_1 \sin i}{\sqrt{(c_1^2-2c_1 c_2 \cos i+c_2^2)}}\end{aligned}\right\} \quad . \quad (22)$$

$$\left.\begin{aligned}p_1&=c_1 \cot \theta_1\\ p_2&=c_2 \cot \theta_2\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In fig. 152, set off from the pitch point P, the lines Pd, Pf perpendicular to the axes, in the directions the wheels are moving at the point P. Take Pd, Pf equal to the surface velocities $a_1 r_1$ and $a_2 r_2$ of the wheels; join df and produce it to meet the axes; then aPb parallel to cg is the line of contact, making angles θ_1 θ_2 with the axes. Draw Pe perpendicular to cg. Then Pe is the common component of the surface velocities, and cg the velocity of transverse sliding of the teeth.[1]

$$c_1 : c_2 : p_1 : p_2 : n_1 \text{ or } n_2$$
$$:: \quad \mathrm{P}d : \mathrm{P}f : \mathrm{P}g : \mathrm{P}c : \mathrm{P}e$$

If Pd, Pf are set off equal to the circumferential pitches of the two wheels, and the same construction is made, then

$$p_1 = \mathrm{P}g\,;\ p_2 = \mathrm{P}c\,;\ \text{and } n_1 \text{ or } n_2 = \mathrm{P}e.$$

143. *Screw gearing when the shafts are at right angles. Worm and Wheel.*—If $i = 90°$, then $\cos\theta_2 = \sin\theta_1$

$$\frac{c_1}{c_2} = \tan\theta_1$$

$$\frac{p_1}{p_2} = \cot\theta_1$$

Hence, $$p_1 = c_2 \text{ and } p_2 = c_1$$

or the axial divided pitch of one wheel is equal to the circumferential pitch of the other.

The most common form of screw gearing is that in which the shafts are at right angles, and a wheel of one thread, or sometimes of two or three threads, works with a wheel of many threads. Then the former is termed a *worm*, and the latter a *worm wheel*. With this arrangement, a high velocity ratio is obtained with a pair of small wheels. If N_1 N_2 are the numbers of revolutions of the worm and wheel, a_1 a_2,

[1] See Rankine's 'Millwork,' p. 160.

their angular velocities, and ν_1 ν_2 the number of threads on each,

$$\frac{N_1}{N_2}=\frac{a_1}{a_2}=\frac{\nu_2}{\nu_1}$$

Thus, if the worm has one thread and the wheel twenty-five, the velocity ratio is twenty-five. Spur wheels for that velocity ratio would have to be at least ten times larger in diameter. The disadvantage of screw gearing of this kind is that the friction and wear is excessive, hence it is rarely used for the continuous transmission of power. If the obliquity of the helices exceeds a certain amount, the wheels are no longer reciprocal; that is, one wheel will drive the other but the second will not drive the first. In that case the motion is prevented by the friction at the point of contact of the teeth. The worm and wheel are commonly so constructed that the worm will drive the wheel, but the wheel will not drive the worm. This is often advantageous, because the gearing remains stationary in any position after being moved.

144. *Friction of worm and wheel.*—Suppose the worm drives the wheel, and that a force P acts at the pitch line of the worm, in the plane of rotation, overcoming a resistance Q acting at the pitch line of the worm wheel in the plane of its rotation. Let θ_1, as before, be the inclination of the worm thread, μ the coefficient of friction, r_1 r_2 the radii of the worm and wheel, and $\nu_1 p_1$ the total axial pitch of the worm :—

$$\frac{P}{Q}=\frac{1+\mu\tan\theta_1}{\tan\theta_1-\mu} \quad . \quad . \quad . \qquad (24)$$

or if ϕ is the angle of repose of metal on metal, so that $\mu=\tan\phi$,

$$\frac{P}{Q}=\cot(\theta_1-\phi) \quad . \quad . \qquad (25)$$

When the worm drives the wheel this ratio must be positive. Hence θ_1 must be less than $90^\circ-\phi$. The ratio of

the useful work done to the power expended, or the efficiency of the pair of wheels is,

$$\eta=\frac{\cot\theta_1}{\cot(\theta_1-\phi)}=\frac{1-\mu\dfrac{\nu_1 p_1}{2\pi r_1}}{1+\mu\dfrac{2\pi r_1}{\nu_1 p_1}} \quad . \quad . \qquad (26)$$

For $\mu=0{\cdot}15$, we get,

$$\eta=\frac{\nu_1 p_1}{\nu_1 p_1+r_1}\text{ nearly} . \quad . \qquad (27)$$

hence the efficiency is greater the less the radius of the worm. Generally $r_1=1{\cdot}5$ to $3\,p_1$. For a one-threaded worm therefore the efficiency is only $\frac{2}{5}$ to $\frac{1}{4}$. For a two-threaded worm $\frac{4}{7}$ to $\frac{2}{5}$; for a three-threaded worm $\frac{2}{3}$ to $\frac{1}{2}$. Since so much work is wasted in friction it is not surprising that the wear is excessive.

145. *Form of screw threads.*—Rankine has pointed out that the sections of the threads, normal to the line of contact, may be similar to involute or epicycloidal teeth, drawn for spur wheels, of radii equal to the radii of curvature of helices normal to the screw threads.

When the shafts are at right angles, it is convenient to draw sections of the teeth of the worm and wheel on a plane perpendicular to the axis of the wheel, and passing through the axis of the worm. Then the sections of the teeth of the worm wheel may be the same as those of a spur wheel of the same radius and circumferential pitch, and the sections of the teeth of the worm may be the same as the teeth of a rack of the same pitch. Fig. 152 shows a worm and wheel, the teeth of which are drawn in this way. The worm here shown is of wrought iron or malleable cast iron, formed in one piece with its shaft. Usually the worm is of cast iron, and when small may be fixed by a pin passing through both worm and shaft. When larger its rotation on the shaft may be prevented by a key, and its tendency to slide along the shaft by collars, one of which may be fixed and the other a

loose collar fixed by a set screw. Sometimes the bearings which support the worm shaft are so arranged as to prevent the endways motion of the worm.

146. *Strength of worm wheels.*—The resultant pressure on the teeth (friction being neglected) is in the direction of the normal to the faces of the teeth at the point of contact, or in the direction in which the normal pitch is measured.

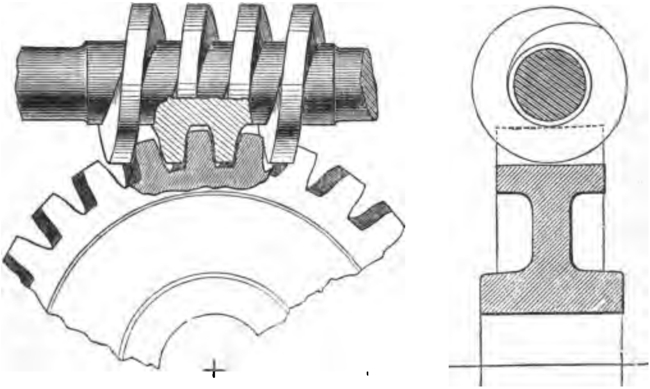


Fig. 153.

Hence it is the tendency to break under the action of the force acting in that direction, which has to be considered in estimating the strength of the teeth.

The worm is usually at least as strong as the worm wheel, hence it is only necessary to consider the strength of the latter. Let n_2 be the normal divided pitch of the worm wheel, c_2 its circumferential pitch, r_2 its radius, and θ_2 the angle between the threads and the axis :

$$n_2 = c_2 \cos \theta_2 = \frac{2 \pi r_2}{v_2} \cos \theta_2$$

Where $\cos \theta_2 = \frac{\text{P}e}{\text{P}f}$ in fig. 151. Let Q be the resistance to rotation at the circumference of the worm wheel. Then the

pressure acting normally to the teeth is $Q_n = \frac{Q}{\cos\theta} = Q\frac{Pf}{Pe}$. The worm wheel is equivalent to a spur wheel resisting the force Q_n at the pitch line, and having the pitch n_2. Hence the normal pitch n_2 may be obtained by the rules for the teeth of spur wheels. Then $c_2 = \frac{n_2}{\cos\theta} = n_2\frac{Pf}{Pe}$.

When the shafts are at right angles, the angle θ_2 is often small, so that $\cos\theta = 1$ nearly. Then the worm wheel is approximately equivalent to a spur wheel resisting the force Q, and having the pitch c_2. Hence, when θ_2 is small, the obliquity of the teeth may be neglected in calculating the pitch. The width of face of the worm wheel is about $1\frac{1}{2}$ times the pitch. In calculating the size of the worm shaft, from the resistance Q overcome, friction should not be neglected. The twisting moment acting on the worm shaft is $\frac{Q\,r_2}{\eta}\ \frac{v_1}{v_2} = Q\,r_2\frac{r_1 p + r_1}{r_2 p_1}$ nearly.

146*a*. *Weight of toothed gearing.*—Let p be the pitch, b the breadth of face, and n the number of teeth of a wheel. Then, its weight in lbs. is, approximately,

$$W = k\,n\,b\,p^2$$

where $k = 0{\cdot}38$ for spur wheels, and $0{\cdot}325$ for bevil wheels. The weight of a pair of wheels is independent of the radii, and depends directly on the H.P. transmitted and the numbers of revolutions of the wheels. The weight of a train of wheels is smaller when the number of pairs of wheels is as small as possible, and when all the pairs, except the quickest running pair, have the greatest practicable velocity ratio.

CHAPTER X.

BELT GEARING.

147. THE term belt, band, or strap is applied to a flexible connecting piece which drives a rotating piece termed a pulley, by its frictional resistance to slipping at the surface of the pulley. Such belts are most commonly of leather; belts liable to be wetted are often of vulcanised india-rubber, of india-rubber cloth, or of gutta-percha. Leather and india-rubber cloth are stronger than vulcanised india-rubber or gutta-percha. These belts are flat belts—that is, they are wide and thin, and they run on pulleys with nearly cylindrical surfaces. Round belts are also used, and at the present time their application is being greatly extended. Such round belts are of hemp rope, of cotton rope, of wire rope, or when small of catgut. The pulleys for round belts have usually V-shaped grooves, in which the belts are placed. Chains are sometimes used in place of belts when great force is transmitted at slow speeds. Then the pulley is toothed, the projections on the pulley fitting the links of the chain and preventing any slipping,

FLAT BELTS.

148. *Velocity ratio in belt transmission.*—A belt is not used in cases where a very exact velocity ratio is necessary. Hence it is generally accurate enough to regard the belt as inextensible. If also there is no slipping of the belt on the pulley, the velocity of the belt and the surface velocities of

the pulleys must all be equal. Let v be the velocity of the belt, d_1 d_2 the diameters of the pulleys, and N_1 N_2 their revolutions per minute—

$$\left.\begin{array}{l}\pi d_1 N_1 = v \\ \pi d_2 N_2 = v\end{array}\right\} \therefore \frac{d_1}{d_2} = \frac{N_2}{N_1} \qquad (1)$$

These equations are in strictness only true when the belt is infinitely thin. When the belt has a thickness δ, the effective diameters of the pulleys are $d_1+\delta$, and $d_2+\delta$. Then,

$$\frac{N_2}{N_1} = \frac{d_1+\delta}{d_2+\delta} \qquad (1\,a)$$

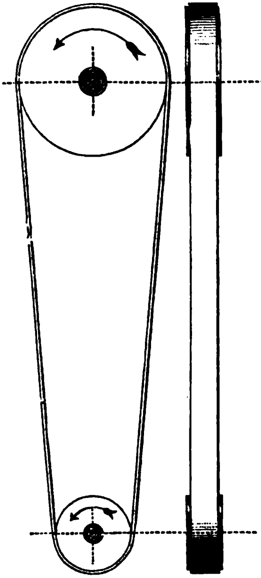


Fig. 154.

Fig. 155.

As the belt thickness is generally small compared with the pulley diameter, δ may be neglected without any great

error, but it should be remembered that, in all questions of velocity ratio in belting, the virtual diameter of the pulley is the diameter measured to the centre of the belt.

149. *Endless Belt.*—When one shaft is driven from another, a pulley is placed on each shaft, and an endless belt is strained over the two pulleys. The belt may be an open belt (fig. 154) or a crossed belt (fig. 155). In the former case the two shafts rotate in the same direction. In the latter case they rotate in opposite directions.

150. *Length of belts.*—Let D and d be the diameters of the two pulleys in inches; C, their distance apart, from centre to centre; L, the length of the belt. Also, let $D+d=\Sigma$, and $D-d=\Delta$.

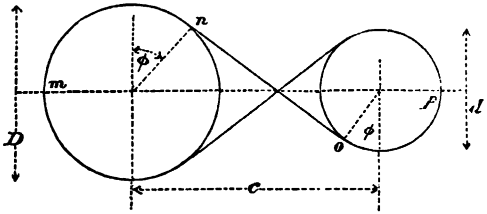


Fig. 156.

For a crossed belt (fig. 156) the total length—

$$=L=2(mn+no+op)$$

$$=\left(\frac{\pi}{2}+\phi\right)D+2c\cos\phi+\left(\frac{\pi}{2}+\phi\right)d$$

$$=\left(\frac{\pi}{2}+\phi\right)\Sigma+2c\cos\phi \quad . \quad . \quad . \quad (2)$$

$$\sin\phi=\frac{D+d}{2c}=\frac{\Sigma}{2c} \quad . \quad . \quad . \quad . \quad (3)$$

The length of the belt is obtained thus:—Calculate the value of $\sin\phi$. From a table of natural sines and cosines find the nearest values of $\cos\phi$ and ϕ, the latter being ex-

pressed in circular measure. Then eq. (2) gives the belt length. If ϕ is found or measured off the drawing in degrees, the circular measure of the angle is obtained by multiplying by 0·0175.

With a crossed belt ϕ depends only on $\text{D}+d$. Hence, if Σ and c are constant for two or more pairs of pulleys, the same belt will run on any pair of pulleys of the set.

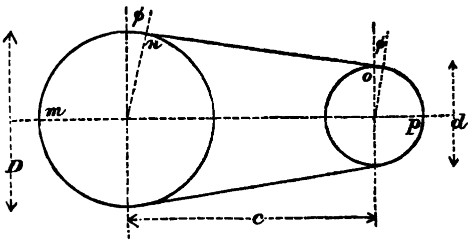


Fig. 157.

When the belt is an open one (fig. 157) the equations are rather less simple—

$$\text{L}=2\,(m\,n+n\,o+o\,p)$$

$$=\left(\frac{\pi}{2}+\phi\right)\text{D}+2\,c\cos\phi+\left(\frac{\pi}{2}-\phi\right)d$$

$$=\frac{\pi}{2}\Sigma+\phi\Delta+2\,c\cos\phi \quad . \quad . \quad . \qquad (4)$$

$$\text{Sin}\ \phi=\frac{\Delta}{2\,c};\ \cos\phi=\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)} \quad . \qquad (5)$$

For an open belt ϕ is generally small, so that,

$$\phi=\sin\phi,\ \text{nearly}$$

$$\therefore \text{L}=\frac{\pi}{2}\Sigma+2\,c\left\{\frac{\Delta^2}{4\,c^2}+\sqrt{\left(1-\frac{\Delta^2}{4\,c^2}\right)}\right\}$$

$$=\frac{\pi}{2}\Sigma+2\,c\left\{1+\frac{1}{8}\,\frac{\Delta^2}{c^2}\right\}\ \text{nearly} \quad . \qquad (6)$$

Hence, if an open belt runs on a pair of pulleys, the sum and difference of whose diameters are Σ_1 and Δ_1 and the same belt is also to run on another pair of pulleys, the sum and difference of whose diameters is Σ_2 and Δ_2, since the length of the belt is the same in the two cases,

$$\frac{\pi}{2}\Sigma_1+2c\left\{1+\frac{1}{8}\frac{\Delta_1^2}{c^2}\right\}=\frac{\pi}{2}\Sigma_2+2c\left\{1+\frac{1}{8}\frac{\Delta_2^2}{c^2}\right\}$$

$$\Sigma_2=\Sigma_1+\frac{\Delta_1^2-\Delta_2^2}{4\pi c} \quad . \quad . \quad . \quad (7)$$

It is accurate enough for practical purposes to calculate the diameters D_2 and d_2 as if the belt were a crossed belt. Then, taking Δ_2=the difference of these diameters, find the value of Σ_2. From that value of Σ_2 recalculate the diameters D_2 and d_2, using eq. (1) or eq. (1*a*).

151. *Speed cones.*—When a shaft running at a constant speed has to drive a machine at several different speeds, sets of pulleys are used which are termed stepped speed cones.

The speed cones (fig. 158) are placed opposite one another, so as to form a series of pairs of pulleys, and by shifting the belt from one pair to another the speed of the machine is altered. In designing these speed cones the ratio of the diameters of each pair depends on the speeds of the shafts, and the sum of the diameters should be so arranged that the same belt will work on any pair of the set without alteration of length.

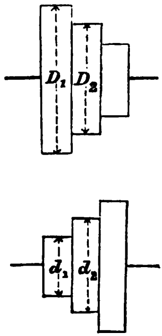


Fig. 158.

Let D_1 d_1 be the diameters of one pair; D_2 d_2 the diameters of another pair. Let N be the number of revolutions of the shaft on which D_1 and D_2 are placed; n_1 and n_2 the revolutions of the other shaft, when the belt is on d_1 and d_2 respectively.

If the belt is a crossed belt, from eq. (1)—

$$\frac{D_1}{d_1}=\frac{n_1}{N} \qquad \frac{D_2}{d_2}=\frac{n_2}{N}$$

also, $$D_1+d_1=D_2+d_2=\Sigma.$$

Hence,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma \\ d_2&=\frac{N}{N+n_2}\Sigma \end{aligned}\right\} \quad . \quad . \qquad (8)$$

If the belt is an open belt, the diameters will be slightly different. Let $\Delta_1=D_1-d_1$; $\Delta_2=D_2-d_2$; $\Sigma_1=D_1+d_1$; $\Sigma_2=D_2+d_2$. If the belt were a crossed belt, we should have,

$$D_2=\frac{n_2}{N+n_2}\Sigma_1\ ;\ d_2=\frac{N}{N+n_2}\Sigma_1$$

and since the diameters for an open belt are but little different,

$$\Delta_2=D_2-d_2=\frac{n_2-N}{N+n_2}\Sigma, \text{ nearly.}$$

Then from eq. (7),

$$\Sigma_2=\Sigma_1+\frac{\Delta_1^2-\Delta_2^2}{4\pi c}$$

And from eq. (1),

$$\frac{D_2}{d_2}=\frac{n_2}{N}$$

Hence,

$$\left.\begin{aligned} D_2&=\frac{n_2}{N+n_2}\Sigma_2 \\ d_2&=\frac{N}{N+n_2}\Sigma_2 \end{aligned}\right\} \quad . \quad . \qquad (9)$$

Hence, the process of designing a set of speed cones is

this :—Having given the speed N of the driving shaft, decide on the speeds n_1, n_2, n_3 . . . . of the driven shaft. Choose a diameter for one of the pulleys of the first pair, and find the diameter of the other by equation (1). The values of Σ_1 and Δ_1 can then be found. From these the diameters of any other pair of pulleys can be found by the equations above.

152. *Resistance to slipping of a belt on a pulley.*—Let fig. 159 represent a belt strained over a pulley and on the point of slipping from T_1 towards T_2. Then the tension T_2 must be greater than the tension T_1, by the amount of the frictional resistance to slipping at the surface of the pulley.

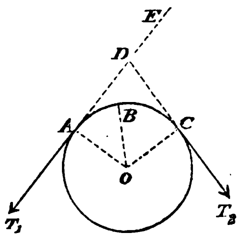


Fig. 159.

Let B be any point of contact, and let the tension at B=T. Let the angle A O B in circular measure be θ; the arc A B$=s$; the radius A O$=r$; the normal pressure of the belt on the pulley estimated per unit of arc =P ; and the coefficient of friction $=\mu$.

Consider a small length, ds of the belt at the point B. The tensions at the ends of that small length are T and $T+dT$, so that $\frac{dT}{ds}$ is the increase of tension, per unit length of the arc of contact. But in unit length of belt the friction is μp,

$$\therefore \frac{dT}{ds}=\mu p$$

The normal pressure which would produce the tension T in the flexible belt is given by the equation

$$\frac{T}{r}=p.$$

Combining these equations and remembering that $ds = r\,d\theta$,

$$\frac{d\mathrm{T}}{r\,d\theta} = \mu\frac{\mathrm{T}}{r}$$

$$\frac{d\mathrm{T}}{\mathrm{T}} = \mu\,d\,\theta$$

$$\int_{\mathrm{T}_1}^{\mathrm{T}_2}\frac{d\,\mathrm{T}}{\mathrm{T}} = \mu\int_0^{\theta}d\theta$$

$$hyp\ log\ \frac{\mathrm{T}_2}{\mathrm{T}_1} = \mu\,\theta \quad . \quad . \quad . \quad (10)$$

Where θ is the angle A O C, or, what is the same thing, the angle C D E, expressed in circular measure, or the arc A C ÷ radius A O. If the angle θ is measured in degrees, it can be reduced to circular measure, by multiplying by $\frac{\pi}{180}$ or by 0·0175.

This equation may be put in the form

$$\frac{\mathrm{T}_2}{\mathrm{T}_1} = \varepsilon^{\mu\theta} \quad . \quad . \quad (11)$$

where $\varepsilon = 2{\cdot}71828$ the base of the system of natural logarithms. Hence, $\mu\,\theta$ is the hyperbolic or natural logarithm corresponding to the number $\mathrm{T}_2 \div \mathrm{T}_1$. As common logarithms are more convenient,

Common log. $\frac{\mathrm{T}_2}{\mathrm{T}_1} = 0{\cdot}434\ \mu\,\theta$ if θ is in circular measure.

$= 0{\cdot}007578\ \mu\,\theta$ if θ is in degrees.

$= 2{\cdot}729\ \mu\,k$ if k is the fraction

of the circumference embraced by the belt. Hence, if the right-hand member of either of these equations is calculated, the value obtained is the logarithm of $\mathrm{T}_2 \div \mathrm{T}_1$. The natural

number corresponding to that logarithm, found by means of a table of logarithms, is the value of $T_2 \div T_1$.

153. *The coefficient of friction.*—The value of μ for belts varies from 0·15 to 0·56 in different cases. For leather belting on iron pulleys, in an ordinary condition of working, $\mu = 0·3$ to 0·4. The experiments of Messrs. Briggs and Towne appear to show that the latter value may safely be taken.[1] For wire rope running on the bottom of a grooved pulley, $\mu = 0·15$, and if the pulley is bottomed with leather or gutta-percha, $\mu = 0·25$.

The following table will give the values of $\frac{T_2}{T_1}$ for all cases likely to occur, with accuracy enough for most practical purposes :—

Tensions on Tight and Slack Sides of Belting.

$\theta =$			$\frac{T_2}{T_1} = \epsilon^{\mu\theta} =$			
In degrees	In circular measure	In fraction of circumference.	$\mu = 0·2$	$\mu = 0·3$	$\mu = 0·4$	$\mu = 0·5$
30	·524	·083	1·110	1·170	1·233	1·299
45	·785	·125	1·170	1·266	1·369	1·481
60	1·047	·167	1·233	1·369	1·521	1·689
75	1·309	·208	1·299	1·481	1·689	1·924
90	1·571	·250	1·369	1·602	1·874	2·193
105	1·833	·319	1·443	1·733	2·082	2·500
120	2·094	·334	1·521	1·875	2·312	2·851
135	2·356	·375	1·602	2·027	2·565	3·247
150	2·618	·417	1·689	2·194	2·849	3·702
165	2·880	·458	1·778	2·372	3·163	4·219
180	3·142	·500	1·875	2·566	3·514	4·808
195	3·403	·541	1·975	2·776	3·901	5·483
210	3·665	·583	2·082	3·003	4·333	6·252
240	4·188	·666	2·311	3·514	5·340	8·119
270	4·712	·750	2·566	4·112	6·589	10·55
300	5·236	·833	2·849	4·808	8·117	13·70

[1] 'Journal of Franklin Institute,' 1868.

154. *Tensions in an endless belt.*—Let an endless belt be strained over two pulleys with an initial tension T_0. At the moment the driving pulley begins to move the belt is stretched on the driving side and the tension increased, whilst the other side of the belt is shortened and the tension diminished. Since the lengthening of the tight and the shortening of the slack side must be equal in amount, the average tension remains unaltered. That is,

$$\frac{T_2+T_1}{2}=T_0 \quad . \quad . \quad . \quad (12)$$

This process goes on till the force T_2-T_1, tending to rotate the driven pulley, is sufficient to overcome its resistance to motion. The driven pulley then rotates, the condition of the belt remaining permanent till the motion ceases again. It is necessary, however, that the initial tension should be sufficient to prevent slipping on either of the pulleys.

Let P=the resistance at the circumference of the driven pulley; V its velocity in feet per second; H the number of indicated horses' power transmitted—

$$P=T_2-T_1$$

$$P\,V=(T_2-T_1)\,V=550\,H$$

$$P=T_2-T_1=\frac{550\,H}{V} \quad . \quad . \quad . \quad (13)$$

If N=number of revolutions per minute, and d=diameter of pulley in inches,

$$P=126{,}000\,\frac{H}{d\,N} \quad . \quad . \quad . \quad (13\,a)$$

155. *Tensions in a belt transmitting a given horse-power.*—From equation 13 obtain the value of P, and from equation

11, or the table corresponding to it, obtain the value of $\frac{T_2}{T_1}$. Then, from eqs. 12 and 13 we get,

$$\left.\begin{aligned} T_2 &= \frac{P}{\frac{T_2}{T_1}-1} \cdot \frac{T_2}{T_1} = Px \\ T_1 &= \frac{P}{\frac{T_2}{T_1}-1} \qquad = Py \end{aligned}\right\} \quad . \quad (14)$$

Table to Facilitate the Calculation of the Belt Tensions.

$\theta =$			Values of $x = \frac{\frac{T_2}{T_1}}{\frac{T_2}{T_1}-1}$ for				Values of $y = \frac{1}{\frac{T_2}{T_1}-1}$ for			
In degrees	In circ. measure	In fractions of circumference	$\mu=0\cdot2$	$\mu=0\cdot3$	$\mu=0\cdot4$	$\mu=0\cdot5$	$\mu=0\cdot2$	$\mu=0\cdot3$	$\mu=0\cdot4$	$\mu=0\cdot5$
30	·524	·083	10·09	6·89	5·29	4·35	9·09	5·88	4·29	3·34
45	·785	·125	6·89	4·76	3·71	3·08	5·88	3·76	2·71	2·08
60	1·047	·167	5·29	3·71	2·92	2·45	4·29	2·71	1·92	1·45
75	1·309	·208	4·35	3·08	2·45	2·08	3·34	2·08	1·45	1·08
90	1·571	·250	3·71	2·66	2·14	1·85	2·71	1·66	1·14	·840
105	1·833	·319	3·26	2·37	1·93	1·67	2·26	1·36	·924	·667
120	2·094	·334	2·92	2·14	1·77	1·54	1·92	1·14	·762	·541
135	2·356	·375	2·66	1·98	1·64	1·44	1·66	·984	·649	·444
150	2·618	·417	2·45	1·84	1·54	1·37	1·45	·840	·541	·370
165	2·880	·458	2·29	1·73	1·47	1·31	1·29	·730	·462	·311
180	3·142	·500	2·14	1·64	1·40	1·26	1·14	·638	·398	·262
195	3·403	·541	2·03	1·56	1·35	1·22	1·03	·563	·345	·223
210	3·665	·583	1·93	1·50	1·30	1·19	·926	·499	·300	·190
240	4·188	·666	1·76	1·40	1·23	1·14	·763	·398	·230	·140
270	4·712	·750	1·64	1·32	1·18	1·10	·639	·322	·179	·105
300	5·236	·833	1·54	1·26	1·14	1·08	·541	·262	·140	·079

156. *Strength of the belt.*—The ultimate strength of the leather used for belting is about 3086 lbs. per sq. in. of section. At the joints the strength is reduced to 1747 lbs. per sq. in. of

belt section when the joint is riveted, and to 960 lbs. per sq. in. of belt section when the joint is laced. The greatest safe working tension (since the belt is subject to only one kind of stress) is about $\frac{1}{3}$rd of these values. Usually a belt has cemented and riveted joints made at the belting factory, and a laced joint, which is made when the belt is put in place, and which serves for tightening up the belt, if it wears slack. Hence, the greatest working tension is that corresponding to the laced joint, and is about 320 lbs. per sq. in. of the belt section.

The thickness of the belt varies from $\frac{3}{16}$ to $\frac{5}{16}$ inch if the belt is a single one, and from $\frac{3}{8}$ to $\frac{3}{4}$ inch if the belt is a double one. Hence, calling f the safe working tension per inch width of belt, and δ the belt thickness,

$$f = 320\,\delta \quad . \quad . \quad . \quad (15)$$

Thickness of Belt $=\delta=$										
$\frac{3}{16}$	$\frac{7}{32}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	$\frac{3}{4}$
Working Tension in lbs. per inch width $=f=$										
60	70	80	100	120	140	160	180	200	220	240

Professor Karl Keller points out that generally thin leather is chosen for narrow belts and thick leather for wide belts, so that if β is the width of the belt, we have on the average,

$$\delta = 0{\cdot}1\sqrt{\beta}$$

and hence,

$$f = 32\sqrt{\beta}$$

Width of Belt in ins. $=\beta$							
2	3	4	6	8	10	12	15
Calculated thickness of Belt $=\delta$							
0·14	·17	·20	·24	·28	·32	·35	·39
Working Tension in lbs. per inch of width $=f$							
45	55	64	78	90	101	110	124

The rule is a good one for small belts, but would be unsafe if applied to very wide ones. For these a definite thickness should be calculated and provided.

157. *Width of Belt for a given maximum tension.*—The greatest tension on the belt is the tension T_2 on the driving side. Then if f is the safe working tension obtained as above, the width of the belt is

$$\beta = \frac{T_2}{f} = \frac{P\frac{T_2}{T_1}}{f\left(\frac{T_2}{T_1} - 1\right)} = \frac{P}{f}x \quad . \qquad (16)$$

where the values of x are given in the preceding table, and P is obtained from eq. 13.

158. *Rough calculations of the size of belts.*—In a great many cases in practice, the belt embraces about 0·4 of the circumference of the pulley on which it is most liable to slip,[1] and the coefficient of friction is at least 0·3. Then, $\frac{T_2}{T_1} = 2$. When this is the case the following simple rules may be used :—

$$\left.\begin{array}{ll}\text{Driving force} & = P = \frac{550\,H}{V} \\ \text{Greatest tension} & = T_2 = 2\,P \\ \text{Initial tension} & = T_0 = 1\tfrac{1}{2}\,P \\ \text{Width of belt} & = \beta = \frac{2\,P}{f}\end{array}\right\} \quad . \quad . \qquad (17)$$

The following approximate table gives the width of belt calculated by these rough rules for one to twenty-five horse-power transmitted, the belt being assumed to be $\frac{7}{32}$nds of an inch in thickness, and carrying safely 70 lbs. tension per inch of width :—

[1] That is, the pulley having the smaller arc of contact.

Velocity of belt in ft. per sec.	Width of belt $\frac{1}{33}$ inch thick when the horses' power transmitted is									
	1	2	3	4	5	7½	10	15	20	25
1	15·7	31·4	...	...	...	...	...	...	...	...
2½	6·3	10·6	18·8	...	...	...	...	...	...	...
5	3·1	6·3	9·4	12·6	15·6	...	...	...	...	...
7½	2·1	4·2	6·3	8·4	10·4	15·6	21·0	...	...	...
10	1·5	3·2	4·7	6·4	7·8	11·8	15·7	23·6	31·4	...
12½	1·3	2·5	3·7	5·0	6·4	9·4	12·6	18·8	25·2	...
15	1·1	2·1	3·1	4·2	5·2	7·8	10·5	15·6	21·0	26·2
20	·79	1·6	2·4	3·2	3·9	5·9	7·9	11·7	15·7	19·6
25	·63	1·3	1·9	2·6	3·1	4·7	6·3	9·4	12·6	15·6
30	...	1·1	1·6	2·2	2·6	3·9	5·2	7·8	10·5	13·1
35	...	...	1·3	1·7	2·2	3·4	4·5	6·8	9·0	11·2
40	...	...	...	1·5	2·0	2·9	3·9	5·9	7·8	9·8
45	...	...	...	...	1·8	2·6	3·5	5·2	7·0	8·8
50	...	...	...	...	1·6	2·4	3·2	4·7	6·3	7·8
60	...	...	...	...	1·3	2·0	2·6	3·9	5·2	6·5
70	...	...	...	...	1·1	1·7	2·2	3·4	4·5	5·6
80	...	...	...	...	...	1·5	2·0	2·9	3·9	4·9
90	...	...	...	...	...	1·3	1·8	2·6	3·5	4·4
100	...	...	...	...	...	1·2	1·6	2·4	3·1	3·9

159. *Influence of the elasticity of the belt on the velocity ratio.*—Let s be the length of belt which runs off either pulley in the unit of time, the belt being measured in its unstrained condition. In working, the length s is extended to s_2 by the tension T_2 on the driving side, and to s_1 by the tension T_1 on the slack side. Since the elongation is proportional to the straining force,

$$s_2=(1+a\,T_2)\,s, \text{ and } s_1=(1+a\,T_1)\,s$$

where a is the elongation of one foot of belt by one pound of tension. The driving pulley receives s_1 feet of belt in the unit of time, and the driven pulley s_2 feet. Hence the velocities of the pulley circumferences are not exactly the same (as assumed in § 148) but are equal to s_2 and s_1 respectively,

$$\left.\begin{array}{l}\pi\,d_1\,N_1=s_1\\ \pi\,d_2\,N_2=s_2\end{array}\right\}\therefore\frac{N_1}{N_2}=\frac{d_2}{d_1}\cdot\frac{s_1}{s_2}=\frac{d_2}{d_1}\,\frac{1+a\,T_1}{1+a\,T_2}\quad . \qquad (18)$$

where N_2 is the number of revolutions of the driving, and N_1 the number of revolutions of the driven pulley. According to M. Kretz, $\frac{1+a\,T_1}{1+a\,T_2}=0{\cdot}975$ for new, and $0{\cdot}978$ for old belts. Hence,

$$a=\frac{{\cdot}00011}{\beta\,\delta} \text{ for old belts.}$$

$$=\frac{{\cdot}00015}{\beta\,\delta} \text{ for new belts.}$$

$\beta\,\delta$ being the area of section of the belt in sq. ins. The velocity of the driven pulley is about 2 per cent. less than it would be if the belt were inelastic. If motion is transmitted through several belts, the loss of velocity due to this cause would become important. This loss of velocity may be termed the *slip* due to elasticity of the belt.

160. *Effect of centrifugal tension on the strength of belts.*—When belts run at high speeds, part of the belt tension is expended in deviating the belt as it passes over the curved surface of the pulley. Hence, a given belt tension produces a less normal pressure on the pulley, and less resistance to slipping, in consequence of the centrifugal force of the belt. The weight of belting is about $w=0{\cdot}43\,\beta\,\delta$ lbs. per foot of length, where β and δ are in inches. The centrifugal force of one foot length of belting is $w\frac{V^2}{g\,r}={\cdot}0134\frac{V^2}{r}$ lbs. The normal pressure on the pulley is $p=\frac{T}{r}-\frac{w\,V^2}{g\,r}$, where the second term becomes unimportant at small velocities, as has been assumed above.

Hence, the greatest tension in the belt is,

$$T_2+\frac{w\,V^2}{g} \quad . \quad . \quad (19)$$

and the belt width must be calculated for that tension instead of for T_2.

Hence, if β is the width of the belt when the centrifugal

tension is neglected, its width when centrifugal tension is allowed for will be

$$\beta_1 = \frac{743\,\beta}{743 - \frac{v^2}{g}} = \frac{23924\,\beta}{23924 - v^2} \quad . \quad (19\,a)$$

The influence of centrifugal tension was first pointed out by Professor Rankine ('Millwork,' p. 532).

161. *Single, double, and combined belting. Joints in belting.*—The leather used for belting is of ox-hide tanned with oak bark, and only the best part of the hide, termed the butt, is used. The butts are cut into strips of the width required, and joined together to form a belt of any required length. The joints are made by paring down the ends of the strip, overlapping them, and cementing them with glue. They are then either sewn, laced, or riveted as an additional precaution. Fig. 160 *C* shows a cemented and laced joint; the overlap is about 7 inches long, and the laces $1\frac{1}{2}$ inch apart, extending an inch beyond the overlap at each end. Sometimes a few rivets are used in addition to the lacing. Fig. 160 *B* shows a cemented and riveted joint, the overlap 6 to 7 inches long, and having about one rivet to $2\frac{1}{4}$ or $2\frac{1}{2}$ sq. ins. of overlap. Fig. 160 *D* shows a laced and riveted joint.

In an endless belt one joint must be uncemented, so that it can be easily broken when the belt requires to be tightened. This joint may be a laced joint, like that previously described, or it may be made with belt screws shown in fig. 160 *A*. These belt screws are of iron with a very flat nut. The length of overlap may be 6 ins., and there may be one screw to 6 or 8 sq. ins. of overlap. This joint is more clumsy than a laced joint, but is very easily broken or made. The laces commonly used are strips of white leather tanned with alum.

When a single belt would be of inconvenient width, a double belt is used. This is made by cementing two strips of leather together, and then sewing them or rivet-

ing them. There may be about one rivet to 3 to 4 sq. ins. of belt. The double belt is more rigid than a single

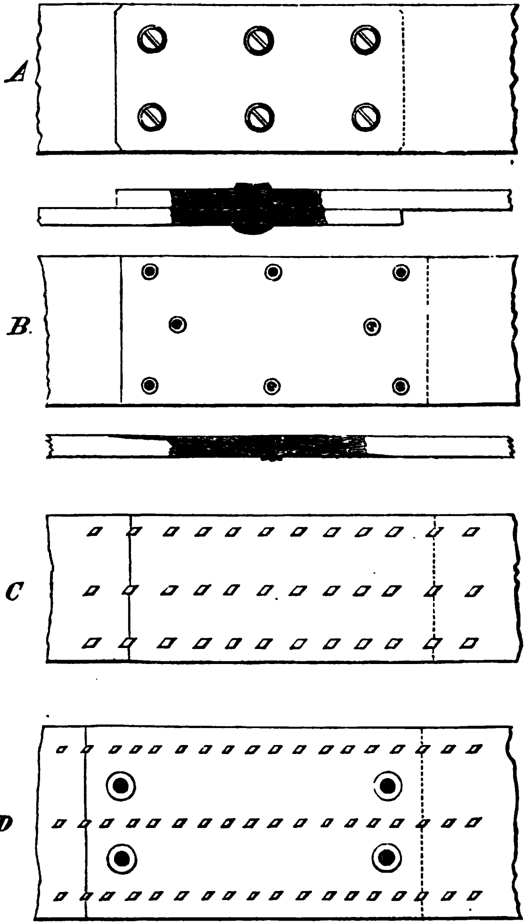


Fig. 160.

belt, and does not work satisfactorily unless there is ample

distance between the pulleys and the pulleys are not too small.

When a very broad belt is required to connect two shafts which are not parallel, (that is, when the belt has a half or quarter twist,) it does not work well, because its rigidity prevents its lying down in contact with the pulleys. It comes in contact with the pulleys on one side only. Messrs. Tullis, of Glasgow, have in such cases employed several narrow belts instead of a single wide one. These run side by side on the same pulleys, and are kept parallel by cross

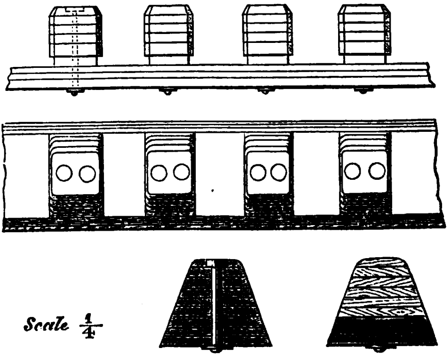


Fig. 161.

strips of leather riveted to them. Thus, for instance, instead of a 12-inch belt, three four-inch belts may be used, connected by cross strips $1\frac{1}{2}$ inch wide, at intervals of about 12 inches. A combined belt of this kind runs quite parallel, and comes much more perfectly in contact with the pulleys than an ordinary belt.

The inside of the leather is rougher than the outer surface, and belts should be so arranged that the rough side is always next the pulleys. Crossed belts and belts passing over guide pulleys require to be twisted in order to keep the same side of the belt next the pulleys.

Fig. 161 shows a peculiar leather belt introduced by

Messrs. Tullis, of Glasgow, and intended to work on pulleys having V-shaped grooves round their circumference. When the grooves have sides inclined at 45°, the adhesion of the belt to the pulley is increased about 2·6 times, so that the grooved pulley is equivalent to a cylindrical pulley with a coefficient of friction, $\mu = 0{\cdot}8$ to $1{\cdot}0$. The V-shaped belt shown in fig. 161 has been used for some years in America. It is made of slices of leather riveted together. The continuous part of the belt consists of three strips about $\frac{5}{8}$ths of an inch in total thickness, and 2 ins. in average width. Hence the belt section is about $1\frac{1}{4}$ sq. in. Several of these belts may be used side by side, precisely in the same way as the rope belts which are described in the next chapter. Messrs. Tullis state that the driving power of the belt is considerably greater than that of an ordinary rope belt.

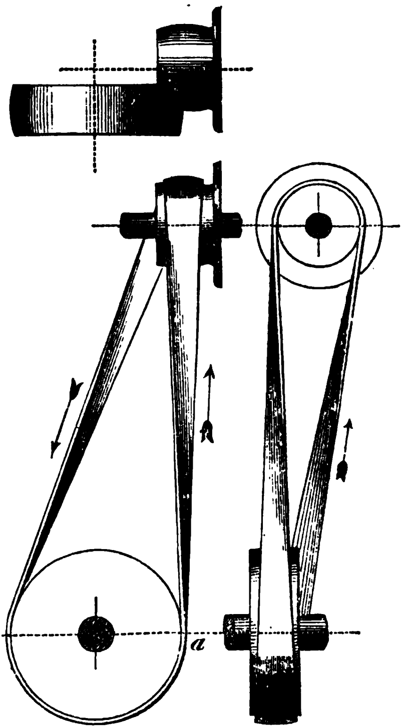


Fig. 162.

162. *Belts connecting shafts which are not parallel.*—When two shafts are not parallel and do not intersect, they may still be connected by an endless belt, provided the pulleys are properly placed. The single

and sufficient condition that the belt may run properly is this :—The point at which the belt is delivered from each pulley must be in the plane of the other pulley. This condition can only be fulfilled for a belt which always runs in one direction.

Fig. 162 shows three views of this arrangement of belting applied to two shafts at right angles. The arrows show the direction of the motion of the belt. If this be followed, it will be found that the point at which the belt runs off each pulley is in the plane passing through the centre of the other pulley. The belt would in this case be said to have a quarter twist.

163. *Guide pulleys.*—When two shafts are not parallel, and whether their directions intersect or not, they may be connected by a single endless belt if intermediate guide pulleys are used. These guide pulleys alter the direction of the belt without modifying the velocity ratio of the shafts. Fig. 163 shows an elevation and plan of an arrangement of pulleys and guide pulleys: *a b* is the intersection of the middle planes of the principal pulleys.

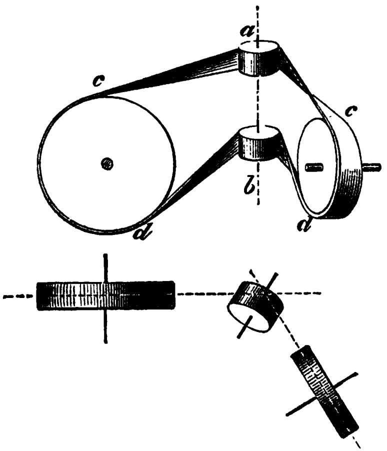


Fig. 163.

Select any two points *a* and *b* on this line, and draw tangents, *a c*, *b d*, to the principal pulleys. Then *c a c* and *d b d* are suitable directions for the belt. The guide pulleys must be placed with their middle planes coinciding with the planes *c a c* and *d b d*. The belt will run in either direction.

Guide pulleys are sometimes used merely to lengthen the belt between two shafts, which are too close together to be connected direct. Fig. 164 shows an arrangement of this kind. The middle planes of the guide pulleys are determined by the method just mentioned. It is, however, possible to place the guide pulleys with their axes parallel. Then the belt must be delivered from each pulley in the plane of the pulley on to which it is running. When this is provided for it will be found that the belt will only run in one direction.

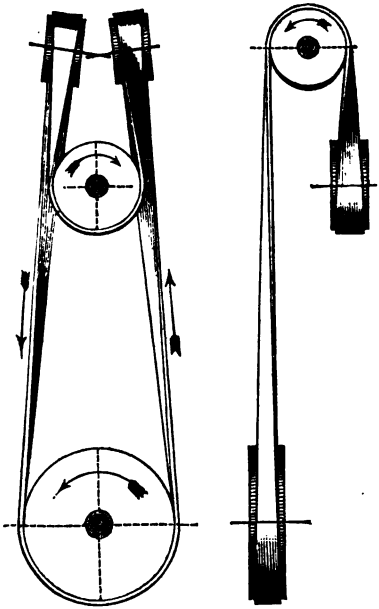


Fig. 164.

Figs. 165 and 166 show two arrangements of belting and guide pulleys for shafts at right angles. If the belts be traced round, it will be found that the rough side of the belt is always next the pulleys. It is to secure this that the belts have a quarter or half twist between the pulleys as shown.

164. *Rounding of pulley rim.*—When a flat belt is placed on a conical pulley, it tends to climb towards the larger end. If the pulley is made of a double conical form, or still better with a rounded rim a little larger at the centre than at the sides, the flat belt keeps its place on the pulley and has no tendency to slip off. The rounding of the rim may be $\frac{1}{2}$ inch per foot of width of pulley, or the section of the rim

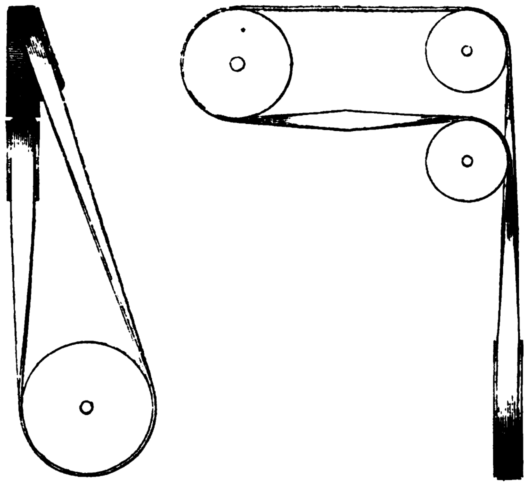


Fig. 165.

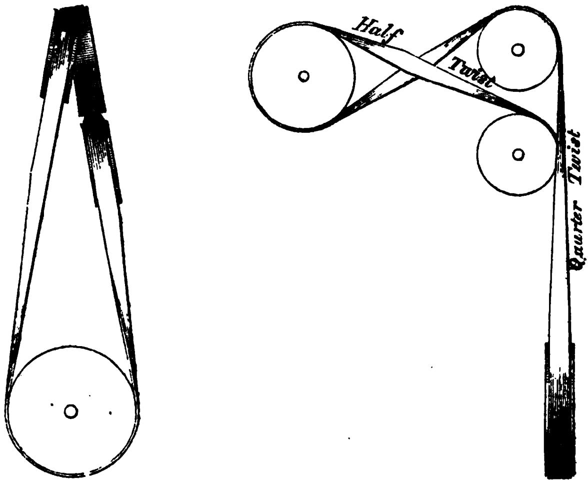


Fig. 166.

may be struck with a radius equal to from three to five times the width of the rim.

165. *Proportions of pulley. Rim of pulley.*—The pulley rim is a little wider than the belt it is intended to carry. Let B=width of rim, β=width of belt. Then,

$$B=\frac{9}{8}\ (B+0{\cdot}4)$$

$\beta=$	2	3	4	5	6	8	10	12
$B=$	2·7	3·82	4·95	6·08	7·2	9·45	10·7	13·95
$=$	$2\frac{3}{4}$	$3\frac{7}{8}$	5	6	$7\frac{1}{4}$	$9\frac{1}{2}$	$11\frac{3}{4}$	14

The form of the rim in section is shown in fig. 167; at the edge the thickness may be

$$t=0{\cdot}7\ \delta+{\cdot}005\ D$$

where D is the diameter of the pulley, and δ the thickness of the belt.

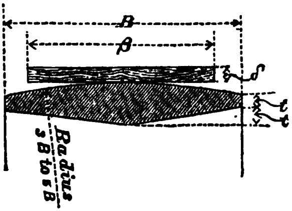


Fig. 167.

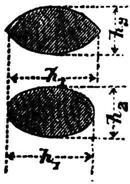


Fig. 168.

The diameter of the larger of two pulleys should not be less than 6 to 8 times the diameter of a wrought-iron shaft suitable for transmitting the power transferred to the belt, and the diameter of the smaller of two pulleys should not be less than about 18 times the belt thickness.

166. *Arms of the pulley.*—The arms of pulleys are of elliptical or segmental section as shown in fig. 168. The latter form of section looks lighter than the elliptical section

and is preferable. For a segmental arm the thickness $h_2 = \frac{1}{2} h_1$ For an elliptical arm the thickness $h_2 = 0{\cdot}4\, h_1$. The arms are either straight or curved. The curved arms

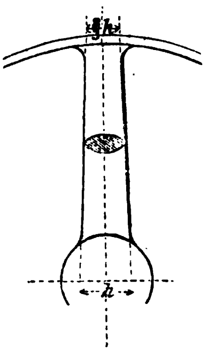


Fig 169.

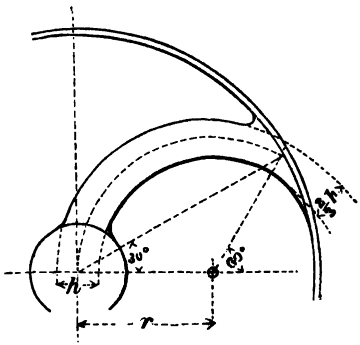


Fig. 170.

are rather less liable to fracture from contraction in cooling, but in other respects the straight arms are preferable, being lighter and stronger. The section of the arms is diminished from the nave to the rim, so that if we put h_1 h_2 for the breadth and thickness of the arm, supposed produced to the centre of the shaft, the breadth and thickness at the rim will be $\frac{2}{3} h_1$ and $\frac{2}{3} h_2$.

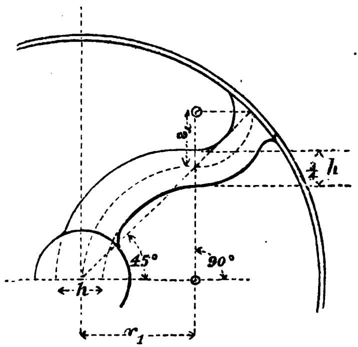


Fig. 171.

Fig. 169 shows an ordinary straight arm, fig. 170 a curved arm, and fig. 171 an **S**-shaped or doubly curved arm. The figures indicate sufficiently the way in which the centre line of the arm is drawn. Let R be the radius of the pulley

measured to the inside of the rim. Then in fig. 170, $r = 0{\cdot}577$ R; and in fig. 171, $r_1 = 0{\cdot}471$ R, and $r_2 = 0{\cdot}236$ R.

Let ν be the number of arms, B the breadth, and D the diameter of the rim. Then,

$$\nu = 3 + \frac{\text{B D}}{150}$$

the nearest whole number being taken.

Width of Pulley B	Diameter of pulley in inches when the number of arms is				
	4	5	6	8	10
3	50	100	150	...	...
6	25	50	75	125	175
12	12	24	36	62	87
18	8	16	24	42	58
24	6	12	18	31	44

The number of arms is really arbitrary, and may be altered if necessary. In calculating the strength of the arms it will be assumed that each arm is equally loaded, and also that each arm may be considered to be fixed at the nave and free at the rim. As these assumptions are only in a rough sense true, a large factor of safety must be allowed. Pulley-arms are also liable to be considerably strained by contraction in cooling. Hence a margin of strength must be allowed to meet this contingency. For these reasons the working stress on the cast-iron will be taken at $f = 2250$ lbs. per sq. in.

If P is the driving force transmitted by the belt, determined by eq. 13, and D is the diameter of the pulley, the greatest bending moment on each arm is—

$$\text{M} = \frac{1}{2} \frac{\text{P D}}{\nu}$$

For an elliptical section of width h (measured at the

centre of the pulley) and thickness 0·4 h, the section modulus (Table IV., p. 35) is

$$\frac{\pi}{32} \times h^2 \times 0{\cdot}4\, h = 0{\cdot}0393\, h^3 \text{ nearly,}$$

and for a segmental section of width h and thickness 0·5 h, the modulus may be taken to be the same. Equating the bending moment and moment of resistance

$$\frac{1}{2}\frac{\mathrm{P\,D}}{\nu} = 0{\cdot}0393\, f h^3$$

$$h = \sqrt[3]{\left(\frac{1}{{\cdot}0786 f} \cdot \frac{\mathrm{P\,D}}{\nu}\right)}$$

and putting $f = 2250$

$$h = 0{\cdot}1781 \sqrt[3]{\frac{\mathrm{P\,D}}{\nu}}. \qquad (20)$$

Since in designing pulleys the driving force P will often be unknown, we may design the arms to resist the maximum driving force which is likely to be transmitted by a belt, the width of which is $\frac{4}{5}$ B. The driving force will be very often half the greatest tension in the belt, and will rarely exceed $\frac{4}{5}$th that tension, except when the belt embraces an unusually large arc. The greatest belt tension may be taken at 70 lbs. per inch width of the belt for single belting, and 140 lbs. for double belting. Hence, P will not exceed 56 and 112 lbs. per inch width of the belt, or 45 and 90 lbs. per inch width of the pulley.

$$\mathrm{P} = 45\ \mathrm{B} \text{ for single belts.}$$

$$= 90\ \mathrm{B} \text{ for double belts.}$$

Inserting this value in the equations above,

$$\left.\begin{aligned} h &= 0{\cdot}6337 \sqrt[3]{\frac{\mathrm{B\,D}}{\nu}} \text{ for single belts.} \\ &= 0{\cdot}798 \sqrt[3]{\frac{\mathrm{B\,D}}{\nu}} \text{ for double belts.} \end{aligned}\right\} \quad (21)$$

These equations agree well with practice. If the arms are of wrought-iron f may be taken equal to 9000 lbs. per sq. in. If the section of the arms is different, the proper section modulus must be substituted for that assumed above.

167. *The nave of the pulley.*—The thickness of the nave may be

$$\delta = 0{\cdot}14 \sqrt[3]{\text{B D}} + \tfrac{1}{4} \text{ (single belt)}$$

$$= 0{\cdot}18 \sqrt[3]{\text{B D}} + \tfrac{1}{4} \text{ (double belt).}$$

The length of the nave, λ, should not be less than $2\frac{1}{2}\,\delta$, and is often $\frac{2}{3}$ B. The key is to be proportioned by the rules in § 67. When the pulley is to run loose on the shaft the nave should be bushed with brass, and the length of the nave should be equal to B.[1] Provision must also be made for lubrication. In large pulleys the nave may be strengthened by wrought-iron rings shrunk on.

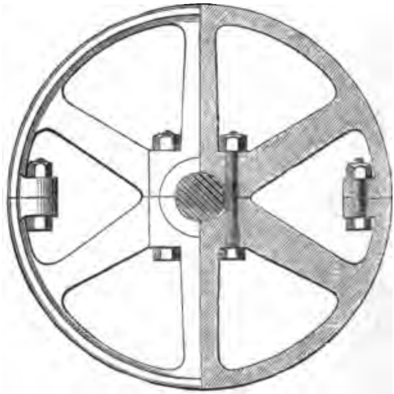


Fig. 172.

168. *Split pulleys.*—When the pulleys are intended to be fixed on shafts which are bossed at the ends, they are often cast in halves. The two halves can then be bolted together on the shaft without dismounting the shaft and without having recourse to cone keys. Fig. 172 shows a pulley of this kind.[2] The net section of the bolt at the rim should be about a quarter the section of the rim,

[1] Box, 'Millgearing,' p. 75.
[2] 'Engineering,' vol. xi. p. 183.

plus $\frac{1}{4}$ sq. in. and that of bolt at the nave about $\frac{1}{4}$ sq. in. plus a quarter the section of the nave calculated as above. The two half-pulleys may be made to grasp the shaft so tightly that relative motion is prevented by friction, and no keys are necessary.

If it is undesirable to cast the pulley in halves, the eye of the pulley must be bored out large enough to pass over the bosses at the ends of the shaft and slightly conical. Then three cone keys, described in § 67, are fitted in the space between the pulley-eye and the shaft. Another plan is to use a conical sleeve, split on one side, like that shown in the drawing of the Sellers's coupling, fig. 108; this is drawn into the eye of the pulley by bolts. In either of these plans the pulley is fixed on the shaft by friction only.

CHAPTER XI.

ROPE GEARING.

169. At the present time, ordinary hemp ropes are being used to replace leather belting and toothed gearing in the transmission of power. For special purposes, similar ropes, made of cotton, are also used. The pulleys for belts of this kind are made with V-grooves round their circumference, each groove having its own rope-belt. When only a small amount of power is to be transmitted, the rope rests on the bottom of the groove, but in most cases the rope rests against the sides of the groove, and is wedged between them, so that the frictional resistance to slipping is very great. The ropes most commonly used are patent ropes of three strands (fig. 173), white or untarred, and from 1 to 2 inches diameter. They are placed on the pulleys with very little initial tension, and the joint is made by splicing the rope. The pressure of the rope on the pulley is chiefly due to its weight. Hence, to secure sufficient frictional adhesion the pulleys should be large, and at a sufficient horizontal distance apart. If the pulleys are vertically over each other the rope must be strained more tightly, and its durability is impaired. Usually the horizontal distance between the pulleys is 20 to 60 feet, whatever their vertical distance may be. The ropes are never strained so tightly as to draw them nearly straight. They hang between the pulleys in catenary curves which approximate to parabolas. It is

Fig. 173.

advisable to have the driving side of the rope on the lower side of the pulleys and the slack side above. Then, in driving, the two sides approach each other, and the arc of contact on the pulleys is increased. The slacker the ropes are, consistently with obtaining sufficient frictional resistance to slipping at the pulleys, the better, because the ropes are less squeezed in the grooves and wear longer.

170. *Strength of ropes.*—The breaking strength of white or untarred rope varies from 7,000 to 12,000 lbs. per sq. in., and is to some extent dependent on the amount of twist given to the rope. The twist diminishes the strength of the rope, but makes it more solid and durable. The working strength may be taken at about $\frac{1}{8}$th of the breaking strength. Hence, the working strength is $f=875$ to 1500 lbs. per sq. in. In the following calculations it is assumed that $f=1200$ lbs. per sq. in.

Let δ be the diameter; γ, the girth; and G, the weight per lineal foot of the rope. The section of hawser-laid rope is about $\frac{9}{10}$ths of the area of the circumscribing circle. Hence,

$$\left.\begin{aligned} \text{Area of section} &= 0{\cdot}9 \times \frac{\pi}{4} \times \delta^2 = 0{\cdot}707\,\delta^2 = 0{\cdot}0716\,\gamma^2 \\ \text{Working strength} &= 0{\cdot}707 f\,\delta^2 = 850\,\delta^2 = 86\,\gamma^2 \end{aligned}\right\} \quad (1)$$

When the rope is wet or tarred, the strength is reduced by about one-fourth.

The weight of ropes, G, in lbs. per foot of length, is given by the following equations:[1]—

$$\left.\begin{aligned} G &= 0{\cdot}2812\,\delta^2 = 0{\cdot}0285\,\gamma^2 \quad \text{dry} \\ &= 0{\cdot}3376\,\delta^2 = 0{\cdot}0342\,\gamma^2 \quad \text{wet or tarred} \end{aligned}\right\} \quad . \quad (2)$$

Hence, for dry ropes, the weight of 3016 feet of rope is equal to the working strength.

[1] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 270.

171. *Ordinary driving force of rope belts.*—In order to ensure durability, the tension in the belt when at work is only a small fraction of the working strength. From data furnished by Messrs. Pearce Brothers, of Dundee, who have erected rope belting extensively, it appears that the difference of tension on the two sides of the belt, or driving force is :[1]—

$$T_2 - T_1 = P = 7{\cdot}81\ \gamma^2 \text{ lbs.} \qquad . \qquad . \quad (3)$$

It will be shown presently that when the belt embraces 0·4 of the circumference of the smaller pulley, the greatest tension is

$$T_2 = 1{\cdot}208\ P = 9{\cdot}43\ \gamma^2$$

Hence the greatest tension is less than $\frac{1}{9}$th of the working strength of the rope.

Table of Weight, Strength, and Driving Force of Rope Belts.

Girth of Rope in ins. γ	Diameter of rope in ins. δ	Weight per foot in lbs. G	Working strength in lbs.	Driving force in lbs.		
				P	K	K_1
$3\frac{1}{8}$	1	·279	842	$76\frac{1}{2}$	0·14	·00061
$4\frac{3}{4}$	$1\frac{1}{2}$	·643	1940	176	0·32	·00140
$5\frac{1}{4}$	$1\frac{11}{16}$	·862	2602	236	0·43	·00188
$6\frac{1}{2}$	$2\frac{1}{16}$	1·204	3633	330	0·60	·00262

172. *Work transmitted by rope belts.*—Since the power which any given rope will transmit is limited, and it is not convenient to use very large ropes, it is necessary in most cases to use several ropes. The pulleys have parallel grooves in which the ropes are placed, sometimes to the number of 20 or 25. Let n be the number of ropes on a pulley ; V, the velocity of the belt in feet per second ; d, the diameter of the pulley in inches ; N, the number of revolutions of the pulley per minute. Then the work transmitted by each belt is

$$P\,V \text{ foot lbs. per second.}$$

[1] In different cases in practice, $P = 6\gamma^2$ to $8\gamma^2$.

Let H be the number of horses' power transmitted,

$$H = \frac{n\,P\,V}{550} = K\,n\,V \qquad . \quad . \quad . \quad (4)$$

Also since
$$V = \frac{\pi\,d\,N}{12 \times 60}$$

$$H = \frac{n\,P\,d\,N}{126100} = K_1\,d\,N \qquad . \quad . \quad (5)$$

where K and K_1 are constants, the values of which are given in the table above.

173. *Friction of rope belting.*—The coefficient of friction for a rope on a metal pulley is $\mu = 0{\cdot}28$. In rope transmission, however, the rope is wedged in the groove of the pulley, and the normal pressure between the rope and the sides of the groove is greater than the force pressing the rope into the groove, in the ratio of cosec. $\frac{\theta}{2} : 1$, where θ is the inclination of the sides of the groove. Hence, the resistance to slipping is the same as on a cylindrical pulley having a coefficient of friction,[1]

$$\mu = 0{\cdot}28 \text{ cosec. } \frac{\theta}{2}$$

In practice $\theta = 45°$, and then $\mu = 0{\cdot}7$.

From the equations in the previous chapter—

$$\left.\begin{array}{ll} \text{Greatest tension } T_2 & = P\,x \\ \text{Tension on slack side} = T_1 & = P\,y \\ \text{Initial tension} & = \frac{1}{2}(T_1 + T_2) = P\,z \end{array}\right\} \quad . \quad (6)$$

And using the above value of μ :—

[1] Some recent experiments by Messrs. Pearce give $\mu = 0{\cdot}57$ to $0{\cdot}88$ for ropes on ungreased grooved pulleys, and $\mu = 0{\cdot}38$ to $0{\cdot}41$, when the pulleys were greased. The former values agree fairly with $\mu = 0{\cdot}7$ assumed above.

Fraction of Circumference of Pulley embraced by Rope.

	0·1	0·2	0·3	0·4	0·5	0·6
$\frac{T_2}{T_1}=$	1·55	2·41	3·74	5·81	9·02	14·00
$x=$	2·82	1·71	1·73	1·21	1·13	1·08
$y=$	1·82	·71	·37	·21	·13	·08
$z=$	2·32	1·21	·87	·71	·63	·58

174. *Pulleys for rope belting.*—The pulleys are usually of cast iron, and when motion is taken from a steam-engine grooves are turned in the rim of the fly-wheel. The diameter of the smallest pulley should not be less than thirty times the diameter of the rope carried by it. The larger the pulley the less is the injury done to the rope by bending and unbending.

Fig. 174 shows the form of the grooves in the pulley-rim and the proportions adopted. The unit for the proportional

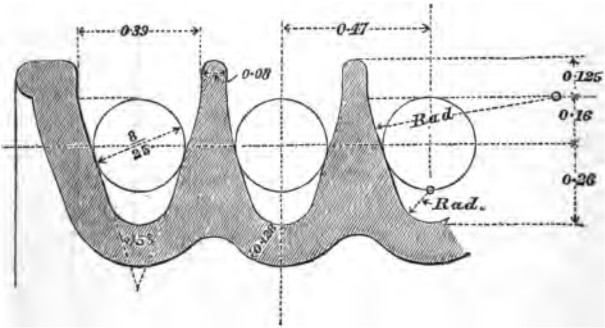


Fig. 174.

figures is γ, the girth of the rope. If the pulley is a guide-pulley merely, the rope should rest on the bottom of the groove. The sides of the groove are usually inclined at 45°. The pulleys are cast in one piece when they are less than 8 feet diameter unless they have to be fixed on shafts which

have bosses at the ends, or require to be fixed after the shafts are in position. When this is the case they are cast in halves, and they are also usually cast in halves when they are from 8 to 12 feet diameter. Larger pulleys are cast in segments and bolted together. The grooves in each pulley must be accurately turned to the same gauge, and of the same diameter. The splices in the rope should be 9 or 10 feet long.

The amount of power which may be transmitted by rope gearing is very great when the conditions are suitable. Thus, suppose the circumferential velocity of an engine fly-wheel is 5000 ft. per minute. Then 20 ropes of $6\frac{1}{2}$ inches girth placed on the fly-wheel would transmit 1000 indicated horses' power. The breadth of the fly-wheel rim in that case would need to be 5 feet. If the speed of the rim were reduced to 4000 feet, 25 ropes would be required. If $5\frac{1}{4}$-inch ropes were used, 20 ropes would transmit 666 horses' power at 5000 feet per minute.

Wire Rope Gearing.

175. Wire ropes have been occasionally used for the transmission of a greater amount of power than would be possible with a weaker material. Sometimes they have been used for direct haulage, and more rarely they have been used like ordinary belts to connect rotating pieces. In the latter case they often gave trouble from the fracture of the whole rope or of individual wires. It may now be asserted that the proper conditions of using wire ropes in those cases were not fulfilled. During the last twenty years a method of wire rope transmission has been in use, on the Continent chiefly, which is perfectly successful, which combines economy of first cost with economy in maintenance, and by which large amounts of motive power can be transferred to great distances, with an efficiency impossible with any other mode of transmission. This mode of transmission was matured by M. G. A. Hirn, and received from him the name of telodynamic transmission.

In belt transmission, we may increase the amount of power transmitted in three ways: by increasing the frictional bite of the pulleys, by increasing the strength of the belt, and by increasing the velocity of the belt. The first principle is applied in ordinary rope transmission, by wedging the ropes in V-shaped grooves in the rim of the pulley. With wire ropes these wedge-grooves cannot safely be adopted, because of the injury done to the rope. On the other hand, wire ropes are enormously stronger than hemp ropes, and if in addition they are run at the highest practicable velocity, a very great amount of power can be transmitted, with comparatively light gearing. The principle of telodynamic transmission is, therefore, to use flexible belts of very great strength on ordinary pulleys, and to work them at very high velocities. Various expedients are necessary in the application of this principle and in securing the greatest possible efficiency, or the least waste of work in friction. The system has proved so successful that power is now frequently transmitted to very great distances with comparatively little loss. That loss is estimated at only $2\frac{1}{2}$ per cent. + 1 per cent. in addition, for every 1000 yards of distance. The method is not suitable when the distance to which the power is to be transmitted is short, and 130 feet has been fixed as the minimum distance for which transmission by wire rope is applicable.[1] At less distances the wire rope is subject to considerable oscillations, which, however, it is possible may be prevented.

176. *Form, strength, and weight of wire ropes.*—The rope used consists of six or more strands wound upon a hemp core. Each strand consists of six or more wires also twisted round a hemp core. Fig. 175 shows the section of a rope, the shaded circles being sections of the wires, and the unshaded portions hemp. The angles of twist are usually 8° to 15° for the strands, and 10° to 25° for the rope. The wire diameter varies usually from $\frac{1}{30}$ to $\frac{1}{12}$ of an inch.

[1] Vigreux, 'Proc. Inst. of Civil Engineers,' xlv. p. 266.

The ropes most commonly used have six strands, each containing six wires and a hemp strand at the centre. For these ropes with 36 wires the diameter of the rope is nearly $9\frac{1}{4}$ times the diameter of the single wires. Ropes of 42 wires are used with the middle hemp core replaced by a strand of six wires, and their diameter is about $10\frac{1}{2}$ times the diameter of a single wire. The number of strands and of wires in each strand is, however, arbitrary, and ropes of 8 strands, each of 10 wires, of 10 strands, each of 9 wires, and various other proportions, are adopted. The relation between the diameter of the rope Δ, the diameter of the wires δ, and the number of wires ν, is given very approximately by the formula

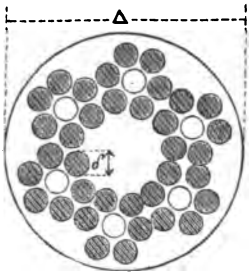


Fig. 175.

$$\frac{\Delta}{\delta} = \frac{\nu}{13} + 7 \quad . \quad . \quad . \quad . \quad (7)$$

The breaking strength of iron wire varies from 85,000 to 108,000 lbs. per sq. in., so that the greatest working stress may safely be taken at 25,000 lbs. per sq. in. of the section of the wires. The breaking strength of steel wire may be taken at 130,000 lbs., and the safe load for such wire at 40,000 lbs. per sq. in.

The weight of wire rope per lineal foot is very nearly

$$= G = 3{\cdot}268\, \nu\, \delta^2 = 1{\cdot}341\, \Delta^2 \text{ lbs.} \quad . \quad . \quad (7a)$$

177. Used as a belt the rope is subjected to two very different straining actions. There is a longitudinal tension due to the tightness with which the belt is strained over the pulleys, to the weight of the rope, and to the work transmitted. There are also stresses of tension and compression, due to the bending of the rope to the radius R of the pulley.

Let f_t be the greatest working stress due to the longitudinal tension of the belt, and f_b the stress due to bending. For those wires which lie on the stretched side of the belt in passing over the pulley, the total stress is

$$f = f_t + f_b \quad . \quad . \quad . \quad . \quad (8)$$

When a cylinder is bent to a radius R, the bending moment at any point is[1]

$$M = \frac{E\,I}{R} = \frac{E\,Z\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (9)$$

The moment of resistance to bending of a circular section of diameter δ (§ 28) is,

$$f_b\,Z$$

Equating these values,

$$f_b = \frac{E\,\delta}{2\,R} \quad . \quad . \quad . \quad . \quad (10)$$

If T is the total longitudinal tension in a rope having ν wires, each of δ inches diameter,

$$f_t = \frac{T}{\frac{\pi}{4}\,\delta^2\,\nu} \quad . \quad . \quad . \quad . \quad (11)$$

Hence the total stress in the most strained wires is

$$f = \frac{E\,\delta}{2\,R} + \frac{T}{\frac{\pi}{4}\,\delta^2\,\nu}$$

hence

$$T = \left(f - \frac{\delta\,E}{2\,R}\right)\frac{\pi}{4}\,\delta^2\nu \quad . \quad . \quad . \quad (12)$$

For a given value of the limiting stress f, T will be a maximum for pulleys of a given radius, when δ is so chosen that,

$$\frac{d\,T}{d\,\delta} = o$$

[1] Compare equation 20, p. 50, and the values of Z given at p. 35.

or when

$$\frac{R}{\delta}=\frac{3E}{4f} \quad . \quad . \quad . \quad . \quad (13)$$

Putting $f=25{,}000$, and $E=29{,}000{,}000$ for wrought iron,

$$\frac{R}{\delta}=870.$$

That is, the longitudinal tension will be a maximum when the diameter of the wires is $\frac{1}{870}$th of the pulley radius. Putting $f=40{,}000$, and $E=30{,}000{,}000$ for steel,

$$\frac{R}{\delta}=560.$$

When the ratio $\frac{R}{\delta}$ varies from these proportions, we have for the greatest safe working stress due to the longitudinal tension,

$$f_t=f-f_b=f-\frac{\delta E}{2R} \quad . \quad . \quad . \quad (14)$$

Sometimes when the rollers are near together, the deflection of the rope will be too small with this tension. The deflection, when the rope is not working, should not be less than 18 inches. If this is the case, a lower value of the working tension should be adopted.

For wrought iron			For steel		
$\frac{R}{\delta}$	f_b	f_t	$\frac{R}{\delta}$	f_b	f
650	22,400	1,600	400	37,500	2,500
700	20,710	3,290	450	33,400	6,600
750	19,400	4,600	500	30,000	10,000
800	18,125	5,875	550	27,300	12,700
850	17,100	6,900	600	25,000	15,000
900	16,110	7,890	650	23,100	16,900
950	15,300	8,700	700	21,430	18,570
1,000	14,500	9,500	750	20,100	19,900
1,100	13,180	10,820	800	18,750	21,250
1,200	12,080	11,920	900	16,666	23,334

It will now be obvious why the working tension is so low in wire rope transmission, being only 7,000 to 9,000 lbs. for iron wire ropes, and correspondingly small for steel wire ropes. In practice the iron wire has proved superior to steel, and the numerical values assumed in the following calculations will relate to iron wire only. The formulas are equally applicable, however, to steel, if suitable values are taken for the safe working stress.

178. *Total longitudinal tension of rope.*—Let T be the greatest tension in any part of the rope, then since the sectional area of the rope is $\frac{\pi}{4}\nu\,\delta^2$,

$$T=\frac{\pi}{4}\nu\,\delta^2 f_t$$

Hence the size of rope for a given total tension is,

$$\delta=\sqrt{\frac{4}{\pi f_t}}\sqrt{\frac{T}{\nu}} \quad . \quad . \quad . \quad (15)$$

$f_t=$	7,000	8,000	9,000	10,000
$\sqrt{\frac{4}{\pi f_t}}=$	0·01349	·01262	·01190	·01128

It has been stated already that δ is usually between $\frac{1}{50}$th and $\frac{1}{12}$th of an inch.

179. *Tension due to centrifugal force.*—The tension due to centrifugal force in a rope weighing G lbs. per foot, and travelling at v feet per second is,

$$C=G\frac{v^2}{g}$$

Inserting the value found in eq. (7*a*) for G,

$$C=3{\cdot}268\,\nu\,\delta^2\frac{v^2}{g}$$

and dividing this by the section of the rope, the intensity of centrifugal tension is—

$$c=0{\cdot}1293\,v^2 \text{ lbs. per sq. in.} \quad . \quad . \quad (16)$$

$v=$	50	60	70	80	90	100
$c=$	323	466	634	828	1047	1293

These stresses must be deducted from the stresses in the table, § 177, p. 238, in order to find the safe tension due to the power transmitted.

180. *Driving force of belt, and power transmitted.*—The equations for the friction of a belt on a pulley given in Chapter X. are equally applicable for an iron wire rope, if proper values are taken for the coefficient of friction.

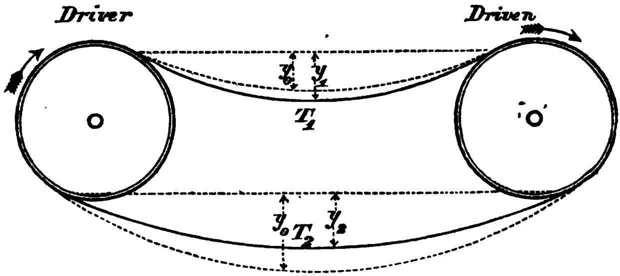


Fig. 176.

Taking $\mu=0{\cdot}24$, and supposing that the belt embraces nearly a semicircle of the pulley, so that $\theta=3$

$$\varepsilon^{\mu\theta}=2 \text{ nearly.}$$

The ratio of the tensions on the tight and slack sides of the belt due to the resistance to slipping is,

$$\frac{T_2}{T_1}=e^{u\theta}=2 \quad . \quad . \quad . \quad (17)$$

The driving force of the belt is the difference of the tensions, that is,

$$P=T_2-T_1$$

$$\left.\begin{aligned} \therefore T_2&=2\,P \\ T_1&=P \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (18)$$

The work transmitted in foot pounds per second is $P\,v$, and if H is the horse-power transmitted,

$$P = \frac{550\,H}{v} \text{ or } H = \frac{P\,v}{550}. \quad . \quad . \quad (19)$$

We may put equation 15 in the form for calculating the size of rope from the driving force, instead of from the total tension. The tension, apart from the bending stress, must not exceed $f_t - c$ lbs. per sq. in., and the total tension due to the work transmitted and the initial tension is T_2 or $2\,P$. Hence,

$$\delta = \sqrt{\frac{2 \times 4}{\pi(f_t - c)}}\sqrt{\frac{P}{\nu}} \quad . \quad . \quad (20)$$

$f_t - c =$	5,500	6,000	7,000	8,000	9,000	10,000
$\sqrt{\frac{8}{\pi(f_t - c)}} =$	0·0215	·0206	·0191	·0178	·0168	·0160

181. *Tightened belt.*—In some cases the diameter of the rope calculated by this rule will prove to be very small. Then it may be convenient to adopt a larger rope than is absolutely necessary. If this is done, either the size of the pulleys may be reduced, if desirable, the rope being capable of bearing a greater bending stress, or the tension in the rope may be increased beyond what is necessary to prevent slipping at the pulleys, with a view of reducing the deflection of the rope between the pulleys.[1] In this latter case, the tension T_2 may be calculated from the size of rope adopted; then T_1 is $T_2 - P$, and from these tensions the curves of the rope may be determined.

182. *Weight of ropes.*—The weight of wire ropes per lineal foot may be taken to be[2]

$$G = 3{\cdot}268\,\nu\,\delta^2 = 1{\cdot}341\,\Delta^2 \text{ lbs.} \quad . \quad . \quad (21)$$

183. *The catenary curve.*—No important error is likely to

[1] Reuleaux, 'Der Constructeur,' p. 398.
[2] Karl Von Ott, 'Proc. Inst. of Civil Engineers,' xlv. p. 271.

be introduced by considering the rope to be perfectly flexible and of uniform section. In that case, the funicular curve in which the rope hangs between the pulleys is known to be the catenary, and the tensions in the rope are due to a distribution of load, vertical and constant per unit length of arc.

Let fig. 177 show the form of the curve in which the rope hangs, and let O be the lowest point of the curve. Take O for origin of co-ordinates, and let $x = O$ A, and $y =$ A B be the abscissa and ordinate of any point B of the curve. Since

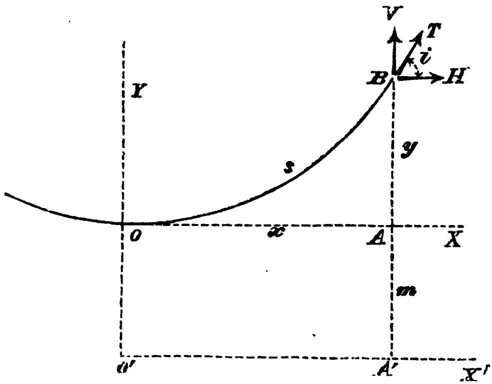


Fig. 177.

the rope is flexible, the tension at any point of the curve must be in the direction of the rope. Let T be the tension at B, and let V and H be its vertical and horizontal components. Let the length of the arc O B$=s$, and let the inclination of the curve at B to the horizontal be denoted by i.

Since G is the weight of a unit length of rope, G s is the weight of O B, and this is equal to the vertical component of the tension at B; hence, G s=V. The other tensions H and T are equivalent to the weight of lengths m and n of rope, m and n being at present undetermined, so that H=G m and T=G n.

The inclination of the rope at B is given by the equations—

$$\left.\begin{aligned}
\cos i &= \frac{dx}{ds} \\
\sin i &= \frac{dy}{ds} = \sqrt{1 - \frac{dx^2}{ds^2}} \\
\tan i &= \frac{dy}{dx} = \frac{\sqrt{\left(1 - \frac{dx^2}{ds^2}\right)}}{\frac{dx}{ds}}
\end{aligned}\right\} \quad . \quad . \quad (22)$$

$$\text{But } \tan i = \frac{V}{H} = \frac{s}{m} \quad . \quad . \quad . \quad (23)$$

Hence,

$$\frac{dx}{ds} = \frac{m^2}{\sqrt{m^2 + s^2}}$$

Integrating, and putting $x = o$, when $s = o$,

$$x = m \text{ hyp. log.} \left\{ \frac{s}{m} + \sqrt{\left(1 + \frac{s^2}{m^2}\right)} \right\} \quad . \quad (24)$$

That is,

$$\frac{s}{m} + \sqrt{\left(1 + \frac{s^2}{m^2}\right)} = e^{\frac{x}{m}}$$

$$s = \frac{m}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right) \quad . \quad . \quad . \quad . \quad (25)$$

$$\frac{s}{m} = \tan i = \frac{dy}{dx}$$

Inserting the value of $\frac{s}{m}$ in equation 25,

$$\frac{dy}{dx} = \frac{1}{2}\left(e^{\frac{x}{m}} - e^{-\frac{x}{m}}\right)$$

Integrating,

$$y = \frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right) + C$$

The constant is determined by the condition that $y=o$, when $x=o$,

$$\therefore y=\frac{m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}-2\right)=\sqrt{s^2+m^2}-m \quad . \quad . \quad (26)$$

This is the equation to the curve termed the catenary, and m is its parameter. For the relation between the tensions at B we have

$$\text{H}=\text{G}\, m \qquad\qquad \text{V}=\text{G}\, s$$

$$\text{T}=\sqrt{(\text{H}^2+\text{V}^2)}=\text{G}\sqrt{(m^2+s^2)}=\frac{\text{G}\, m}{2}\left(e^{\frac{x}{m}}+e^{-\frac{x}{m}}\right)$$

$$=\text{G}\,(y+m) \quad . \quad . \quad . \quad . \quad (27)$$

From this last equation it is seen that the tension, at any point of the rope, is equal to the weight of a length of the rope, $y+m$, equal to the ordinate y of the point added to the parameter m. If $O\,O'=m$ (fig. 177) and $O'\,X'$ is drawn horizontally through O', then the tension at any point B is the weight of a length B A′ of the rope.

At the vertex O of the curve the tension is horizontal, and equal to the weight G m of a length $O\,O'$ of the rope. But this is the same as the horizontal component of the tension at B. Hence the horizontal component of the tension at any point is equal to the horizontal tension at the vertex of the curve.

184. *Approximate equations.* — Introducing the value $\tan i=\frac{s}{m}$ in eq. 24, we get [1]

$$x=m \text{ hyp. log.}\left(\frac{\sin i+1}{\cos i}\right)$$

$$\frac{x}{m}=\text{hyp. log.}\,(1+\sin i)-\tfrac{1}{2}\text{ hyp. log.}\,(1-\sin^2 i)$$

$$=\tfrac{1}{2}\text{ hyp. log.}\ \frac{1+\sin i}{1-\sin i} \quad . \quad . \quad . \quad . \quad (28)$$

But hyp. log. $\frac{1+\sin i}{1-\sin i}=2\left(\sin i+\tfrac{1}{3}\sin^3 i+\ .\quad .\quad\right)$

[1] Keller's 'Treibwerke,' p. 201.

Or when i is small, neglecting the terms containing powers higher than the first—

$$\text{Hyp. log. } \frac{1+\sin i}{1-\sin i}=2 \sin i$$

$$\therefore x=m \operatorname{Sin} i \quad . \quad . \quad . \quad (29)$$

Using this value in equations 23 and 24,

$$s=\frac{x}{\cos i}=y \frac{\sin i}{1-\cos i} \quad . \quad . \quad (30)$$

$$y=x \frac{1-\cos i}{\sin i \cos i} \quad . \quad . \quad . \quad . \quad (31)$$

$$y+m=\frac{m}{\cos i} \quad . \quad . \quad . \quad . \quad (32)$$

$$\frac{2x}{y+m}=\sin 2 i \quad . \quad . \quad . \quad . \quad (33)$$

These equations enable all problems relating to the form of the rope to be solved.

185. *Case I. Horizontal transmission.*—Let the supporting points of the rope be at the same level, and at a distance l apart, and let the total tension $T=T_2+C=2P+C$ be known.

From equation 27, we get $y+m=T \div G$ at the points of support. Since in this case the curve is symmetrical about its lowest point, $x=\frac{1}{2}l$. Hence at the points of support,

$$\left.\begin{array}{l} \operatorname{Sin} 2 i=\dfrac{2x}{y+m}=\dfrac{Gl}{T} \\[2ex] \text{The parameter}=m=(y+m) \cos i=\dfrac{T}{G} \cos i \\[2ex] \text{The deflection}=y=\dfrac{T}{G}-m \\[2ex] \text{The length of the rope}=2s=2y\dfrac{\sin i}{1-\cos i} \end{array}\right\} \quad . \quad (34)$$

Conversely if the deflection y at the centre is given in place of the greatest tension, and also the half span x,

$$y = \frac{m}{\cos i} - m = \frac{m}{\sqrt{1 - \frac{x^2}{m^2}}}$$

$$m = \frac{x^2}{4y} + x\sqrt{\frac{x^2}{16\,y^2} + \tfrac{1}{2}} \text{ nearly} \quad . \ (35)$$

Then the other values can be found as before.

186. *Case II. Inclined transmission. The points of support* B′ B″ *not at the same level.*—This case is best solved by approximation. We may assume the length $s' + s''$ of the rope to be the same as if the points B′ B″ were at the same level and the same distance apart.[1] Let T′, the tension at B′, be given, and also the length B′ B″ $= l$, and the difference of

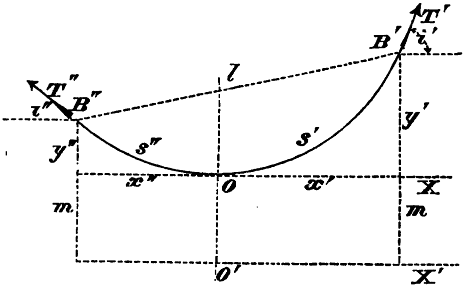


Fig. 178.

level $y' - y'' = h$. Calculate first the length of rope $2s$ from the equations above, putting T¹ for T, and assuming the pulleys to be l feet horizontally apart.

Then on the assumption above,

$$s' + s'' = 2\,s$$

$$y' + m = \frac{T'}{G}$$

$$y'' + m = \frac{T'}{G} - h = T''$$

[1] Reuleaux, 'Der Constructeur'; Keller, 'Treibwerke.'

By equation 26,

$$(y'+m)^2=m^2+s'^2$$

$$(y''+m)=m^2+s''^2$$

$$s'^2-s''^2=(y'+m)^2-(y''+m)^2$$

$$s'-s''=\frac{(y'+m)^2-(y''+m)^2}{2\ s}$$

Having now obtained $s'+s''$ and $s'-s''$, it is easy to find s' and s''. Let i' i'' be the inclinations of the ropes at the points of support. The vertical forces at B′ B″ are

$$\text{V}'=\text{G}\ s' \text{ and } \text{V}''=\text{G}\ s''$$

$$\text{Sin } i'=\frac{\text{V}'}{\text{T}'} \text{ and } \sin i'=\frac{\text{V}''}{\text{T}''} \qquad . \qquad . \quad (36)$$

The value of T″ being given above. Hence i' and i'' can be found,

$$\left.\begin{aligned} m&=\frac{s'}{\tan i'}=\frac{s''}{\tan i''} \\ x'&=m \sin i' \\ y'&=\frac{m}{\cos i'}-m \end{aligned}\right\} \qquad . \qquad . \quad (37)$$

From these values of x' and y' the position of the vertex of the curve can be found.

187. *Deflection for which the longitudinal tension is a minimum.*—From the equations $y=\frac{m}{\cos i}$, and $\sin i=\frac{x}{m}$ we get

$$y=\tfrac{1}{2}\ \frac{x^2}{m} \text{ nearly}$$

But

$$\text{T}=\text{G}\ (y+m)=\text{G}\left(y+\frac{x^2}{2\ y}\right) \text{ nearly.}$$

This will be a minimum for different values of the deflection, when,

$$\frac{d\,\mathrm{T}}{d\,y}=1-\frac{x^2}{2\,y^2}=o$$

or when,

$$y=\frac{x}{\sqrt{2}}=0{\cdot}7\,x$$

The tension increases to infinity for $y=o$ and for $y=\infty$.

188. *Tensions in the sloping wire rope.*—Let T'_2 T'_1 be the tensions on the tight and slack sides of the rope at the upper pulley, T''_2 and T''_1 those at the lower pulley,

$$\left.\begin{aligned}\mathrm{T}'_2&=2\,\mathrm{P}+\mathrm{C}\\ \mathrm{T}'_1&=\ \ \mathrm{P}+\mathrm{C}\\ \mathrm{T}''_2&=2\,\mathrm{P}+\mathrm{C}-\mathrm{G}\,h\\ \mathrm{T}''_1&=\ \ \mathrm{P}+\mathrm{C}-\mathrm{G}\,h\end{aligned}\right\} \quad . \quad . \quad (38)$$

where h is the difference of level of the pulleys.

189. *To draw the curve of the rope.*—In drawing the curve of the rope, which is often necessary to determine the space it will occupy, it is sufficiently accurate to substitute for the catenary curve a common parabola. Divide the abscissa O A and the ordinate A B, of any point, into an equal number of equal parts. Join O_1 O_2 O_3, and from 1′ 2′ 3′ draw verticals. These verticals will intersect the corresponding sloping lines, in points situated in a parabola.

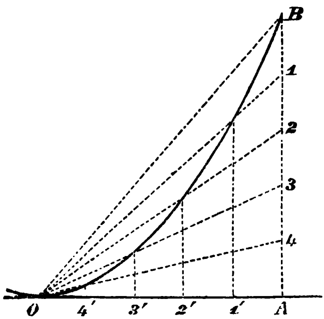
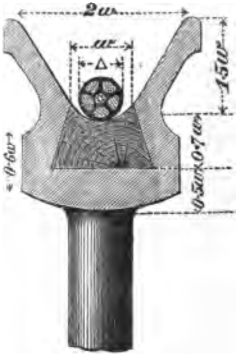


Fig. 179.

190. *Pulleys for wire rope transmission.*—Wire ropes will not support without injury the lateral crushing which occurs

when the rope rests against the sides of V-shaped grooves. Hence, it is necessary to construct the pulleys with grooves so wide, that the rope rests on the rounded bottom of the pulley. It was found by Hirn that the wear of the rope was greatly diminished, and at the same time the frictional resistance to slipping was increased, by lining the bottom of the groove of the pulley with gutta-percha or wood. The gutta-percha is softened and hammered into the groove, which is dovetailed in section. The wood may be inserted in short blocks, through a lateral opening, which is afterwards covered by a metal plate. More recently, leather has been found to succeed better than either wood or gutta-percha. The leather is cut into strips and placed in the groove on edge. Fig. 180 shows the section of a pulley rim. The unit for the proportional figures is $w = \Delta + \frac{1}{2}$, where Δ is the diameter of the rope.



$w = \Delta + \frac{1}{2}$

Fig. 180.

The pulleys are often of cast iron, with cross-shaped arms, which may be calculated in the same way as the arms of toothed wheels. Sometimes they have oval curved arms like those of ordinary pulleys, and sometimes the arms are of round bar iron. These are cut to the right length and tinned at the ends. They are then placed in the sand mould, and the rim and nave cast round them. Such arms are usually placed sloping in the plane of the axis of the pulley, the slope being alternately in opposite directions. The pulley is thus rendered rigid enough to resist accidental lateral forces.

It has already been proved (eq. 14) that the radius of the pulley must not be less than

$$R = \frac{2(f - f_t)}{\delta E}$$

Or, when $f=25{,}000$, and the tension f_t, due to the work transmitted and the centrifugal force, does not exceed 8,000 lbs. per sq. in.,

$$R=900\ \delta \text{ nearly.}$$

The pulleys commonly used are 12 to 15 ft. diameter.

When the distance to which the power is transmitted is great, intermediate guide or supporting pulleys are introduced to lessen the deflection of the rope. The supporting pulleys for the tight side of the belt must be of the same size as the principal pulleys, those for the slack side may be smaller, in the ratio

$$\frac{R'}{R}=\frac{f-\frac{1}{2}f_t+\frac{3}{2}c}{f-f_t}$$

where f is the total stress in the rope, f_t the stress due to the longitudinal tension, including centrifugal force, c the stress due to centrifugal force.

The pulleys are supported on shafts which rest in pedestals on masonry piers or timber trestles.

191. *Velocity of the rope.*—The rope is run at the highest safe velocity. That velocity is determined by the liability of the pulleys to burst, under the action of the centrifugal force, when the speed exceeds a certain limit. Let G be the weight of a cubic foot of cast iron, v the velocity of the pulley rim in feet per second, a its sectional area in square feet, R its radius in feet, and f its tensile strength in lbs. per sq. ft. The tension in the rim due to centrifugal force is,

$$\frac{Gav^2}{g}$$

The resistance of the rim is fa. Equating these,

$$v=\sqrt{\frac{gf}{G}} \quad . \quad . \quad . \quad (39)$$

Thus, putting $G=450$ lbs., $f=4500\times 144$,

$$v=215 \text{ ft. per sec.}$$

The actual speed is never as high as this, a larger margin of safety being necessary. Usually the speed of the rope is from 60 to 100 feet per second.

Fig. 181 shows three arrangements of a wire-rope transmission.

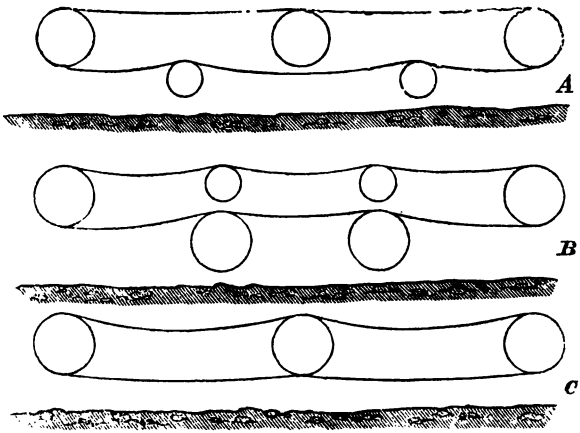


FIG. 181.

In A and B guide or supporting pulleys are used. The upper part of the rope is the driving side in A, and the lower part in B; C is the arrangement adopted by Ziegler at Frankfort for transmitting 100 H P a distance of 984 metres.

192. *Loss of work in friction.*—The loss of work in friction may be taken to be about

$$h = \cdot 01\, H\, n + \cdot 025 \quad . \quad . \quad . \quad (40)$$

where H is the total horse-power transmitted; n the number of thousand yards to which it is transmitted; and h the loss of work in horses' power.

The following table contains data taken from a paper by Achard:[1]—

[1] 'Annales des Mines,' viii. p. 229; 'Proc. Inst. of Civil Engineers,' xlv. p. 267.

Wire Rope Transmission.

Locality	Rope			Pulleys		H. P. transmitted	Total distance		Velocity of belt ft. per sec.
	Diam. Δ ins.	Diam. of wires δ in ins.	No. of wires ν	Diam. 2 R in ft.	Distance apart in ft.		Horiz.	Vert.	
Oberursel	0·59	·06	36	12·3	394	94	3,153	146	73·8
Schauffhausen[1]	0·95	·072	80	14·75	333 to 456	326	1,997	...	61·87
Fribourg	0·97	·070	90	14·75	502	300	2,510	269	65

[1] There are two cables. If one breaks, the other is capable of transmitting the power.

CHAPTER XII.

LINKWORK.

CRANKS AND LEVERS.

193. CRANKS are levers fixed on shafts which rotate continuously or through a limited angle. The simplest form of crank is the ordinary winch handle, used for driving rotating pieces by hand. Most commonly the crank forms part of a linkwork arrangement for changing reciprocating into rotative motion.

HAND LEVERS AND WINCH HANDLES.

Fig. 182 shows an ordinary straight lever for working machinery by hand. The part grasped by the hand may be $1\frac{1}{4}$ inch in greatest and $1''$ in smallest diameter, and 5 inches long. Let P be the force exerted at the handle, and l the length of the lever. Then P l (nearly) is the greatest bending moment on the arm. Let b be the width and h the thickness of the arm at its largest part. Then,

$$\tfrac{1}{6}\, b^2\, h f = \mathrm{P}\, l$$

$$b^2\, h = \frac{6\, \mathrm{P}\, l}{f}$$

Let the greatest force, P, exerted by a man be taken at 84 lbs.; and let $f = 9{,}000$ lbs. per sq. in. for wrought-iron. Then,

$$b^2\, h = \tfrac{1}{18}\, l \text{ nearly} \quad . \quad . \quad . \quad (1)$$

If $h = \frac{3}{4}$ inch, $b = 0{\cdot}27\sqrt{l}$. If the flat part of the lever is of

uniform thickness, its least width should be half its greatest width, the case corresponding with Case I. Table VI. Let d=diameter of shaft on which the lever is keyed ; n=dis-

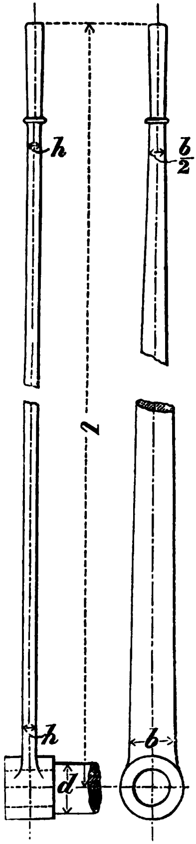


Fig. 182.

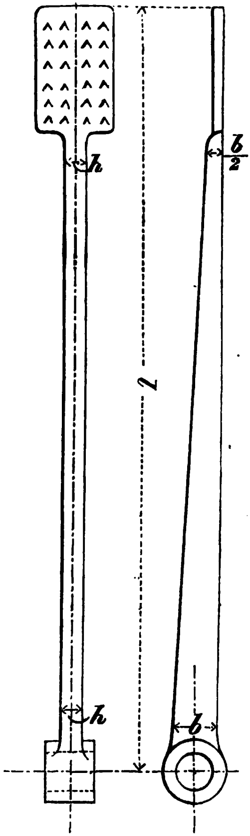


Fig. 183.

tance from centre of lever to centre of nearest bearing of shaft. Then the shaft is subjected to a twisting moment P l and a bending moment P n, and its strength is determined

by the rules in § 42 and § 88. The equivalent bending moment is P $(0{\cdot}7\,n+0{\cdot}48\,l)$ nearly. Hence,

$$d=0{\cdot}0947\sqrt[3]{\{P(1{\cdot}4\,n+0{\cdot}96\,l)\}}$$
$$=0{\cdot}42\sqrt[3]{(1{\cdot}4\,n+0{\cdot}96\,l)} \quad . \quad . \quad . \quad (2)$$

The part in the eye of the lever may have a diameter $=0{\cdot}42\sqrt[3]{l}$. The eye of the lever may have a thickness $=0{\cdot}3\,d$ and a length $=1$ to $1\frac{1}{4}\,d$.

Fig. 183 shows a foot lever. The foot plate is about 8 ins. by 5 ins., and $\frac{5}{8}$ in. thick. In designing this lever P may be taken at 180 lbs. Then,

$$\left.\begin{array}{l} b^2\,h=\frac{1}{8}\,l \\ d=\cdot54\sqrt[3]{(1{\cdot}4\,n+0{\cdot}96\,l)} \end{array}\right\} \quad . \quad (3)$$

194. Fig. 184 shows a winch handle or cranked lever. When this is intended to resist the full force of one man, P

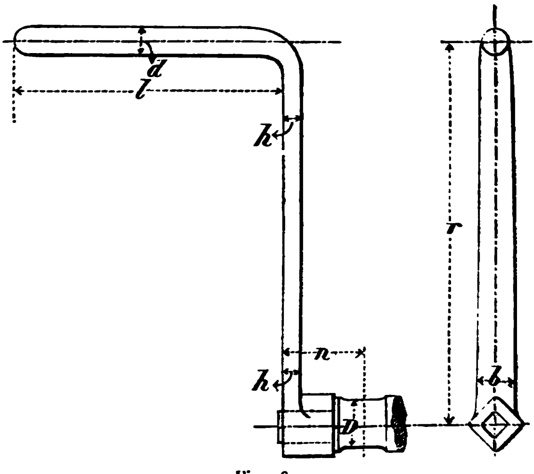


Fig. 184.

may be taken at 84 lbs., and if worked by two men, P=168 lbs. The mean effort per man in continuous work

is only 15 to 30 lbs. The radius r is usually 16 or 17 ins., and the height of the shaft from the ground may be 3 ft. to 3 ft. 3 ins. The length of handle l may be 10 or 12 ins. for one man, and 20 ins. for two men. The pressure on the handle may be taken to act at $\frac{2}{3}$rds of the length. The greatest bending moment at the handle is $\frac{2}{3}$ P l. Then, its diameter should not be less than

$$d = 0{\cdot}0947 \sqrt[3]{\tfrac{4}{3}\,\mathrm{P}\,l} = 0{\cdot}1042 \sqrt[3]{\mathrm{P}\,l} \quad . \quad (4)$$

or say $1\frac{1}{16}$ inch for one man and $1\frac{1}{2}$ inch for two men. The journal of the shaft is subjected to a twisting moment P r, and a bending moment P $(\frac{2}{3}\,l + n)$. The equivalent bending moment (§ 42) is P $(0{\cdot}6\,l + 0{\cdot}9\,n + 0{\cdot}4\,r)$ nearly. Then,

$$\mathrm{D} = 0{\cdot}0947 \sqrt[3]{\{\mathrm{P}\,(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)\}}$$

$$\left.\begin{aligned} &= 0{\cdot}42 \sqrt[3]{(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)} \text{ for one man} \\ &= 0{\cdot}54 \sqrt[3]{(1{\cdot}2\,l + 1{\cdot}8\,n + 0{\cdot}8\,r)} \text{ for two men} \end{aligned}\right\} (5)$$

For the part in the eye of the crank, the term $1{\cdot}8\,n$ may be omitted. The greatest bending moment on the arm is P r, and the twisting moment $\frac{2}{3}$ P l nearly. Hence the equivalent bending moment is P$(0{\cdot}9\,r + {\cdot}27\,l)$. If b is the breadth and h the width of the arm at the larger end,

$$b^2\,h = \frac{6\,\mathrm{P}}{f}\,(0{\cdot}9\,r + 0{\cdot}27\,l)$$

$$\left.\begin{aligned} &= {\cdot}056\,(0{\cdot}9\,r + 0{\cdot}27\,l) \text{ for one man} \\ &= {\cdot}112\,(0{\cdot}9\,r + 0{\cdot}27\,l) \text{ for two men} \end{aligned}\right\} \quad (6)$$

Either b or h may be selected and the other obtained from the formula. If the arm is of uniform thickness, its least breadth should not be less than $\frac{1}{2}\,b$, or less than $2\sqrt{\frac{\mathrm{P}\,l}{f\,h}}$ or $0{\cdot}19\sqrt{\frac{l}{h}}$ for one man, and $0{\cdot}27\sqrt{\frac{l}{h}}$ for two men. Thickness of eye of crank, $0{\cdot}3$ D; length of eye, $1\frac{1}{4}$ D.

ENGINE CRANKS.

195. Engine cranks are of cast- or wrought-iron. A single crank consists of a nave bored to receive the crank shaft, an arm, and a crank pin. If the crank pin is a separate piece, it is fitted into an eye formed at the small end of the crank. Disc cranks have plain circular discs, instead of the ordinary crank arm, and they have the advantage of being nearly balanced with respect to the crank shaft. A double crank is used when the crank pin cannot be placed at the end of the crank shaft. An eccentric is a crank of peculiar form. It is essentially a crank, with a crank pin, the radius of which is greater than the sum of the crank and crank shaft radii.

196. *Crank and connecting rod. Forces acting on the crank pin.*—Usually the crank is driven by the pressure on a piston, transmitted to it through a connecting rod. The path of the crank pin is $2\pi R$ in one revolution, while the path of the piston is $4R$. Hence the mean driving pressure on the crank pin is less than the mean pressure on the piston, in the ratio of $4 : 2\pi$, or $2 : \pi$. The resultant pressure on the crank pin is, however, at times, much greater than the mean pressure.

Suppose that the varying piston pressure is represented by an indicator diagram. Then, for any position of the piston, the corresponding crank position can be found, and the pressure on the crank pin at that moment. In fig. 185 let O A be the crank and A B the connecting rod. Let C and D be the positions of the crosshead at the beginning and end of the stroke, and let an indicator diagram of the piston pressure be drawn on C D. Then B G represents the pressure P' on the piston when the piston is at B. It is required to find the corresponding resultant pressure on the crank pin. Set off O E along the crank and equal to B G. Draw E F parallel to the connecting rod. Then O F is the effort on the

crank pin or pressure in the direction of motion.[1] Along the tangent at A take A H = O F, and at H draw H K perpen-

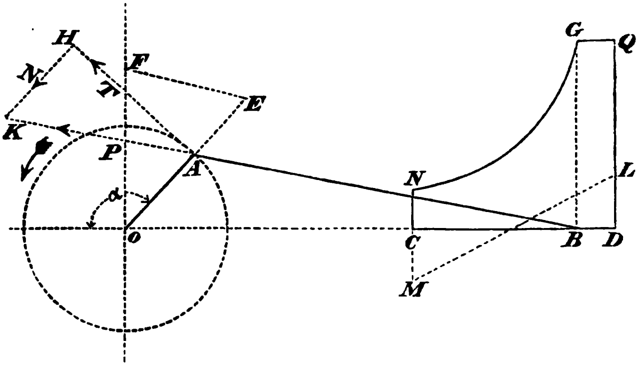


Fig. 185.

dicular to A H, and meeting the direction of the connecting rod in K. Then A K = P is the resultant pressure on the crank pin, and A H = T and H K = N are its components, tangential and normal, to the path of the crank pin.

If this construction is made for several positions of the crank, it will be seen that the tangential component T vanishes when the crank is at the dead point, and, except when the pressure on the piston diminishes before half-stroke, it reaches its maximum when the crank and connecting rod are at right angles. Its maximum value, if R = crank radius and L = connecting rod length, is,

$$T_{max} = P' \frac{\sqrt{R^2 + L^2}}{L} \quad . \quad . \quad . \quad (7)$$

Since L is usually 4 to 5 times R,

$$T_{max} = 1{\cdot}02 \text{ to } 1{\cdot}03\ P' \quad . \quad . \quad . \quad (7a)$$

[1] It is easy to show that O E, O F are proportional to the radii drawn to the instantaneous axis of the connecting rod. Then if V and v are the velocities of the crank pin and piston $\frac{V}{v} = \frac{OE}{OF}$. Consequently, if P and T are the efforts at the piston and crank pin $\frac{P'}{T} = \frac{V}{v} = \frac{OE}{OF}$.

The radial component N vanishes when the crank and connecting rod are at right angles, and is greatest and equal to P′ when the crank is at the dead point.

197. *Effect of the inertia of the reciprocating parts.*—If the crank pin moves uniformly under the control of the fly-wheel, the reciprocating parts are accelerated during nearly the first half of the stroke, and retarded during the remainder. Hence, the inertia of these parts balances part of the piston pressure during the first half of the stroke, and diminishes the pressure on the crank pin. During the remainder of the stroke, the crank pin pressure is increased by the inertia of the reciprocating parts. The acceleration is greatest at the beginning and end of the stroke, and is zero when the piston is nearly at mid-stroke. Hence the inertia of the moving parts does not much affect the pressure on the crank pin when the crank and connecting rod are at right angles, but may greatly alter it when the crank is at the dead points.

Let W = weight of reciprocating parts in lbs.; R = radius of crank in ft.; V = velocity of crank pin; P′ = piston pressure at the beginning of the stroke; P″ = piston pressure at the end of the stroke. Then when the inertia of the reciprocating parts is taken into account, the crank pin pressures at the beginning and end of the stroke are

$$N' = P' - \frac{W V^2}{g R} \text{ and } N'' = P'' + \frac{W V^2}{g R}. \qquad (8)$$

and the crank must be capable of resisting the greater of these. It must also resist P′ simply, because if the engine happens to move slowly, the effect of the inertia is very small.

Set off D L, C M, each equal to $\frac{W}{g} \cdot \frac{V^2}{R}$, and join L M. Then the pressure transmitted to the connecting rod at any point of the stroke is given, approximately, by measuring the vertical ordinate from L M instead of C D.

198. *General case. Straining action on crank arm.* Let fig. 186 represent a crank in any position, and let P be the total pressure on the crank pin. Resolve P into a tangential component P, and a radial component N. Let $a\,b$ be any section of the arm at a distance r from the centre of crank pin, and let m be the distance between centre lines of crank pin and crank arm. Then the straining actions at $a\,b$ which require to be considered are :—

(*a*) A direct compression (or tension) equal to N.
(*b*) A bending moment $\mathrm{N}\,m$ in the plane of the arrow B.
(*c*) A bending moment $\mathrm{T}\,r$ in the plane of the arrow A.
(*d*) A twisting moment $\mathrm{T}\,m$.

To take into account all these straining actions in several positions of the crank would be laborious. Generally it is sufficient to estimate the strength of the crank in two positions, when the crank is at the dead point and when the crank and connecting rod are at right angles. In the former case, T vanishes and N becomes equal to the greatest piston pressure, or to $\mathrm{N}''=\mathrm{P}''+\dfrac{\mathrm{W}\,\mathrm{V}^2}{g\,\mathrm{R}}$, which will also for simplicity be denoted by N simply. In the latter case N vanishes and T is equal to 1·02 or 1·03 times the piston pressure.

199. *Strength of the crank.*—Let N be the radial pressure when the crank is at the dead point, and T the tangential pressure when the crank and connecting rod are at right angles. Let further

d, l = diameter and length of crank pin.
D, L = diameter and length of crank-shaft journal.
d', l', t' = internal diameter, length and thickness of small eye of crank.
$d''\ l''\ t''$ = internal diameter, length, and thickness of large eye of crank.
h, b = thickness and width of arm at any section $a\,b$; the same letters with one accent referring to the section of the arm supposed produced to the centre

of small eye, and with two accents the section produced to the centre of large eye.

R = crank radius.

m = distance from centre line of crank pin to centre line of crank arm.

n = distance from centre line of crank pin to centre line of crank-shaft journal.

The crank pin and crank-shaft journal are first designed by the rules in Chapter VII. For the section of the crank

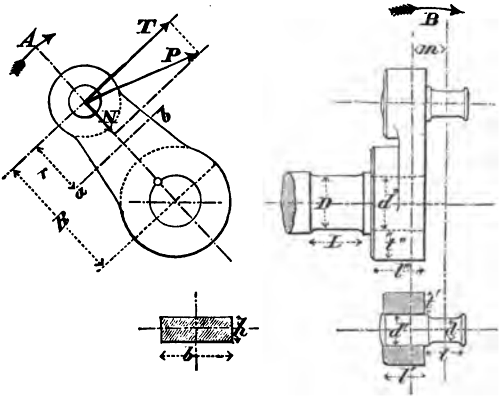


Fig. 186.

arm we have, when the crank is at the dead point, a direct tension or compression N and a bending moment N m. Then the greatest stress is (§ 41),

$$f = \mathrm{N}\left(\frac{1}{h\,b} + \frac{6\,m}{b\,h^2}\right)$$

$$b = \frac{\mathrm{N}}{f}\left(\frac{1}{h} + \frac{6\,m}{h^2}\right) \quad . \quad . \quad . \quad (9)$$

or,

$$h = 2{\cdot}65\sqrt{\frac{\mathrm{N}\,m}{f\,b}} \text{ nearly} \quad . \quad . \quad (9a)$$

If this straining action only were considered, the crank arm would be of uniform section throughout. Hence this equation is chiefly useful for determining the breadth and thickness of the arm at the small end, where the straining action due to the force T is least important.

When the crank and connecting rod are at right angles, there is a bending moment T r, and a twisting moment T m. Combining these, the equivalent bending moment is (§ 42),

$$=0{\cdot}91\ \mathrm{T}\,r+0{\cdot}41\ \mathrm{T}\,m \text{ nearly.}$$

The bending is parallel to the plane of rotation, so that the modulus of the section is $\frac{1}{6}\,b^2\,h$. Then

$$\tfrac{1}{6}\,b^2\,hf=0{\cdot}41\ \mathrm{T}\,r+0{\cdot}41\ \mathrm{T}\,m$$

$$b=\sqrt{\left\{\frac{6\,\mathrm{T}}{f\,h}(0{\cdot}91\,r+0{\cdot}41\,m)\right\}} \quad . \quad (10)$$

$$\text{or, } h=\frac{6\,\mathrm{T}}{f\,b^2}(0{\cdot}91\,r+0{\cdot}41\,m)\,. \quad . \quad . \quad (10a)$$

Select a value for h or b. Then the greater of the values given by equations 9 and 10, or 9a and 10a, is the proper value for the remaining dimension. It will often be sufficient to use equation 9 or 9a to determine b' or h'; equation 10 or 10a to determine b'' or h''; and the sides of the crank arm may be drawn as planes.

When the crank is of cast-iron the arm may be trough-shaped (fig. 188), and is then somewhat lighter than when it is rectangular. Let b and h be the width and thickness of a rectangular arm, and let $b_1\,h_1$ and $b_2\,h_2$ be the dimensions of an arm of trough-section of equivalent strength. Then if flexure is in the plane of rotation,

$$b^2\,h=\frac{b_1^3\,h_1-b_2^3\,h_2}{b_1}$$

Let $\qquad b_1=b$

$$b^3\,h=b^3\,h_1-b_2^3\,h_2$$

Let $b_2 = x\, b$

$h_2 = y\, h_1$

Then $$h_1 = \frac{h}{1 - x^3 y} = c\, h \quad . \qquad . \qquad . \quad (11)$$

where c has the following values :—

$y=$	$x=$ 0·6	·65	·7	·75
·6	1·15	1·20	1·26	1·34
·65	1·16	1·22	1·29	1·38
·7	1·18	1·24	1·32	1·42
·75	1·19	1·26	1·35	1·46

At the section at the centre of the small eye of the crank the flexure is at right angles to the plane of rotation, and the

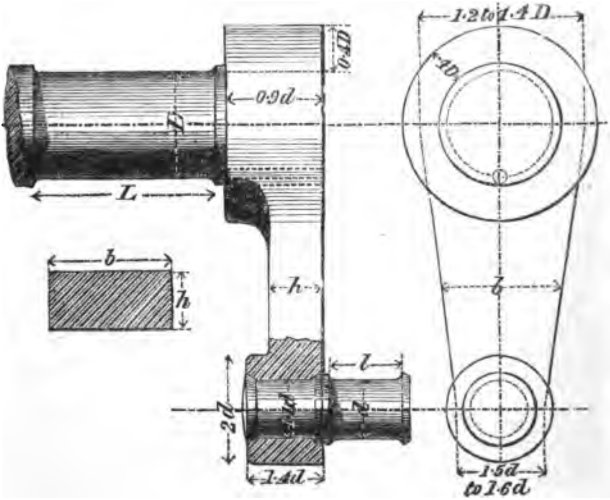


Fig. 187.

feathers strengthen the section very little. Hence the section there may remain unchanged, the feathers being allowed to diminish towards that end.

When a crank has a T-form of section, it is but little strengthened by the feather, but it is more easily cast.

200. *Proportions of cranks.*—The crank is shrunk on to the crank shaft, and the crank pin is also fixed in the same way and riveted cold. The key in the crank shaft may have a breadth $=\frac{1}{3}$ D, and a thickness $\frac{1}{6}$ D, for small cranks, and $\frac{1}{4}$ D and $\frac{1}{8}$ D for large cranks.

Fig. 187 shows a wrought-iron crank with a section of the arm. The arm is sometimes tapered and the back face

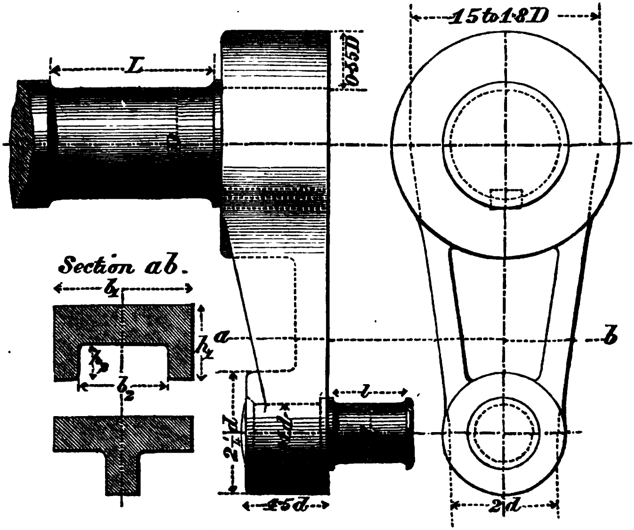


Fig. 188.

of the arm is then rounded, so that it forms, in fact, part of a slightly conical surface, turned in the lathe. Fig. 188 shows a cast-iron crank, with sections of arms both trough-shaped and T-shaped.

Eccentrics.

201. An eccentric is a modified crank, chiefly employed to drive the slide valve of steam-engines. It is really

a crank and connecting rod, with a crank pin enlarged, so as to include the crank shaft within its section, the radius of the eccentric being greater than the sum of the crank and crank-shaft radii. The eccentric consists of a sheave, which is virtually a crank pin, and a strap and rod which is virtually equivalent to a connecting rod. The sheave is most commonly of cast iron, and is often cast in two parts connected by bolts. In very hard-worked eccentrics the sheave may be of wrought iron, case-hardened. When the sheave is in two parts, the smaller may be of wrought and the larger of cast iron. The strap is in two parts, and is prevented from slipping sideways by a flange or flanges, or it has internally a spherical surface fitting on the sheave. The strap is of brass, of cast iron lined with brass, or of wrought iron lined with brass or with white metal. The friction of the eccentric is much greater than that of a crank, and it is therefore not used where ordinary cranks can be applied.

The distance between the centres of the crank shaft and eccentric sheave is termed the eccentricity, the radius, or the half stroke of the eccentric. Let this be denoted by r, and let d be the diameter of the shaft on which the eccentric is fixed. Then the least diameter suitable for the eccentric sheave is about

$$= \mathrm{D} = 1{\cdot}2\, d + 2\, r + \tfrac{3}{4}.$$

Professor Reuleaux has pointed out that the width of the eccentric sheave, or virtual length of the crank pin, should be the same as the length of an ordinary crank pin for the same work. The width b can then be decided by the rules for journal lengths in § 82. There is, however, great difficulty in applying these rules, because the force which the eccentric has to overcome cannot be very accurately ascertained.

202. *Friction of the slide valve.*—Let a be the area of the back of the valve subjected to the steam pressure, and p the

steam pressure in lbs. per sq. in., reckoned above atmospheric pressure in the case of non-condensing engines, and above zero in the case of condensing engines.

Then the frictional resistance is ordinarily taken to be

$$F = \mu p a,$$

where μ is about 0·15 for smooth surfaces, such as slide-valve surfaces, not well lubricated. But it is possible that the steam may insinuate itself partially, or over the whole extent of the faces of the valve, which are in contact with the surface of the valve chest; and in that case the downward pressure of the steam on the back of the valve at those parts would be neutralised by the upward pressure of the layer of steam between the surfaces, and the friction would be due to the pressure on the remainder of the valve only. According to some experiments of Mr. Thomas Adams, steam does so insinuate itself, so long as the intensity of pressure between the valve and steam chest faces is less than the steam pressure, but when the pressure is greater, as must be the case with ordinary slide valves, this layer of steam is squeezed out, and then the coefficient of friction is found to have a much higher value, so that in the formula above we ought to take $\mu = 0·2$ to 0·35, the value being greater as the pressure and temperature of the steam is greater.

The friction which the eccentric has to overcome may be assumed to be proportional to $p\,a$, and the unit for the following proportional dimensions will therefore be taken,

$$= k = c\sqrt{p\,a} \quad . \quad . \quad . \quad . \quad (12)$$

where c appears to have on the average the value $\frac{1}{80}$ to $\frac{1}{100}$ for stationary and marine engines, and $\frac{1}{100}$ to $\frac{1}{120}$ for locomotives.

203. *Radius of eccentric.*—Let w be the greatest width of port opened to steam; l, the lap of the valve; r, the radius of the eccentric,

$$r = w + l,$$

w is in some cases the whole width of the steam port, but in quick-running engines the opening to steam is less than the opening to exhaust. This is secured by making w about $\frac{2}{3}$rds of the width of the port. The external lap l may vary from $\frac{1}{8}$th of the width of the port to the whole width of the port, according to the amount of expansion required.

204. *Proportions of sheave.*—The width b of the bearing surface of the sheave may be taken equal to $2\,k$, or $2\frac{1}{4}\,k$.

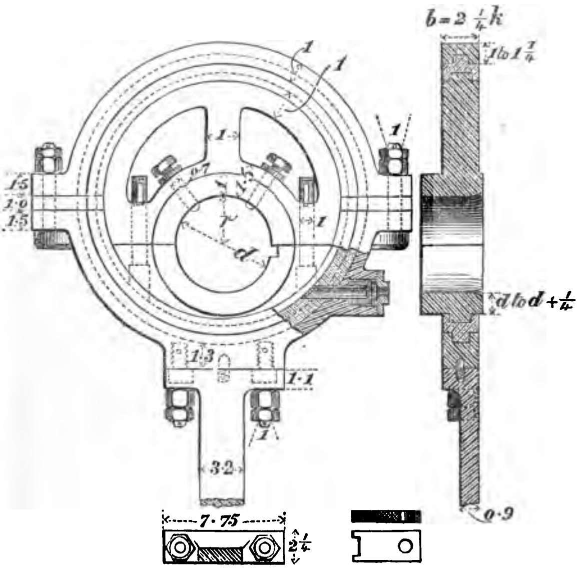


Fig. 189.

The diameter of the bolts connecting the two parts of the sheave may be $0{\cdot}85\,k$ to k, and the cotter in these bolts may have a width equal to their diameter, and a thickness equal to $\frac{1}{4}$ of their diameter. The set screws may be $0{\cdot}7\,k$ diameter, or if there is only one it may be k in diameter. If the sheave has flanges to retain the strap, their projection may be $0{\cdot}4\,k$, and their thickness $0{\cdot}3\,k$. The smaller part of

the sheave, when it is in two parts, is generally of wrought-iron in small eccentrics. The least thickness of this part, where the shaft comes near the edge of the sheave, should be at least k for wrought-iron and $1\frac{1}{4}\,k$ for cast-iron.

Proportions of strap.—The strap thickness varies very much. For wrought-iron it may be from ·5 k in large to k in small eccentrics. For gun-metal it should be from ·625 k to $1\frac{1}{4}\,k$. For cast-iron from ·75 k to 1·5 k. The brass lining may be about $\frac{1}{8}$th the strap thickness in large eccentrics, and in other cases its thickness may be $\frac{D}{40}+\frac{1}{8}$.

When the strap is recessed to fit a projection on the sheave, the depth of the recess may be $\frac{5}{7}$th of the thickness of the brass, and its width 0·3 b. The corresponding recess in the brass may be of the same depth, and its width, $b-\frac{5}{8}$ to $b-\frac{7}{8}$.

Proportions of eccentric-rod.—The eccentric-rod is very commonly attached to the eccentric-strap by a T-end, and has at the other an eye to receive the pin of the valve rod. At its smaller or eye end, it may have a width 1·8 k, and a thickness 0·55 k. It tapers in width about $\frac{1}{2}$ inch per foot of length to the T-end, the thickness being constant. The bolts in the T-end may be of the same size as the strap-bolts.

Connecting Rods.

205. Connecting rods are most commonly used to transmit to a crank the pressure of a piston. When they connect two cranks they are often termed coupling rods. In stationary engines the connecting rod is sometimes of cast-iron; in other cases it is almost always of wrought-iron. Cast-iron rods are cross-shaped in section; wrought-iron rods are circular in section in most cases, but for engines running at high speeds their depth in the plane of rotation should be greater than their breadth. To secure this the

section of the rod may be rectangular, or approximately rectangular. The extremities of a connecting rod are fitted with arrangements for receiving journals.

A connecting rod may be subjected to tension or to compression only; and in that case, when it is open to choice, it is preferable to arrange the machine so that the rod may be in tension. Rods in compression, which have a journal at each end, are in the position of the rod in Case II., Table VII., and their strength is to be calculated by the rule there given, if the ratio of length to diameter exceeds the value given in § 39. Most connecting rods are subjected to reciprocating stress, alternately compressive and tensile. Then their least section must be sufficient for the tension, and their greatest section, near the middle of their length, must be sufficient for the compression.

206. *Forces to which connecting rods are subjected.*—Let P_1 be the force transmitted to the end of a connecting rod, due either to steam pressure on a piston or to any other kind of load. Let θ be the angle between the direction of P_1 and the axis of the rod. Then the force acting along the axis of the rod is $P = P_1 \sec \theta$. Hence, if a connecting rod is n times the length of the crank (usually $3\frac{1}{2}$ times to 6 times), the thrust along the connecting rod, at its greatest obliquity, is $\frac{\sqrt{n^2+1}}{n} P_1$, or about $1{\cdot}03$ to $1{\cdot}08\ P_1$. In addition to this load, the connecting rod is subjected to straining actions due to the inertia of the parts connected with it, and to its own weight and inertia. In quick-running engines these straining actions become of importance. In slow-moving machines they may be neglected.

Suppose the connecting rod to be an ordinary engine connecting rod, attached at one end to a rotating crank, of radius R in ft., and at the other to a reciprocating cross-head. The crank pin moves, nearly uniformly, with a velocity V, under the control of the fly-wheel; and the cross-head and parts connected to it (piston, piston-rod, slide

blocks, etc.), have, in consequence, a varying velocity. Let W be the total weight of the parts which reciprocate, inclusive of half the weight of the connecting rod itself. When the crank is near the dead points, the resistance to acceleration is $\pm W \frac{V^2}{g R}$. Hence the thrust in the connecting rod will be $P_1 - W \frac{V^2}{g R}$ at the beginning, and

$$P_1 + W \frac{V^2}{g R} \quad . \quad . \quad . \quad (13)$$

at the end of the stroke, where for P_1 is to be put in each case the corresponding piston pressure. The greater of the two values is to be taken. In a non-expansive engine the greatest thrust will be at the end of the stroke, but in an expansive engine it is not always so.

When the connecting rod and crank are nearly at right angles, the former is subjected to a transverse bending action due to its resistance to acceleration in a direction perpendicular to that of the motion of the piston. According to Grashoff, the bending action is greatest at $\frac{6}{10}$ths of the length of the rod from the cross-head end, and consequently, that is the point at which the rod should have the greatest diameter. The rod is sometimes tapered uniformly from the cross-head end to the crank-pin end, and in quick-running engines this is better than making the diameter greatest at the centre of the rod. Let w be the average weight of the rod in lbs. per inch of length; l, the length of the rod between the centres of the journals in ins.; R, the radius of the crank in inches; V, the velocity of the crank pin in feet per second. Then the greatest bending moment due to the swaying of the rod is

$$M = 0{\cdot}82 \frac{V^2}{R} \frac{w\, l^2}{g} \quad . \quad . \quad (14)$$

The stress due to this moment is $f_i = \frac{M}{Z}$, where Z is the modulus of the section of the rod. Hence,

$$\left.\begin{aligned} f_i &= 4{\cdot}92\ \frac{V^2}{R}\ \frac{w\,l^2}{g\,b\,h^2} \text{ for a rectangular rod} \\ &= 8{\cdot}351\ \frac{V^2}{R}\ \frac{w\,l^2}{g\,d^3} \text{ for a round rod} \end{aligned}\right\} \quad . \quad (15)$$

d being the diameter, *b* the breadth at right angles to the plane of motion, and *h* the depth in the plane of motion.

Let G be the weight of a cubic inch of the material of the rod ($=0{\cdot}261$ lb. for iron.) Then, $w = G\,b\,h$ or $\frac{\pi}{4} G\,d^2$. Hence,

$$\left.\begin{aligned} f_i &= 1{\cdot}28\ \frac{V^2}{R}\ \frac{l^2}{g\,h} \text{ for a rectangular rod} \\ &= 1{\cdot}712\ \frac{V^2}{R}\ \frac{l^2}{g\,d} \text{ for a round rod} \end{aligned}\right\} \quad . \quad (16)$$

If P is the pressure acting along the rod determined as above, the stress due to that pressure is

$$\left.\begin{aligned} f_t &= \frac{P}{b\,h} \text{ for rectangular rods} \\ &= \frac{4\,P}{\pi\,d^2} \text{ for round rods} \end{aligned}\right\} \quad . \quad . \quad (17)$$

And the total stress is $f_t + f_i$, which must not exceed the safe limit of stress for the material of the rod. So far as the bending stress is concerned, the breadth has no influence on the strength. It is for this reason that in locomotive coupling-rods the depth is made greater than the breadth.

It would render the rules for connecting rods too complicated, to introduce the bending stress in the ordinary formulæ for proportioning them. But it is desirable, when a rod has been proportioned for the load on the piston, with a

factor of safety which allows for other straining actions, to test whether that allowance is sufficient, by examining the stress when inertia is taken into account.

207. *Strength of connecting rods.*—Connecting rods are often proportioned in an empirical way, their diameter being made proportional to the piston diameter. Thus, cast-iron rods for stationary engines have a section at the centre equal to about 0·056 of the piston area; locomotive rods are $\frac{1}{6}$ to $\frac{1}{5}$ the piston diameter; marine engine rods are $\frac{1}{8}$ to $\frac{1}{6}$ of the piston diameter. Hence, a close agreement of theoretical rules with actual practice is not to be expected. For very long rods, and when the bending stress, due to inertia, is neglected, the strength is determined by Rule II. in Table VII. But most commonly the ratio of length to diameter is such that the rules in § 39 are applicable.

Let P be the greatest longitudinal thrust transmitted through the rod; l, the length of the rod between the centres of the end journals; I, the moment of inertia of the section of the rod at the centre; A, the area of that section; n, the factor of safety. Then for long rods,

$$n\,\text{P}=\pi^2\,\frac{\text{E I}}{l^2}$$

$$\left.\begin{aligned}\text{P}&=57{,}240{,}000\,\frac{\text{I}}{l^2}\text{ for wrought-iron}\\&=27{,}970{,}000\,\frac{\text{I}}{l^2}\text{ for cast-iron}\end{aligned}\right\}\quad .\qquad (18)$$

For a rod having a circular section of diameter d, or a rectangular section, the smaller dimension of which is h and the greater $b=c\,h$,

$$\left.\begin{aligned}d&=\alpha\sqrt{\{l\sqrt{\text{P}}\}}\\h&=\beta\sqrt{\left\{l\sqrt{\frac{\text{P}}{c}}\right\}}=\gamma\sqrt{\{l\sqrt{\text{P}}\}}\end{aligned}\right\}\quad .\qquad (19)$$

where $\alpha=0·02443$ for wrought-iron, and 0·0292 for cast-iron;

β=0·0214 for wrought, and 0·0256 for cast iron; and γ has the following values:—

$c=$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
$\gamma=$·0214	·0203	·0194	·0187	·0181 for wrought iron.	
=·0256	·0242	·0231	·0222	·0215 for cast iron.	

For ordinary rods it is better to use the equations in § 39. Then

$$\left.\begin{aligned} P &= \frac{8500\,A\,I}{\cdot 00009\,A\,l^2+1} \text{ for wrought iron} \\ &= \frac{2840\,A\,I}{\cdot 00027\,A\,l^2-1} \text{ for cast iron} \end{aligned}\right\} \quad . \quad (20)$$

These equations are in an inconvenient form for determining the diameter from the thrust. The following formulæ, which have been obtained by applying Poncelet's rules for approximation, are extremely simple, and give very approximately the same results.

For connecting rods of circular section and diameter d,

$$\left.\begin{aligned} d &= 0\cdot 01363\sqrt{\{l\sqrt{P}+0\cdot 79\,P\}} \text{ for wrought iron} \\ &= 0\cdot 03394\sqrt{\{l\sqrt{P}-0\cdot 033\,P\}} \text{ for cast iron} \end{aligned}\right\} \quad (21)$$

For connecting rods of rectangular section, the lesser dimension being h, and the greater $b=c\,h$,

$$\left.\begin{aligned} h &= 0\cdot 01194\sqrt{\left\{l\sqrt{\frac{P}{c}}+0\cdot 81\,\frac{P}{c}\right\}} \text{ for wrought iron} \\ &= 0\cdot 02974\sqrt{\left\{l\sqrt{\frac{P}{c}}-0\cdot 034\,\frac{P}{c}\right\}} \text{ for cast iron} \end{aligned}\right\} \quad (22)$$

If these equations are put in the form

$$h=C\sqrt{\{l\sqrt{P}\pm k\,P\}} \quad . \quad . \quad . \quad (23)$$

$c=$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	
$C=$	·01194	·01129	·01079	·01039	·01004	wrought iron.
$=$	·02974	·02813	·02687	·02586	·02501	cast iron.
$k=$	·81	·72	·66	·61	·57	wrought iron.
$=$	·034	·030	·028	·026	·024	cast iron.

Other forms of cross-section than the circular and rectangular are occasionally used. For cast-iron rods, a cross-shaped section with rounded internal corners has been adopted. The modulus of this section is nearly the same as that of a square, the angles of which coincide with the middle points of the arms of the cross. For locomotive coupling-rods of wrought iron, a double-T or I-shaped section has been used, formed by slotting out the sides of a solid bar. By thus lightening the rod, the bending stresses due to the vertical oscillation of the rod are diminished.

208. *Diameter of connecting rod calculated from initial steam pressure on piston.*—Let P_1 be the load on the piston, due to the initial steam pressure. Suppose the diameter of the rod calculated for the load $P = m P_1$, where m is a factor of safety, intended to allow for the neglected straining actions, due to acceleration parallel and perpendicular to the direction of the piston's motion. Then, for engines of a given class, working in similar conditions, m may be assumed constant; and for different classes of engines, its value may be determined by examining the diameters which have been used in practice. More simply still, if d or h is the diameter or depth calculated by the formulæ above, when P_1 is put for P, then $d\sqrt[4]{m}$ and $h\sqrt[4]{m}$ will be the dimensions which should actually be taken, to allow for the straining actions additional to P_1, which have been left out of the reckoning.

$m=$1·0	1·25	1·5	2·0	3·0	4·0	5·0	7·5	10
$\sqrt[4]{m}=$1·0	1·06	1·12	1·2	1·3	1·4	1·5	1·6	1·8

For locomotives it appears that $m=1·25$ to $1·5$. For

stationary and marine engines m is much greater, being often 5, and ranging from 4 to 10 in different cases.

Connecting-rod Ends.

209. *Proportions of steps.*—The ends of connecting rods are designed to receive crank pins or neck journals, and are

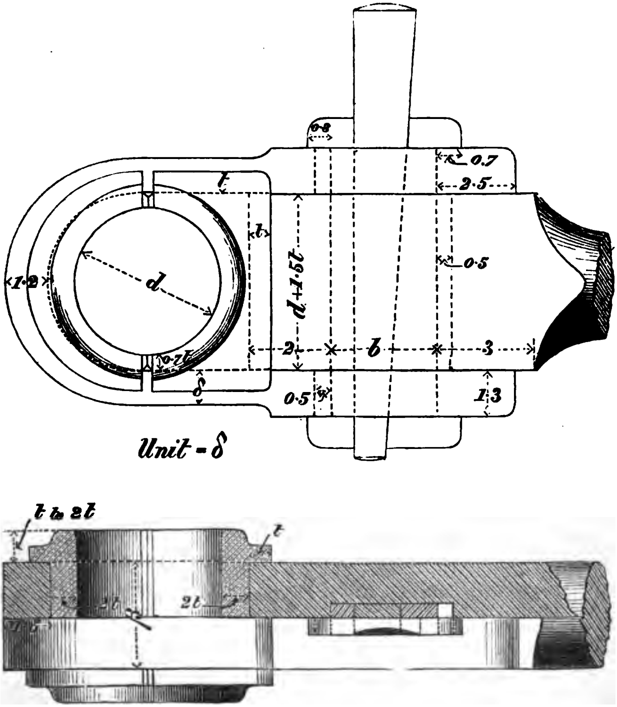


Fig. 190.

fitted with gun-metal steps similar to those used for pedestals. The unit for the proportional numbers relating to the steps in connecting rods is

$$t = 0{\cdot}08\, a + \tfrac{1}{8} \quad . \quad . \quad . \quad (24)$$

where d is the diameter of an ordinary crank pin supporting the thrust transmitted by the connecting rod. When the connecting rod is attached to a journal of greater size than is sufficient, for the thrust of that connecting rod only,

$$t = \cdot 007\sqrt{P_1} + \tfrac{1}{4} \quad \text{to} \quad \cdot 012\sqrt{P_1} + \tfrac{1}{4}\,. \qquad (24a)$$

The flanges of the steps are of very variable thickness, but very often the space between the flanges of the steps in which the connecting-rod end is placed is $\frac{7}{10}$ths of the length of the journal.

210. *Strap end.*—Fig. 190 shows a very common form of connecting-rod end, having a loose strap enclosing the steps. This strap is kept in place by gibs and cotter. It will be seen that when the cotter is tightened up to neutralise the wear of the brasses, the rod is shortened in length. The strap is of wrought iron or cast steel, and its total section (on two sides of the rod end) is $2\,\beta\,\delta$ sq. ins. Let P_1 be the initial steam pressure on the piston, and let as above $m\,P_1$ be the greatest load on the strap, due to all causes of straining action, m being a factor of safety. Then

$$\beta\,\delta = \frac{m\,P_1}{2\,f}\,. \qquad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$= \cdot 000055\, m\,P_1 \text{ for wrought iron.}$$

$$= \cdot 000037\, m\,P_1 \text{ for cast steel.}$$

The factor of safety m appears to be large in most cases, being 3 to 4 for locomotives and 8 to 10 for other engines. If a value is selected for β, then δ can be determined.

The total combined section of gibs and cotter is about $1\tfrac{1}{4}\,\beta\,\delta$ when they are of the same material as the strap. If b is their total width, and $n\,b$ their thickness, then

$$b = \sqrt{\left(1 \cdot 25\,\frac{\beta\,\delta}{n}\right)}$$

where n is usually $\frac{1}{4}$ to $\frac{1}{5}$. The inclination of the sides of the cotter is $\frac{1}{2}$ inch per foot on each side, but if a set screw

is used to lock the cotter, then the inclination may be 1 inch per foot. The proportions of the other parts may be obtained from the numbers given on the figure, the proportional unit being δ.

211. *Box end.*—Fig. 191 shows a connecting-rod end having no loose strap. The brass steps have a thickness $2\,t$ opposite the key, and $t_1 = 6\,t - \frac{1}{2}$ next the key. At the sides the thickness is reduced to t. The thickness and overlap of the flanges of the steps may be $\frac{1}{5}\,l - \frac{1}{8}$, so that the width of the box may be $\beta = \frac{3}{5}\,l + \frac{1}{4}$. The flanges of the steps are partially removed on one side to allow their insertion in place. The thickness δ of the sides of the box may have the same value as the thickness of strap in the last case. The mean breadth of the cotter is

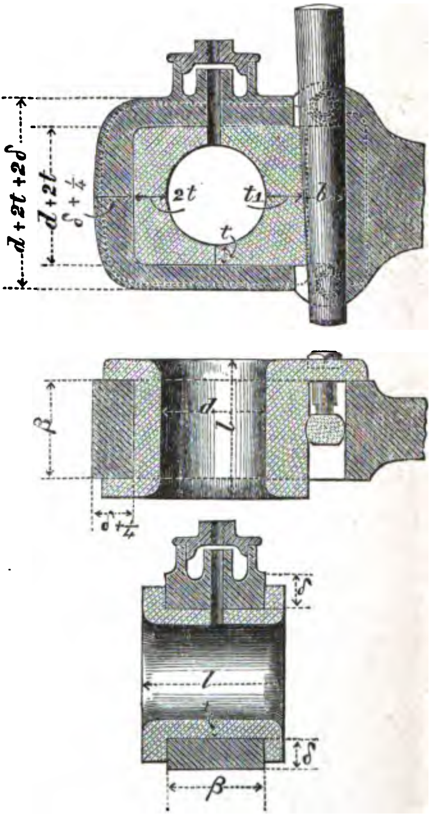


Fig. 191.

0·6 β, and its thickness 0·3 β, and it tapers 1 in 12 on each side. It is secured by two set screws; diameter of set screws

=cotter thickness. Unlike the last form, this connecting rod is lengthened when the cotter is tightened. But it may

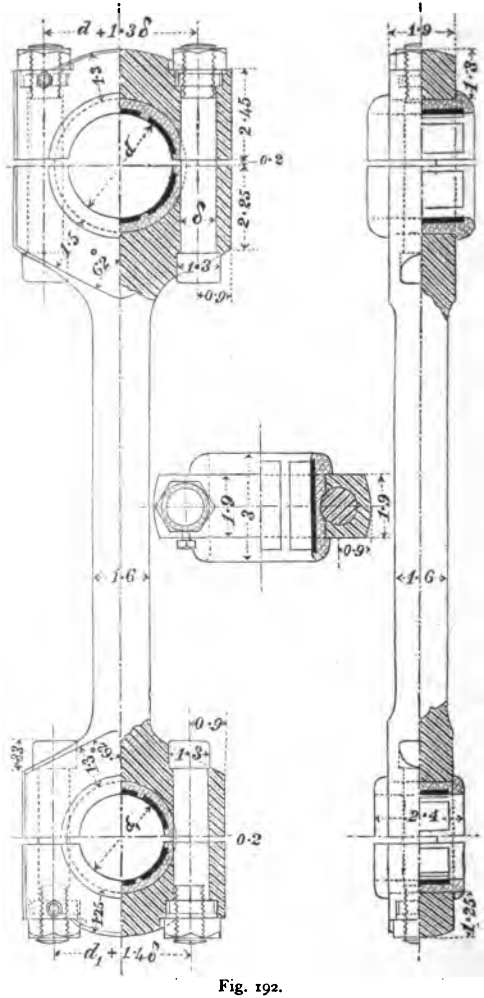


Fig. 192.

be arranged with the cotter on the other side of the brass steps, and then it is shortened by tightening the cotter. A coupling rod should have one end arranged in the former and one in the latter method. Then the length of the rod is not much altered by tightening the cotters. The proportional unit for this figure is δ.

212. *Marine engine connecting-rod end.*—Fig. 192 shows another form of connecting-rod end. This is of simple and massive form, and is often used in marine engines. The

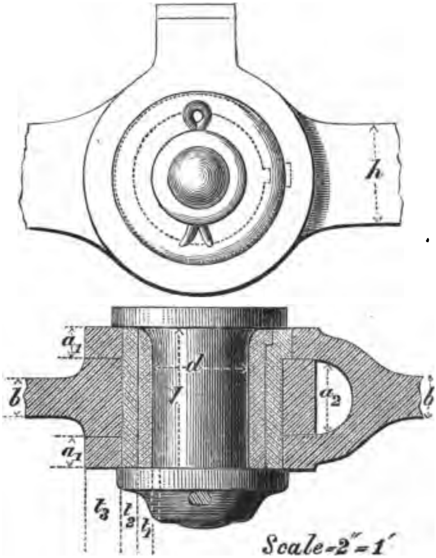


Fig. 193.

bolts here correspond to the strap in fig. 190. Hence, if δ is the diameter of these bolts,

$$\delta = \cdot 0084 \sqrt{(m\ P_1)} = k \sqrt{P_1} \quad . \quad . \quad . \ (26)$$

$m=$	4	5	$7\frac{1}{2}$	10
$k=$	·0177	·0198	·0243	·0280

The numbers on the figure are proportional to δ. The brasses are lined with white metal or Babbitt's metal cast in shallow recesses.

213. Fig. 193 shows a coupling-rod joint which may serve as an example of a journal bearing where there is not a great amount of motion and wear. This joint is intermediate in construction between a common knuckle-joint and a connecting-rod end. It has bushes to diminish friction and wear, but these are not divided, so that there is no adjustment after wear has taken place. The crank pin turns in a brass bush, which is protected by an outer steel bush. Both brass and steel bush are fixed in the forked rod end by small snugs, and the solid rod end turns on the steel bush The pin in a joint of this kind is often larger than is necessary for strength, because, by using a large pin with a small intensity of pressure between the rubbing surfaces, there is less danger of squeezing out the lubricant. The pin is of steel. The proportions may be

$$t_1 = t_2 = 0{\cdot}1\, d + \tfrac{1}{8}.$$

$$a_1 = 0{\cdot}3\, d.$$

$$a_2 = 0{\cdot}8\, d.$$

$$t_3 = \frac{b\,h}{4\,a_1}.$$

CROSS-HEADS.

214. Cross-head is the name given to the part which connects together the piston rod and connecting rod of a steam engine, and with which is also connected the guiding arrangement either of slide blocks or parallel motion bars. It consists essentially of a socket to which the piston rod is keyed, and a journal, or two journals, on which the connecting rod works. In the former case the connecting rod has a single end, in the latter it is forked. Generally there

are arrangements for attaching the slide blocks to the cross-head.

The connecting rod works with less velocity of rubbing on the cross-head than on the crank-pin journal. Hence the former is of less length than the latter. Cross-head pins are usually neck journals, with a length equal to their diameter only. Putting d and l for the diameter and length of the cross-head journal for a single-end connecting rod, we have from eq. 10, p. 120,

$$d=l=\sqrt{\frac{1\cdot28}{f}}\;\sqrt{P},$$

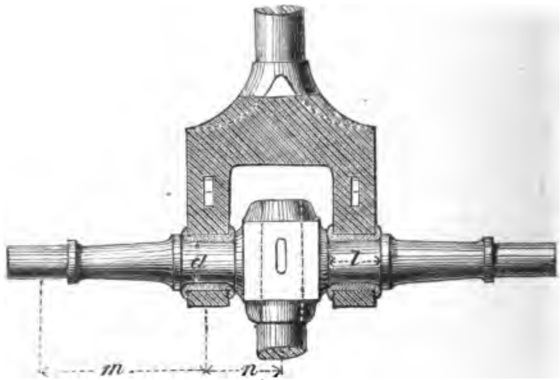
where P is the maximum thrust in the connecting rod. If for P is put the piston pressure only, then the value of d so obtained must be multiplied by a factor of safety to allow for the additional straining action which has been neglected. In locomotive cross-heads that factor may be $1\frac{1}{4}$ to $1\frac{1}{2}$, and in stationary and marine engines $1\frac{1}{2}$ to 2.

215. *Forms of cross-heads.*—Only cross-heads for engines with slides will here be considered. The form of the cross-head depends primarily on the arrangement of the slides. There may be—(1) four slide bars, two on each side of the cross-head; (2) two slide bars in the plane in which the connecting rod oscillates; (3) a slipper slide on one side only of the cross-head.

Fig. 195 shows a simple cross-head for an arrangement of four slide bars. The cross-head is of wrought iron, cottered to the piston rod, and having a forked end embracing the connecting rod. A pin passing through the cross-head forms a neck journal for the connecting rod, and at the same time two end journals on which the slide blocks are fixed. The slide blocks are simple cast-iron blocks. In large engines these blocks have brass faces on the rubbing surfaces. The pin must be fixed in the jaws of the cross-head by a small key, shown in the end view, which prevents the rotation of the pin. For the connecting rod

journal $d=l$. The unit for the proportions of the other parts of the cross-head is d.

Fig. 194.



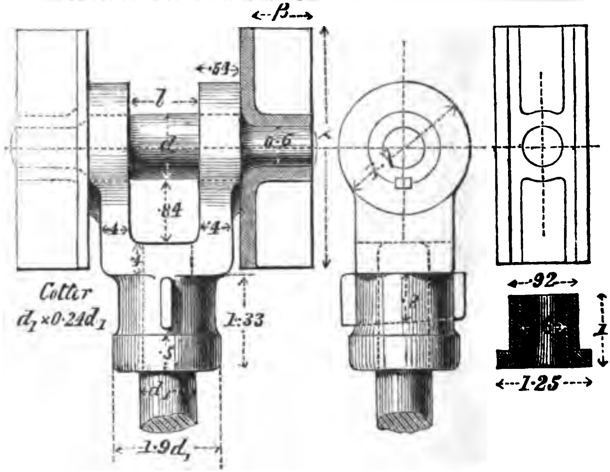


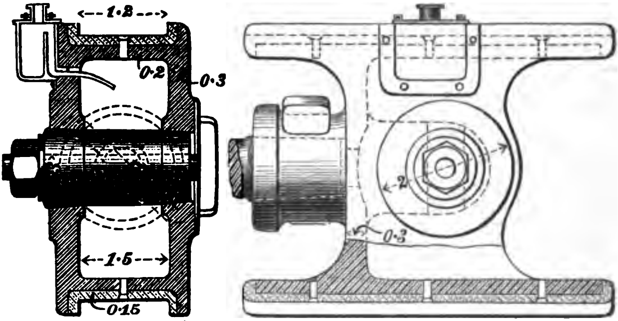
Fig. 195.

Fig. 194 shows the form of the cross-head pin when the cross-head has a single end and the connecting rod is

forked. Each connecting-rod end is designed as above described, but for half the total thrust in the rod.

Figs. 196, 197, show two forms of cross-head applicable

Fig. 196.



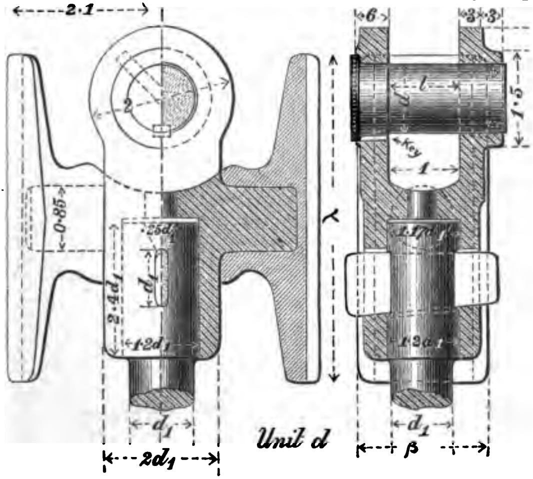


Fig. 197.

when there are two slide bars in the plane of oscillation of the connecting rods. The piston-rod socket is propor-

tioned to the piston-rod diameter, d_1. In both these examples the piston rod is enlarged at the cross-head end. This involves a split stuffing-box. The unit for the remaining parts is the cross-head pin diameter, *d*. In fig. 196 the cross-head is entirely of wrought iron, except the brass faces attached by set screws to the rubbing surfaces. The cross-head pin is kept in place by a T-headed bolt, which passes completely through it. The ends of the pin are tapered, and rotation of the pin is prevented by friction of the tapered parts.

In fig. 197 the cross-head of wrought iron and the slide blocks are separate, and of cast iron. The cross-head pin is kept in place by a split pin, and rotation is prevented by a small key inserted on one side.

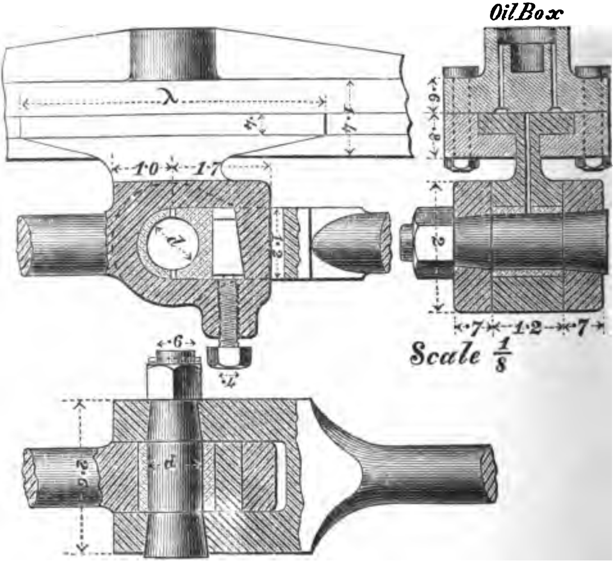


Fig. 198.

Fig. 198 shows a cross-head designed by Mr. Stroudley

for a slipper slide.[1] The cross-head is of wrought iron forged in one piece with the piston rod and slide block, and the connecting rod is forked at the end, and embraces the cross-head. The steps of the cross-head pin are of gun-metal, or of case-hardened wrought iron, and are tightened by a wedge and set screw. The cross-head pin, of case-hardened wrought iron, is fixed to the jaws of the connecting rod. In this case the slipper slide block is over the cross-head. In most cases, however, it is underneath it.

SLIDE BARS.

216. In most link-work arrangements it is necessary to guide the ends of some of the bars, so as to constrain them to move in straight lines. This can be done by an arrangement of links forming what is termed a parallel motion. Into the construction of parallel motions no elements enter which have not already been discussed. A parallel motion may be made to guide a given point with great accuracy, and with very little friction. On the other hand, it is a somewhat complicated arrangement, and if the links alter in length, in consequence of wear, it no longer properly answers its purpose. Hence parallel motions have been to a great extent superseded by a simpler arrangement of straight guiding surfaces termed slides. The only objection to these is that they involve more loss of work in friction than parallel motions. Slides are very commonly employed to guide the end of the piston rod at the point where the thrust is transmitted to the connecting rod.

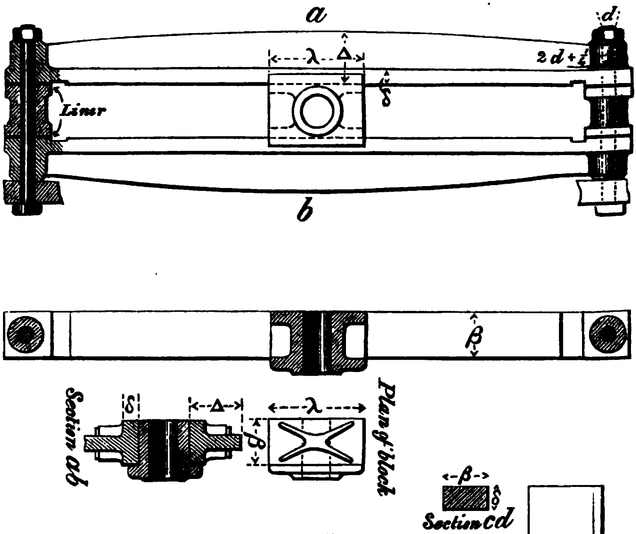
Fig. 199 shows two ordinary cast-iron slide bars with the slide block between them. The bars are of T section, and are spaced apart at the ends by distance pieces. Thin washers or liners are introduced between the distance pieces

[1] 'Engineering,' ix. p. 65.

and the bars, so that when the bars and slide block are worn, the bars can be brought closer together.

The bars are notched at the ends, and the slide block

Fig 199.



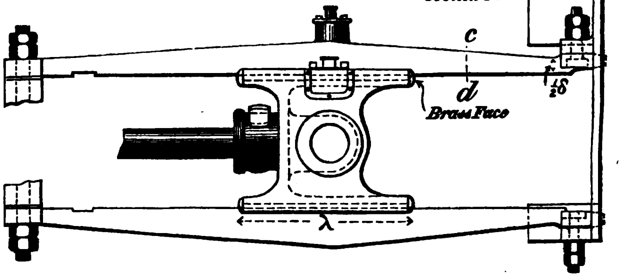


Fig. 200.

passes the edge of the notch at each stroke. This prevents the formation of a ridge at the end of the stroke, in consequence of the wear of the bar. Ample provision must be

made for lubricating the bars. The slide blocks may be of cast iron or of gun metal. They fit on journals at the end of the cross-head pin. The arrangement will be understood, if the cross-head and slide blocks in fig. 194 are compared with the slide block and slide bar in fig. 199.

Fig. 200 shows slide bars of wrought iron (sometimes case-hardened) or steel slide bars. In the arrangement here shown, the bars are above and below the cross-head. The bars are rectangular in section, and thickest at the centre, where the thrust is greatest. The cross-head here shown has brass faces. With this kind of arrangement, in large engines, provision is made to neutralise the wear of the bars by separating the surfaces of the slide blocks.

A slipper slide is sufficiently shown in the arrangement already described, fig. 198. The slide block is a T-shaped piece, forged in one with the cross-head, and this is guided in a groove formed by a flat slide bar and two L-shaped bars. Other forms of slide bar are used, the sliding surfaces being sometimes wedge-shaped and sometimes cylindrical.

217. *Wearing surfaces of Slide Blocks.*

Let P = total piston pressure in lbs.

Q = thrust of slide blocks on slide bars.

$a = \beta\,\lambda$ = area of slide block surface.

r = crank radius.

l = connecting rod length.

v = mean velocity of piston.

The pressure of the slide blocks on the slide bars is greatest when the crank and connecting rods are at right angles. Then

$$Q = P\frac{r}{l} \text{ nearly} \qquad . \quad . \quad . \ (27)$$

It is for this thrust that the strength of the bar must be calculated. Since, however, the obliquity of the connecting

rod is constantly varying, the mean pressure between the block and slide bar is

$$Q_m = P \frac{0{\cdot}785\, r}{\sqrt{(l^2 - 0{\cdot}617\, r^2)}} = k\, P \text{ very nearly} \quad . \quad (28)$$

$\frac{l}{r}=$	$3\frac{1}{2}$	4	5	6	7
$k=$	·2303	·2003	·1590	·1320	·1129

Since the second term in the bracket is in most cases small compared with the first,

$$Q_m = \frac{\pi}{4}\frac{r}{l} P \text{ nearly} \quad . \quad . \quad . \quad (29)$$

The work wasted in friction is

$$T = Q_m\, v = \frac{\pi}{4}\frac{r}{l} P\, v \text{ foot lbs. per second.}$$

The heat produced is

$$H = \frac{T}{J} = \frac{\pi}{3088}\frac{r}{l} P\, v \text{ units per second ;}$$

and supposing that h units are dissipated per second by conduction through the metal pieces from each unit of area of the slide block,

$$a = \frac{\pi}{3088\, h}\frac{r}{l} P\, v \quad . \quad . \quad . \quad (30)$$

Hence it would appear that the slide block surface should be proportional to the ratio $\frac{r}{l}$, to the mean velocity of the piston, and to the mean pressure on the piston, or for engines working in similar conditions, to the area of the piston.

Let ω=area of piston, a=area of slide block surface, β=width of slide bars, λ=length of slide block.

For locomotives, $a = \frac{1}{4}\,\omega$ on the average, a little more area being allowed in express engines, and a little less in goods'

engines. For ordinary land engines working at ·60 lbs. pressure, $a=\omega\frac{r}{l}$; and for large marine condensing engines, $a=\frac{1}{3}\omega\frac{r}{l}$ to $\frac{1}{2}\omega\frac{r}{l}$. If there are two slide blocks and two sets of slide bars, $2\beta\lambda=a$. When there is only one set of bars, $\beta\lambda=a$.

218. *Strength of the Slide Bar.*—Let m and n be the distances from the centre of the connecting rod eye to the points of support of the slide bar, when the crank and connecting rod are at right angles. Then the greatest bending moment on the slide bar, immediately under the connecting rod, is

$$\mathrm{P}\frac{r}{l}\cdot\frac{m\,n}{m+n} \qquad . \qquad . \qquad . \ (31)$$

Hence, if the section of the bar is rectangular, of breadth β and thickness δ,

$$\tfrac{1}{6}\beta\,\delta^2 f=\mathrm{P}\frac{r}{l}\cdot\frac{m\,n}{m+n},$$

$$\delta=\sqrt{\left\{\frac{6}{f}\cdot\frac{r}{l}\cdot\mathrm{P}\frac{m\,n}{(m+n)\,\beta}\right\}}$$

$$=k\sqrt{\left\{\mathrm{P}\frac{m\,n}{(m+n)\,\beta}\right\}} \qquad . \qquad . \qquad . \ (32)$$

The limiting stress should be taken at 6,000 lbs. for wrought iron or steel to allow for the straining actions due to reaction, and to secure stiffness; and at about 3,000 lbs. for cast iron. Hence,

$\frac{l}{r}=$	$3\frac{1}{2}$	4	5	6	
$k=$	·0169	·0158	·0141	·0129	for wrought iron,
$=$	·0239	·0224	·0200	·0183	for cast iron.

The T-shaped section for cast iron is more rigid, but not much stronger than if the feather were omitted.

When the slide bars are horizontal, the weight of the connecting rod, cross head, &c., rests on the lower bar. If, then, the engine runs only in one direction, it may be arranged so that the thrust, due to the pressure transmitted, acts on the upper bar. Then the weight and thrust partially neutralise each other, and friction and wear is diminished. If the engine runs in both directions, but more constantly forwards than backwards, the surface of the slide block, which receives the thrust when running forwards, is often greater than that which receives the thrust when running backwards. This is the case in fig. 198. When the engine runs forward, the thrust is upward. When running backward, the thrust is downward.

CHAPTER XIII.

PISTONS, STUFFING-BOXES, VALVES, AND COCKS.

219. A piston, or plunger, is a sliding piece which is either driven by fluid pressure or acts against fluid pressure as a resistance. Pistons and plungers are commonly circular in section, and are guided by cylindrical bearing surfaces, so as to reciprocate in a straight path. But other forms of piston are occasionally used.

A plunger is a single-acting piston—that is, a piston receiving the action of the fluid on one face only—and it is guided, not by the cylinder itself, but by a stuffing-box in the cylinder cover. The bearing surface of the plunger therefore requires to be longer than the stroke. The stuffing-box forms the only joint requiring attention to keep it staunch, and it is accessible without removing the plunger. A piston is equivalent to a short plunger entirely contained within the cylinder and guided by it. The force is transmitted through a piston rod of relatively small area. Hence the piston has two faces on which the fluid pressure can act, and it is usually double-acting. With a piston there are two joints requiring to be kept staunch, one within the cylinder, and one where the rod passes through the cylinder cover. A large hollow piston rod is termed a trunk. The pistons of pumps are often termed buckets.

220. *The volume swept through* by an ordinary piston is the product of the transverse section of the piston normal to the direction of the motion, and the length of its path. With an incompressible fluid, such as water, the volume swept through

is the volume of water lifted, in the case of a pump, or acting on the machine, in the case of a pressure engine or ram.

Work done on a Piston.—The work done on a piston by fluid pressure is the product of the volume swept through by the piston and the intensity of the fluid pressure. If the fluid pressure is variable, the mean intensity of the fluid pressure is to be taken. If the work is to be in foot lbs., the volume swept through may be in cub. ft. and the pressure in lbs. per sq. ft., or the volume swept through in units of 12 cub. inches and the pressure in lbs. per sq. inch.

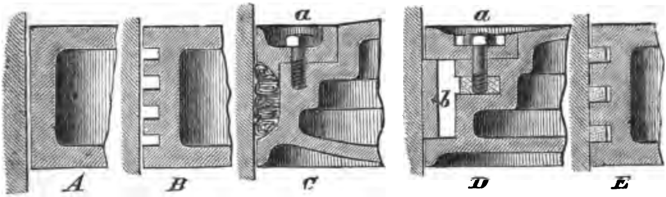
Velocity of Piston.—Ordinarily a piston drives or is driven by a crank, rotating with nearly uniform velocity. Then the motion of the piston is approximate harmonic motion, varying from rest at each end of the stroke to a maximum near mid stroke. The acceleration is greatest at the beginning of the stroke, vanishes near mid stroke, and changes sign and increases to another maximum at the end of the stroke.

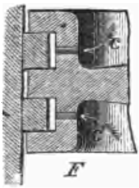
Influence of the weight of the Piston on the Crank Pin pressure.—When a piston is driven by a constant pressure, it is generally desirable to make the piston as light as possible, because the inertia of the piston causes the crank pin effort to be more irregular than it otherwise would be. When, however, the pressure on the piston varies, the inertia of the piston may be used to diminish the variation of the crank pin effort, and to make the total pressure on the crank pin nearly uniform.

221. *Construction of Pistons.*—Various arrangements have been adopted to diminish the leakage between the piston and the sides of the cylinder in which it slides. The piston may be simply turned to fit the cylinder accurately, (A, fig. 201); but, however good the fit at first, the wear of the cylinder and piston will gradually enlarge the clearance between them, and the leakage will steadily increase. If a series of recesses are cut round the piston circumference,

(B, fig. 201), the leakage for any given width of clearance space is less, because the fluid loses its energy of motion, at each sudden enlargement of the section of the annular space between the piston and cylinder, through which it is escaping. Pistons of this kind are used for quick running pumps, where a small leakage is not very prejudicial. Leakage may be prevented by placing, in a recess in the piston, a packing of gasket or tallowed rope (C, fig. 201). This soft and elastic packing is compressed against the

Fig. 201.





cylinder by a junk ring, shown at *a*, which is fixed by studs or set screws. As the gasket wears away, it can be replaced, and thus the permanent staunchness of the piston is secured. Pistons of this kind are not now much used for steam cylinders, though they are still employed for air pumps and cold water pumps. The objection to them is that the repacking of the piston is troublesome, and the friction of the piston is considerable. To diminish the wearing away of the gasket, a face ring or spring ring, shown at *b*, was introduced (D, fig. 201), made of cast iron and divided on one side, to allow it to expand to the cylinder diameter as it wore away. The space behind the spring ring was at first filled by gasket packing, but it was found better to substitute steel springs for gasket, which retain their elasticity much longer, and press the spring ring outwards quite as effectively. In small pistons, the elasticity

of the spring ring itself is sufficient to maintain contact with the cylinder. Various arrangements of this kind have been used. Sometimes the spring ring is a cast iron ring, of uniform or varying thickness. Ramsbottom's rings are shown at E, fig. 201. These consist of a continuous spiral steel ring of about 3 coils, or of 3 separate steel rings, each split on one side. The rings are initially of larger size than the cylinder, and, when compressed within it, press outwards with sufficient force to prevent leakage. Cast iron answers best perhaps for small spring rings. It retains its elasticity till the ring is half worn through. Cast-iron rings are sometimes of uniform thickness, but very often they are one-half thicker at the middle of the ring than at the ends where the ring is split. At F, fig. 201, is shown an arrangement for admitting the steam pressure in the cylinder to the back of the rings. In principle this is a good arrangement, but, in this form, it does not succeed very well, and is not very often adopted. Bramah's cup leather is a perfectly successful application of the same principle. For pumps and blowing cylinders, wood blocks have been used to replace the spring ring.

222. *Strength of Pistons.*—Pistons are of a complicated form, and it is not easy to determine their strength theoretically. If the piston were a simple metal disc, supported at the centre and uniformly loaded, the greatest stress would be

$$f = k \frac{d^2}{t^2} p \quad . \quad . \quad . \quad (1)$$

where d is the diameter of the piston, t its thickness, p the greatest difference of the pressures on the two sides, estimated per unit of area, and k is a constant. Putting $f =$ 8,000 for wrought iron and 3,000 for cast iron, we get

$$\left.\begin{aligned} t &= \cdot 0051\, d\sqrt{p} \text{ for wrought iron} \\ &= \cdot 0083\, d\sqrt{p} \text{ for cast iron} \end{aligned}\right\} \quad . \quad . \quad (2)$$

These values of t will be taken as empirical units for the proportions of pistons. Since, however, the form of pistons varies greatly, and also the conditions under which they work, the draughtsman should not depend solely on the following proportional figures, but should deduce the

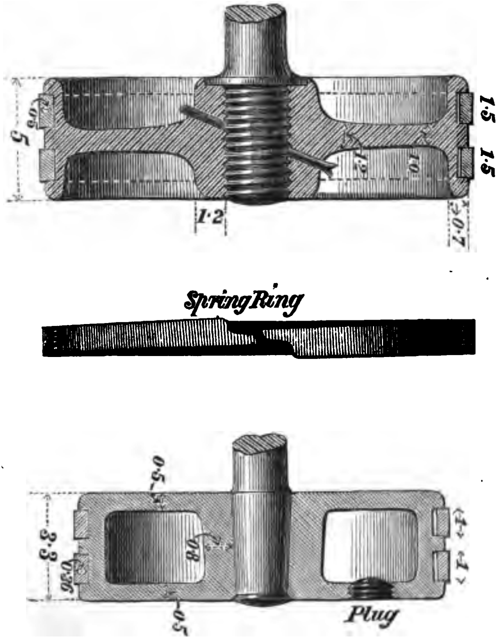


Fig. 202.

proportional figures for himself, from good examples of pistons of a similar kind to the one he is designing.

223. *Locomotive Pistons.*—Fig. 202 shows two forms of piston used in locomotives. One is constructed chiefly of

chiefly of wrought iron. Wrought iron is

e engineers on account of its toughness and

strength. But cast iron is much cheaper and answers well. The spring rings in both cases are of cast-iron and require no springs or packing. These rings are of uniform section, about $1\frac{1}{2}$ inches wide by $\frac{1}{2}$ inch thick, in pistons of average size. The split is made with a half lap, to prevent leakage at that point. The rings are sprung into the recesses in the piston, and should be so placed that the splits in the two rings are on opposite sides of the piston. This equalises the wear of the cylinder. A small screw is sometimes used to prevent the rings turning round in the grooves. The piston rod is screwed into the wrought-iron piston and fixed by a split pin. In the case of the cast-iron piston, the rod is slightly coned at the end, and, when in place, is riveted over. The holes filled by screw plugs are intended for the removal of the sand core after casting.

Fig. 203 shows another locomotive piston. In this three spiral springs are placed behind the spring ring, and assist the elasticity of the latter in keeping the piston tight. A brass tongue-piece prevents leakage at the joint in the spring ring. The piston rod has a strong taper to enable it to be easily removed, and it is secured by a screwed end and nut. The spiral springs are so placed as to prevent the body of the piston bearing on the bottom side of a horizontal cylinder.

224. *Stationary Engine Pistons.*—Fig. 204 shows one form of stationary engine piston. It is made of cast iron, with a junk ring to confine the metallic packing. The packing consists of three cast-iron rings of the sectional form shown. The outer rings are turned $\frac{1}{16}$ inch larger than the cylinder diameter, and are split. The inner ring may or may not be split. By screwing down the junk ring the two outer rings are forced outwards, as they slide down the conical surfaces of the inner ring, and thus any desired amount of pressure can be obtained between the piston and cylinder. The inner ring has sometimes been made in the form of a spiral spring. It then presses the outer rings

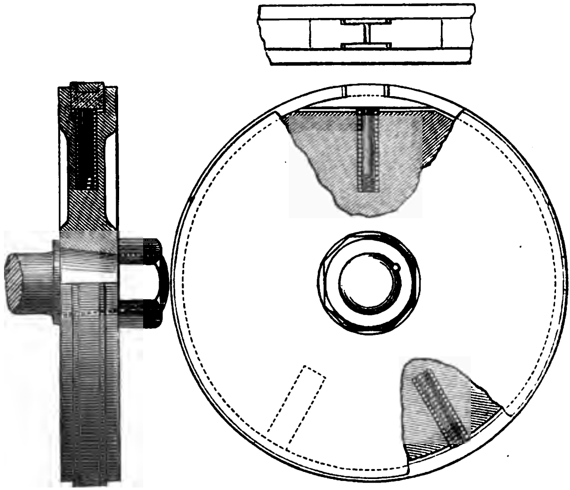


Fig. 203.

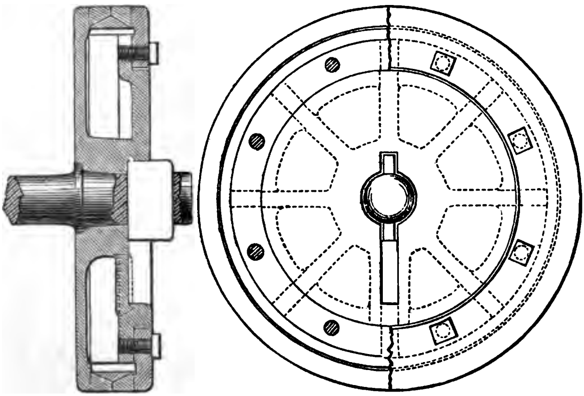


Fig. 204.

both apart and outwards. In this piston the rod is tapered at the end and fixed by a cotter.

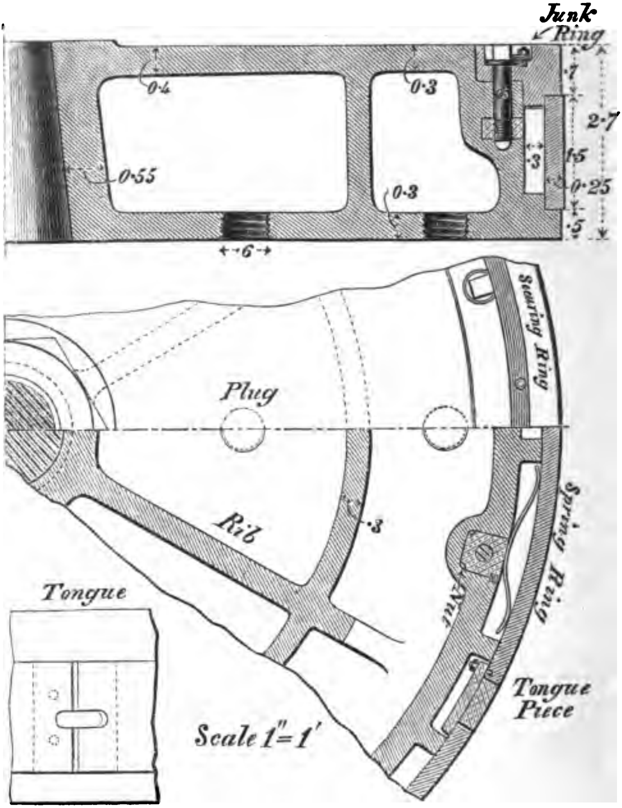


Fig. 205.

225. *Marine Engine Pistons.*—Marine engine pistons are often of very large size, and are usually of cast iron, of a box-shape and stiffened by numerous ribs. Fig. 205 shows a piston of this kind. The spring ring of cast iron is of

uniform thickness. Leakage at the split is prevented by a brass tongue-piece, fixed to one end of the spring ring by screws. The spring ring is pressed outwards by numerous plate-springs, placed in recesses cast in the rim of the piston. The spring ring and springs are kept in place by a junk ring. This last is attached to the piston by bolts, which have brass nuts placed in recesses behind the plate-springs. To prevent these bolts slacking back, in consequence of the vibration of the piston, various locking arrangements are used. In the piston shown (a type used by Messrs. Humphreys and Tennant), a securing ring bears against the heads of all the junk ring bolts. This ring is attached to the piston by studs, the nuts being fixed by split pins.

In this piston the rod is tapered and the piston is secured to it by a large nut on the upper side of the piston. When such a piston works in a horizontal cylinder, a block placed between the spring ring and piston body keeps the latter from bearing on the cylinder.

226. *Hydraulic Pistons.*—Fig. 206 shows a combined piston and plunger with a cup-leather arrangement for preventing leakage. The fluid pressure acting on the flexible leather cups, aided by their own elasticity, makes an exceedingly staunch joint, whatever the pressure may be. The cup-leathers are so arranged that one acts when the piston moves in one direction, the other when the piston moves in the reverse direction.

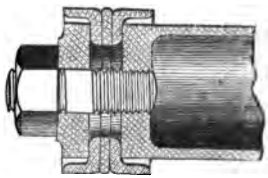


Fig. 206.

Stuffing-boxes.

227. Stuffing-boxes are used to prevent leakage of steam or water, at the points where moving parts pass

through the sides of vessels containing fluids. Thus a stuffing box is used where the piston rod of an engine passes through the cylinder cover, or where a rotating shaft passes through a centrifugal pump case. In ordinary stuffing-boxes, soft packing is used to prevent leakage, but various forms of metallic packing have also been employed.

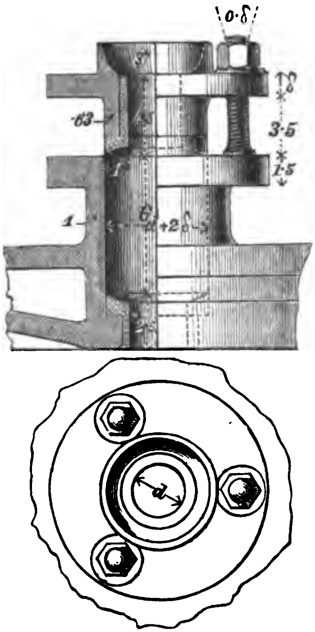


Fig. 207.

In fig. 207 is shown a stuffing-box for a vertical rod, and in fig. 208 a stuffing-box for a horizontal rod. In both cases the stuffing-box is cast on the cylinder cover. The stuffing-box is larger than the rod which traverses it, by the space necessary for the soft packing. At the bottom of the box there is a brass bush, which, being softer than the rod, preserves the latter from injury. When the bush has worn oval, it is easily replaced by a new one. To keep the packing in place and to compress it sufficiently to prevent leakage, a loose piece termed a gland is used. This is entirely of brass (fig. 208) or bushed with brass (fig. 207), and often has an oil-box formed in it (fig. 208). The gland is forced down on the soft packing by two or more bolts or studs.

228. *Proportions of Stuffing-box and Gland.*—Let d be

the diameter of the rod or shaft traversing the stuffing-box. The diameter δ of the gland bolts may be $=\frac{1}{4}d+\frac{1}{4}$, if there are two; and $=\frac{1}{5}d+\frac{1}{4}$, if there are three. Then δ will be taken as the unit for the proportions of the box. The thickness of packing in the box is very variable. In ordinary

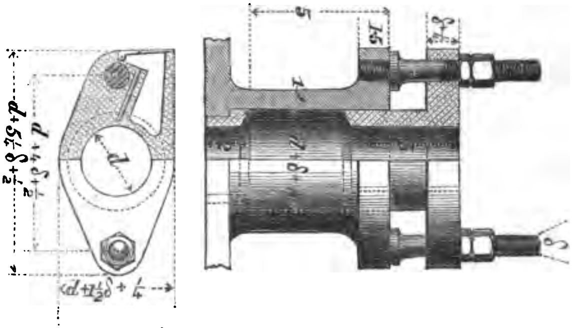


Fig. 208.

stuffing-boxes, not of very great diameter, the packing thickness varies from $\frac{1}{2}\delta$ to δ. But in large stuffing-boxes for trunks or hollow piston rods a less thickness is employed. The length of the box is also variable. The greater the length of box, the less frequently will it be necessary to renew the packing. On the other hand, the space available for the stuffing-box is sometimes restricted. From 5δ to 8δ is an average length. The thickness of the stuffing-box flange may be $1\frac{1}{4}\delta$ to $1\frac{1}{2}\delta$, and the thickness of the gland flange may be δ for cast iron, or $1\frac{1}{4}\delta$ for brass. If an oil-box is cast in the flange, the thickness is somewhat greater. The length of gland may be $\frac{3}{4}$ to $\frac{4}{5}$ the stuffing-box length. The thickness of the stuffing-box should not be less than $\frac{1}{8}\sqrt{d}$ or less than $\frac{3}{8}\delta$. The length of the brass bush may be about $2\frac{1}{2}\delta$, but when the stuffing-box serves to guide the rod, as in oscillating engines, a much greater length of bush is used.

229. *Cup-leather Packing.*—When great hydraulic pressure is to be resisted, a peculiar packing is used, invented by Bramah, and already alluded to. The leakage is prevented by a flexible leather ring, kept in contact with the piston rod or ram on one side and the cylinder on the other by the fluid pressure. The leather is moulded into an annular shape in plan and to a U-shape in section (fig. 209.) This ring is placed in a recess in the cylinder or in a stuffing-box, in such a way that the fluid has free access to its interior; the fluid pressure acting within the ring presses it against the plunger and the sides of the recess, and this, aided by the elasticity of the ring, makes a perfectly tight joint. When the cup-leather is large, it is provided with an internal brass ring, and a thin guard ring of brass on the edge most liable to wear. A packing of hemp or cotton is used as a bed for the leather.

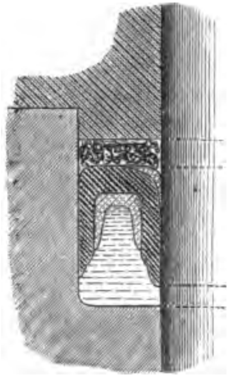


Fig. 209.

VALVES.

230. In all machinery put in motion by the action of a fluid (water or steam) or employed in pumping fluids, valves are required to regulate the admission and discharge of the fluid. With reference to the mode in which the motion of valves is obtained, they may be divided into four classes: (1) Valves opened and closed by hand; (2) Valves opened and closed by independent mechanism; (3) Valves opened and closed by mechanism so connected with the machine as to render the times of opening and closing synchronous with the motions of the machine; (4) Valves opened and closed by the action of the fluid.

The mode in which a valve is actuated does not affect its construction, and a more convenient division, for the present purpose, depends on the way in which the valve moves relatively to its seat. Thus we have: (1) flap or butterfly valves, which rotate in opening; (2) lift valves, or puppet valves, which rise perpendicularly to the seat; (3) sliding valves, which open by moving parallel to the seat.

231. *Flap or Butterfly Valves.*—Fig. 210 shows the simplest form of flap-valve, formed of a leather disc, strength-

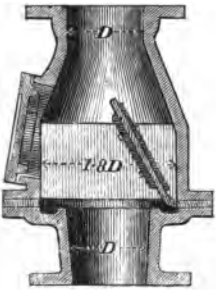


Fig. 210.

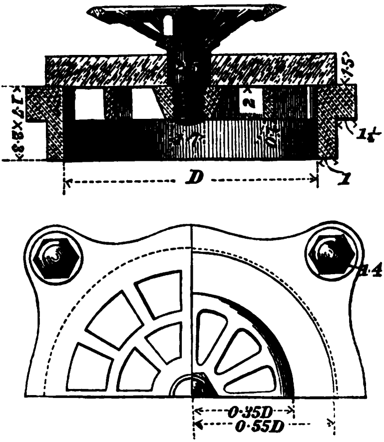


Fig. 211.

ened and stiffened by two plates of iron, of brass or of lead, which at the same time give weight enough to the valve to close rapidly, when the pressure beneath it ceases. A butterfly valve consists of two flap valves placed hinge to hinge, or sometimes edge to edge. In the latter position the direction of motion of the fluid is less interfered with. The flap is sometimes entirely of brass, as in the case of air-pump foot valves, where leather would not be sufficiently

durable. Valves of this kind are most commonly lifted by the fluid.

Indian-rubber Disc-valve.—A form of valve very extensively used for condensers and pumps, consists of a circular disc of Indian-rubber, secured by a bolt at the centre, and resting on a brass grid which forms the seating. The Indian-rubber being flexible lifts easily from the grating, when any fluid pressure is applied beneath it, and closes again readily, and without violent shock, when the reflux begins. To prevent the Indian-rubber rising too high, a perforated guard plate is placed over the valve. Figs. 211 and 212 show two of these valves. In one the valve seat is attached to the cast-iron casing of the condenser by bolts, and the Indian-rubber and guard plate are attached to it by a stud. In the other the seating, Indian-rubber and guard plate are all secured by the same central bolt, which bears against a cross-bar on the other side of the casing to that on which the valve is placed. In each of these figures the valve and guard-plate are removed from one half of the plan, in order to show the grating on which the valve rests. The Indian-rubber should not be too thin; $\frac{3}{4}$ inch to $\frac{7}{8}$ inch thickness is

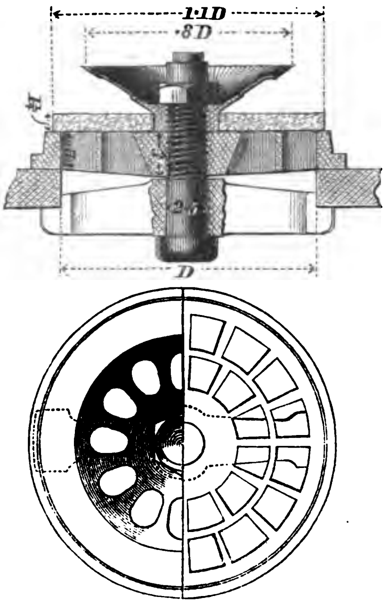


Fig. 212.

sufficient, and the apertures of the grating should be so small that there is no great flexure of the Indian-rubber when resting on the grating. Moderate-sized valves of this description answer better than large ones. It is more satisfactory to use several valves of 7 inches to 9 inches diameter than to use a single large one.

The throttle valve used on many engines, which consists of a circular or square metal disc, capable of turning about a shaft passing through it in the direction of a diameter, is a kind of double-flap valve. The disc is placed in a pipe, and closes the passage-way when placed across the pipe, whilst it offers little resistance when parallel to the axis of the pipe. This valve is an imperfect equilibrium valve, the pressure on one half partly balancing the pressure on the other, so that the force required to move the valve is only equal to the difference of these two pressures. The equilibrium is exact, however, only while the valve is shut or so long as there is no sensible current passing it. If a rapid current is established, the pressure on that half of the valve which first deflects the current is greater than on the other half, thus tending to close the valve.

232. *Lift or Puppet-valves.*—These are very various in form, the simplest being a circular disc, usually of metal, with a flat or bevelled edge, which fits a circular metal seating. These valves are generally placed with the axis of the valve vertical, so that their weight tends to keep them closed, but they may be otherwise placed if springs or rods are used to close them. In order that the valve may open an annular space, equal in area to the circular passage under the valve, the lift must be equal to one-fourth of the valve's diameter. This lift is sometimes objectionable, because, in closing, the valve acquires a considerable velocity and there is a shock at the moment of closing. This is not only prejudicial from the vibration it occasions, but it leads to the destruction of the faces of the valve and seat. Hence with simple lift-valves the lift is often restricted to a less

amount than would otherwise be desirable, and then the resistance to the passage of the fluid is increased.

Fig. 213 shows a conical disc valve and casing. The valve is guided in rising and falling by three feathers which fit the cylindrical part of the seating, and are shown in the plan of the valve. The lift of the valve is limited by a projection on the cover of the casing. The fitting part, or face of the valve, should be narrow, as it is then more easy to make it tight. It must, however, present area enough to resist deformation by the hammering action of the valve. The inclination of the face of the valve is usually 45°

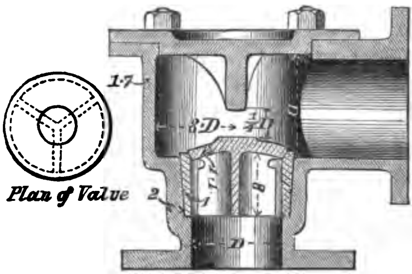


Fig 213.

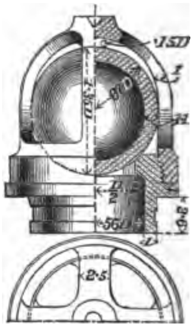


Fig. 214.

with the axis of the valve. Conical disc valves may either be actuated by the fluid pressure or by hand. In the latter case they are opened and closed by a screwed rod.

Fig. 214 shows a ball valve, which acts in precisely the same way as a disc valve, except that as the surface of the ball is accurately spherical, it fits the seating in every position. The only guide required is, therefore, an open cage, which limits the play of the valve. Such valves are often used for small fast-running pumps. To lighten the ball it is often made hollow.

The proportions of valves depend partly on the diameter. Thus the area of the waterway must be constant, and

the linear dimensions of the casing are proportional to the valve's diameter. But the thicknesses are in most cases excessive as regards strength, especially in small valves, and do not increase in the same proportion as the diameter. For these the empirical proportional unit

$$t = \frac{1}{5}\sqrt{\text{D}}$$

will be adopted, where D is the diameter of the valve.

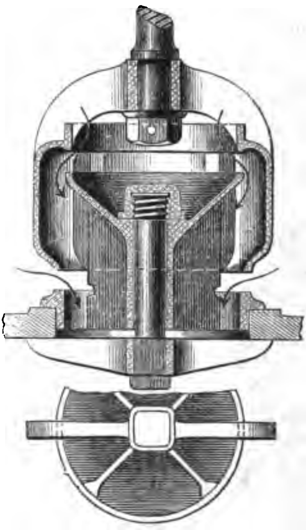


Fig. 215.

233. *Double Beat or Cornish Valve.* — The objection to a great lift in metal valves has already been mentioned. In the double beat valve, two valve faces are obtained in the same valve, and two annular spaces are opened when the valve lifts. For a given area of opening, the lift is only about one-half that of a simple lift valve of the same diameter. Fig. 215 shows a Cornish valve for a pumping engine. This valve is raised and lowered by a cam acting on an arrangement of levers. The lower seating is carried directly by the steam-chest. The upper seating is carried by four feathers or radiating plates cast with the lower seating. The valve itself is ring-shaped. Since the two valve faces are nearly of the same diameter, another subsidiary advantage is gained in this form of valve. The valve is pressed down on its seat, partly by its weight, partly by the steam pressure acting on one side of it. If the valve were a simple disc-valve, the steam pressure would act on an area $\frac{\pi}{4}$ D² where D is the diameter of the

valve. As the valve is annular, however, the steam presses only on the area $\frac{\pi}{4}(D_1^2 - D_2^2)$ where D_1 and D_2 are the diameters of the two faces.

234. *Sliding Valves.*—Sliding valves are more commonly used than any others for stop-valves, which are opened and closed by hand; they may be divided into two classes: (1) those with plane faces and seats; (2) those with cylindrical or slightly conical faces and seats. The former class includes engine slide-valves and the sluices, often of very large size, which are used as stop valves on water mains. The latter class includes the hand-worked valves commonly known as cocks.

235. *Engine Slide Valves.*—Of the various valves used to effect the distribution of steam to steam-engine cylinders, the slide-valve is by far the most frequently adopted. The full treatment of the action of the slide-valve is beyond the scope of this treatise, and a short description of the most simple form of slide-valve is all that can be attempted. In its simplest form the slide-valve consists of a dish-shaped rectangular piece, the face of which is very accurately planed and scraped to a true surface. It slides upon a seating formed in the steam-chest, which is also an accurate plane, and in which are formed the passages through which the steam passes, termed the *ports*. The slide-valve is pressed down on the seating by the steam pressure on its back, which is greater than the pressure on its face, because part of the lower surface of the valve communicates with the atmosphere or the condenser. The section of the valve is D-shaped, as shown in figure 216, and it has two flat faces, in section, which, when the valve is in its middle position, cover the two passages leading to the two ends of the cylinder, or steam-ports; at the same time the hollow part of the valve covers the middle passage through which the steam is discharged, and which is termed the exhaust-port. When the valve moves in either direction from its middle position, it uncovers one steam-port and allows

steam to pass to one end of the cylinder, whilst, at the same time, the hollow part of the valve passes over the other steam-port and puts it in communication with the exhaust-port. The reciprocating motion of the slide-valve, which opens the ports alternately, is effected by an eccentric, which may be regarded as a very short crank, keyed on the same shaft as the engine crank. It is obvious that the travel of the valve each way from its mid-position is equal to the radius of the eccentric, unless modifying arrangements are interposed.

In the earliest slide-valves the width of the faces of the valve was sensibly equal to the width of the steam-ports. Then, the moment the valve passed its mid-position, it began

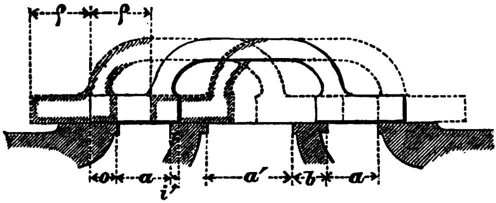


Fig. 216.

to open one port to steam and the other to exhaust. Apart from a circumstance to be mentioned presently, the steam-piston must be at the end of its stroke at the moment steam begins to be admitted on one side and exhausted from the other. It follows that with this form of valve, the valve is at mid-stroke when the steam piston is at the end of its stroke. Consequently the eccentric must be at right angles to the crank.

It was discovered, however, that with this arrangement the steam entered and left the cylinder with difficulty at the beginning of each stroke, in consequence of the very gradual opening of the slide-valve. To afford a wider opening to the steam, it was found necessary that the valve should be already a little open at the beginning of a stroke. To secure this it is only necessary to fix the eccentric a

little more than 90° in advance of the crank. The width of the port open at the beginning of a stroke is termed the *lead* of the valve.

Next it was found desirable to make the faces of the valve wider than the steam-ports, so that when the valve is placed in its middle position, the valve faces overlap the edges of the ports. The width of the overlap on the steam edge of the valve is termed the outside lap, and the width of the overlap on the exhaust edge of the valve is termed the inside lap. The former is generally greater than the latter. The valve must be open to the extent fixed for the lead, when the piston is at the end of the stroke, and the crank connected with it on one of its dead centres. To attain that position, the valve must have moved from its middle position a distance equal to outside lap + lead. The eccentric must therefore have passed its middle position by the angle necessary to move the valve through that distance. It will therefore be in advance of a position at right angles to the crank, by an angle termed the angular advance of the eccentric. Two important advantages are obtained in a valve thus arranged: (1) The steam is cut off from the cylinder before the end of the stroke, and the slide-valve acts as an expansion valve. (2) By making the outside lap greater than the inside lap, the lead to exhaust is made greater than the lead to admission, by the amount of the difference of the laps. Now, the greater freedom of exhaust thus secured reduces the back pressure on the piston in a very important degree. There is a limit to the amount of lap which can be used with an ordinary slide-valve. To whatever extent the opening of the exhaust is made earlier, to the same extent the closing of the exhaust is made earlier also. If the exhaust is closed too soon, steam is retained in the cylinder and compressed, as the piston returns, into the small clearance space at the end of the cylinder. This action is termed cushioning. A moderate amount of cushioning is useful, but excessive cushioning would be prejudicial. To prevent this the

outside lap is usually not greater than is sufficient to close the steam-port at $\frac{5}{8}$ths of the stroke. When more expansion is wanted, a double slide-valve or some other arrangement is used.

Fig. 216 shows a section of a slide valve and of the steam ports, taken parallel to the direction in which the valve moves. The dotted lines show the positions of the valve at the ends of its stroke, in either position completely uncovering one port to exhaust, and partially uncovering the other to steam.

236. *Area of Steam-ports.*—The area of the steam-ports must be so arranged that the mean velocity of the steam does not exceed 80 to 100 feet per second. Let Ω be the piston area and ω the area of each steam-port, then the ratio of port area to piston area is as follows :—

Piston velocity in feet per minute	$\frac{\omega}{\Omega}$
200	·04
300	·055
400	·07
600	·10

For locomotives which run at a high, but variable speed, $\omega = \cdot 07\Omega$.

Let l be the length and a the width of each steam-port. Then $al = \omega = \left(\frac{\omega}{\Omega}\right)\Omega$. The proportions of the port are variable ; the length l may be from 0·5 to 0·8 of the cylinder diameter, and the ratio $\frac{l}{a}$ from 4 to 9. Let D be the cylinder diameter, and let $l = x$ D, $a = y$ D. Then suitable values of x and y are given in the following table :—

Piston speed	x	y	x	y	x	y
200	·4	·078	·5	·062	·6	·052
300	·5	·086	·6	·072	·7	·062
400	·6	·091	·7	·078	·8	·068
600	·7	·112	·8	·098	·9	·087

The whole width of the steam-port is opened to exhaust, but often only 0·6 to 0·9 of the width to steam.

237. *Proportions of the Slide-valve*:—

Let a=width of steam port.

na=greatest width opened to steam.

o=outside lap.

i=inside lap.

e=lead ; e'=inside lead.

b=width of bar between steam and exhaust ports.

a'=width of exhaust port.

ρ=half travel of valve, or radius of eccentric.

r=radius of crank or half stroke of engine.

ϵ=ratio of eccentric radius to length of eccentric rod.

ξ=distance valve has travelled from its mid-position when the crank has moved through an angle ϕ from the dead point.

l=distance piston has travelled from beginning of stroke, at the same moment.

θ=angle of advance of eccentric, so that the eccentric is $90°+\theta$ in advance of the crank.

The width b of the bars is fixed empirically. In small engines it may be$=\frac{a}{2}+\frac{1}{4}$, and in large engines it is determined almost entirely with reference to convenience of casting, and should be at least equal to the cylinder thickness. The inside lap, i, is generally small, and may be from $\cdot075a$ to $\cdot1a$. The outside lap, o, is determined by the point at which steam is to be cut off. Very commonly o is about $0.25a$. Then, if the valve and eccentric are directly connected,

$$\rho=n\,a+o=a+i \quad . \quad . \quad . \quad (3)$$

If these equations are satisfied o and i are not independent when a and $n\,a$ are fixed.

The lead e may be from $\frac{1}{16}\rho$ in slow to $\frac{1}{8}\rho$ in fast engines.

The angular advance of the eccentric, θ, is determined by the equation—

$$\text{Sin } \theta = \frac{o+e}{\rho} \text{ nearly} \quad . \quad . \quad . \quad (4)$$

This determines θ, and then the angle between the crank and eccentric radius is $90° + \theta$. The following table gives some values of θ :—

$\frac{e}{\rho}=$	When $\frac{o}{\rho}=$ ·1	·15	·2	·25	·3	·35
$\frac{1}{16}$	9° 21′	12° 16′	15° 13′	18° 12′	21° 15′	24° 22′
$\frac{3}{32}$	11 10	14 6	17 5	20 6	23 11	26 21
$\frac{1}{8}$	13 1	15 58	18 58	22 2	25 10	28 22

The equation is not quite exact, because of the obliquity of the eccentric rod, but the error is not great.

The inside lead $= e' = \rho \sin \theta - i$. The inside and outside lead and lap are connected by the equation, $o + e = i + e'$.

The width a' of the exhaust port must be equal to or greater than $2\rho - b$.

The width of the hollow under the valve (measured parallel to the direction of the valves' motion) $= 2(a + b - \rho) + a'$.

Travel of valve and corresponding crank angle when the influence of the obliquity of the eccentric rod is neglected.—Let a line through the two dead centres of the crankpin circle be termed the line of stroke. Generally this line is also parallel to the axis of the cylinder. If the obliquity of the connecting rod is neglected, the valve is in its mid position when the eccentric radius is at right angles to the line of stroke. Let that position be termed, for shortness, the mid position of the eccentric. As the eccentric moves through an angle α from its mid position, the valve travels a distance

$$\xi = \rho \sin \alpha \quad . \quad . \quad . \quad . \quad (5)$$

which will be + or − according as a lies between 0° and 180°, or between 180° and 360°, a being measured in the direction of motion of the crank. Since the eccentric is $90° + \theta$ in advance of the crank,

$$a = \phi + \theta,$$

where ϕ is the angle through which the crank has moved from its position at the beginning of the stroke. Hence

$$\xi = \rho \sin(\phi + \theta) \quad . \quad . \quad . \quad (6)$$

The opening of the port to steam is

$$w = \xi - o = \rho \sin(\phi + \theta) - o,$$

and the opening of the port to exhaust is

$$w' = -\xi - i = -\rho \sin(\phi + \theta) - i.$$

When admission begins and when steam is cut off, $w = O$; and when exhaust or compression begins, $w' = O$. Inserting these values, we obtain four values of the crank angle for each edge of the value and for one revolution of the engine.

For			$(\phi + \theta)$ lies between
Admission	$w = o$	$\sin(\phi_1 + \theta) = \frac{o}{\rho}$	0° and 90°
Cut off		$\sin(\phi_2 + \theta) = \frac{o}{\rho}$	90° and 180°
Release	$w' = o$	$\sin(\phi_3 + \theta) = -\frac{i}{\rho}$	180° and 270°
Compression		$\sin(\phi_4 + \theta) = -\frac{i}{\rho}$	270° and 360°

From these equations the values of $\phi + \theta$, and therefore of ϕ, can be obtained. The angles are connected by the relations $\phi_2 = 180° - \phi_1$; $\phi_4 = 180° - \phi_3$.

The following form of the same equations is sometimes more convenient.[1]

Since $\quad \text{Sin } \theta = \dfrac{o + e}{\rho}, \; o = \rho \sin \theta - e.$

[1] Resal, 'Mécanique Générale,' vol. iv., p. 288.

Hence,

$$w = \xi - o = e + \rho \left\{ \sin (\phi + \theta) - \sin \theta \right\}.$$

For admission and cut off, $w = O$, and we get

$$\text{Sin } (\varphi + \theta) = \sin \theta - \frac{e}{\rho};$$

For admission, $\phi_1 = 2\pi - \dfrac{e}{\rho \cos \theta}$;

For cut off, $\phi_2 = \pi - 2\theta + \dfrac{e}{\rho \cos \theta}$.

The angles are in circular measure in these equations, and can be reduced to degrees by multiplying by $\dfrac{180}{\pi}$ or by 57·3.

Similarly, since $i + e' = o + e$,

$$i = \rho \sin \theta - e'$$

$$w' = -\xi - i = e' - \rho \left\{ \sin (\phi + \theta) + \sin \theta \right\}.$$

For release and compression, $w' = O$; then

$$\text{Sin } (\phi + \theta) = -\sin \theta + \frac{e'}{\rho};$$

For release, $\phi_3 = \pi - \dfrac{e'}{\rho \cos \theta}$;

For compression, $\phi_4 = 2\pi - 2\theta + \dfrac{e'}{\rho \cos \theta}$.

238. *Position of piston for given crank angles when the obliquity of the connecting rod is neglected.*—If l is the distance the piston has travelled from the beginning of its stroke, when the crank has revolved through the angle ϕ, measured from the dead point, then if the obliquity of the connecting rod is neglected

$$l = r\,(1 - \cos \varphi) \qquad . \quad . \quad . \quad (7)$$

where $\cos \phi$ is negative, if ϕ lies between 90° and 270°. By inserting the values of φ, obtained above, we obtain approximately the position of the piston for admission, cut off,

release, and expansion. As, however, the obliquity of the connecting rod sensibly affects the position of the piston, it is better to set off the positions of the crank corresponding to the above values of ϕ on a diagram drawn to scale, and then by laying off the connecting rod length, the position of the piston is found exactly.

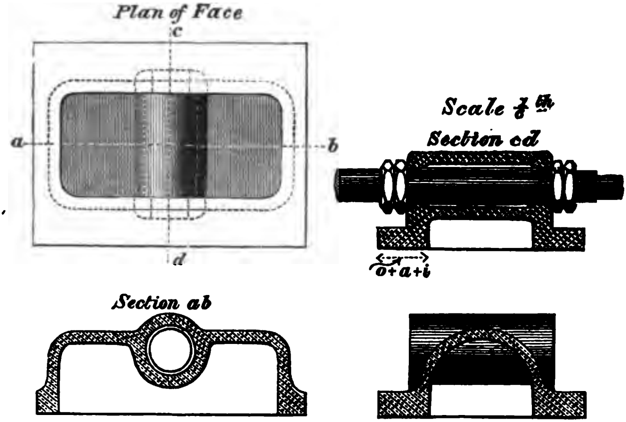


Fig. 217.

Crank angles corresponding to given ratios of expansion.—Let l_2 be the travel of the piston corresponding to the crank angle ϕ_2. Then $\frac{l_2}{2r}$ is the ratio of expansion. The following table gives the relation between these quantities, when the obliquity of the connecting rod is neglected :—

$\frac{l_2}{2r}$ = 0·4	0·45	0·5	0·55	0·6	0·65
ϕ_2 = $78\frac{1}{2}°$	83	90	96	$101\frac{1}{2}$	$107\frac{1}{2}$
$\frac{l_2}{2r}$ = 0·7	0·75	0·8	0·85	0·9	0·95
ϕ_2 = $113\frac{1}{2}°$	120	127	$134\frac{1}{2}$	143	154

The action of the slide-valve is most conveniently examined

by the aid of a geometrical construction due to Prof. Zeuner.[1]

Fig. 217 shows an ordinary locomotive slide-valve.

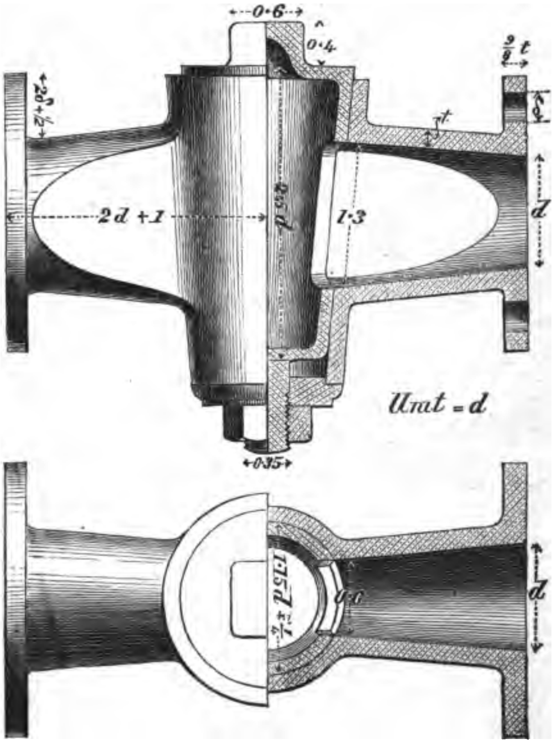


Fig. 218.

Cocks.

239. The term cock is sometimes used for any valve opened or closed by hand, but it is more properly restricted to valves which are nearly cylindrical, and which rotate in

[1] Treatise on Valve Gears.

seatings of the same figure. In ordinary cocks the seating is a hollow, slightly conical casing, and the valve, which is termed a plug, fits accurately in the seating. The passage-way for the fluid is formed through the plug. By rotating the plug in one direction its apertures are made to coincide with the entrance and discharge orifices of the casing. The cock is then open. By rotating it in the other direction the holes in the plug are brought over blank parts of the casing and the cock is closed. The slight taper given to the plug enables it to be accurately fitted, by turning and grinding, to its seating, and it can, from time to time, be refitted. Each time it is refitted the plug sinks a little lower in the casing. If the plug were cylindrical this refitting would be impossible. The objection to the use of cocks in many cases, especially for pipes of large size, is that a good deal of power is required to move them, and this is partly due to the conical form, which increases the friction.

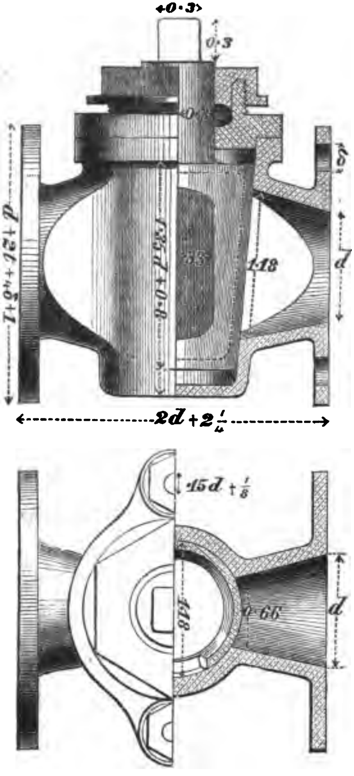


Fig. 219.

The simplest cocks have a solid plug, which is kept in place by a screwed end. When the cock is small, the casing has a screwed socket on

one side and a screwed end on the other, for the attachment of the cock to the pipes with which it is connected. But in larger cocks, the inflow and outflow orifices are provided with flanges.

For small brass cocks, with socket and spigot ends, the following proportions may be adopted:—

Diameter of waterway of cock$=d$
Diameter of plug at centre$=1\cdot15d+\frac{1}{4}$
Height of hole in plug$=1\cdot3d$
Width of hole in plug$=0\cdot6d$
Total length of tapered part of plug$=2\cdot5d$ to $3d$
Side of square for handle$=0\cdot7d$
Height of square for handle$=0\cdot4d$
Thickness of metal$=0\cdot2d+\frac{1}{16}$
Diameter of plug screw$=0\cdot35d$
Diameter of screwed end$=d+\frac{5}{16}$
Internal diameter of socket end$=d+\frac{3}{16}$
Total length$=3\cdot3d$
Taper of plug$=1$ in 12 to 1 in 9 on each side.

For cocks with flanged ends, like that shown in fig. 218, the proportions are the same. When the cock is not very small the thickness is best obtained from the rule—

$$t=\tfrac{1}{8}\sqrt{d}+\tfrac{3}{16} \text{ for cast iron.}$$

$$=\tfrac{1}{12}\sqrt{d}+\tfrac{3}{16} \text{ for brass.}$$

Some proportions are marked on the figure.

240. Large cocks connected with boilers, and in situations where failure would be dangerous, are best made with closed ends, as shown in fig. 219. The proportions of cocks of this description are a little different.

Diameter of waterway$=d$
Thickness of plug (brass)$=0\cdot12\sqrt{d}+\frac{1}{8}$
„ (cast-iron)$=0\cdot18\sqrt{d}+\frac{1}{4}$
Thickness of shell (brass)$=0\cdot18\sqrt{d}+\frac{1}{8}$
„ (cast-iron)$=0.25\sqrt{d}+\frac{1}{4}$

The shell may be reduced to the same thickness as the plug in parts which do not require to be turned.

Diameter of plug at centre$=1{\cdot}18d$

Size of openings in plug$=1{\cdot}18d\times 0{\cdot}66d$

Overlap of plug at top and bottom$=0{\cdot}08d+0{\cdot}4$

Depth of stuffing-box$=\frac{1}{8}d+\frac{1}{2}$

Depth of gland$=\frac{1}{20}d+\frac{1}{4}$

Diameter of studs in cover, $\frac{1}{8}d+\frac{1}{8}$

Taper of plug$=1$ in 12 on each side.

Some other proportions are marked on the figure.

ADDENDUM.

The following are the sizes of bolts and nuts according to the Whitworth Standard, as revised some two years since. The exact sizes are given in decimals, and the nearest approximate sizes in sixty-fourths of an inch.

Diameter of bolts	Width of nuts across flats		Height of boltheads	
$\frac{1}{8}$	·338	$\frac{21}{64}$ *f*	·1093	$\frac{7}{64}$
$\frac{3}{16}$	·448	$\frac{29}{64}$ *b*	·1640	$\frac{5}{32}$
$\frac{1}{4}$	·525	$\frac{33}{64}$ *f*	·2187	$\frac{7}{32}$
$\frac{5}{16}$	·6014	$\frac{19}{32}$ *f*	·2734	$\frac{17}{64}$
$\frac{3}{8}$	·7094	$\frac{45}{64}$ *f*	·3281	$\frac{21}{64}$
$\frac{7}{16}$	·8204	$\frac{53}{64}$ *b*	·3828	$\frac{3}{8}$ *f*
$\frac{1}{2}$	·9191	$\frac{29}{32}$ *b*	·4375	$\frac{7}{16}$
$\frac{9}{16}$	1·011	$1\frac{1}{64}$ *b*	·4921	$\frac{31}{64}$ *f*
$\frac{5}{8}$	1·101	$1\frac{3}{32}$ *f*	·5468	$\frac{35}{64}$
$\frac{11}{16}$	1·2011	$1\frac{13}{64}$ *b*	·6015	$\frac{19}{32}$ *f*
$\frac{3}{4}$	1·3012	$1\frac{19}{64}$ *f*	·6562	$\frac{21}{32}$
$\frac{13}{16}$	1·39	$1\frac{25}{64}$ *b*	·7109	$\frac{45}{64}$ *f*
$\frac{7}{8}$	1·4788	$1\frac{31}{64}$ *b*	·7656	$\frac{49}{64}$
$\frac{15}{16}$	1·5745	$1\frac{37}{64}$ *b*	·8203	$\frac{13}{16}$ *f*
1	1·6701	$1\frac{43}{64}$ *b*	·875	$\frac{7}{8}$
$1\frac{1}{8}$	1·8605	$1\frac{55}{64}$ *f*	·9843	$\frac{63}{64}$
$1\frac{1}{4}$	2·0483	$2\frac{3}{64}$ *f*	1·0937	$1\frac{3}{32}$
$1\frac{3}{8}$	2·2146	$2\frac{7}{32}$ *b*	1·2031	$1\frac{13}{64}$
$1\frac{1}{2}$	2·4134	$2\frac{13}{32}$ *f*	1·3125	$1\frac{5}{16}$
$1\frac{5}{8}$	2·5763	$2\frac{37}{64}$ *b*	1·4128	$1\frac{27}{64}$
$1\frac{3}{4}$	2·7578	$2\frac{3}{4}$ *f*	1·5312	$1\frac{17}{32}$
$1\frac{7}{8}$	3·0183	$3\frac{1}{16}$ *f*	1·6406	$1\frac{41}{64}$
2	3·1491	$3\frac{5}{32}$ *b*	1·75	$1\frac{3}{4}$
$2\frac{1}{8}$	3·337	$3\frac{11}{32}$ *b*	1·8523	$1\frac{55}{64}$
$2\frac{1}{4}$	3·546	$3\frac{35}{64}$ *b*	1 9687	$1\frac{31}{32}$
$2\frac{3}{8}$	3·75	$3\frac{3}{4}$	2·0781	$2\frac{5}{64}$
$2\frac{1}{2}$	3·894	$3\frac{57}{64}$ *f*	2·1875	$2\frac{3}{16}$
$2\frac{5}{8}$	4·049	$4\frac{3}{64}$ *f*	2·2968	$2\frac{19}{64}$
$2\frac{3}{4}$	4·181	$4\frac{3}{16}$ *b*	2·4062	$2\frac{13}{32}$
$2\frac{7}{8}$	4·3456	$4\frac{11}{32}$ *f*	2·5156	$2\frac{33}{64}$
3	4·531	$4\frac{17}{32}$ *b*	2·625	$2\frac{5}{8}$

The thickness of the nuts is in every case the same as the diameter of the bolts : *f* = full, *b* = bare.

INDEX.

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